

This homework is due on Wednesday, February 13, 2019, at 11:59PM.
Self-grades are due on Saturday, February 16, 2019, at 11:59PM.

1. Complex Numbers

A common way to visualize complex numbers is to use the complex plane. Recall that a complex number z is often represented in Cartesian form.

$$z = x + jy \text{ with } \operatorname{Re}\{z\} = x \text{ and } \operatorname{Im}\{z\} = y$$

See Figure 1 for a visualization of z in the complex plane.

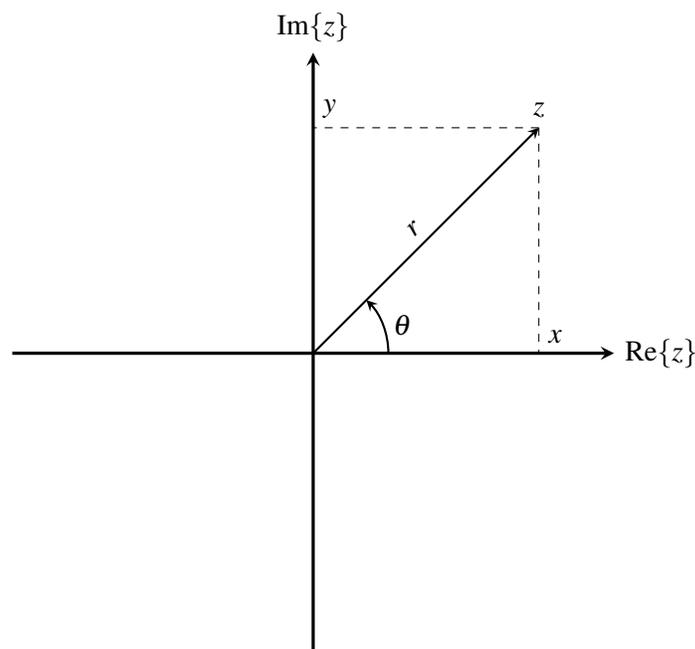


Figure 1: Complex Plane

In this question, we will derive the polar form of a complex number and use this form to make some interesting conclusions.

- Calculate the length of z in terms of x and y as shown in Figure 1.** This is the magnitude of a complex number and is denoted by $|z|$ or r . *Hint.* Use the Pythagorean theorem.
- Represent x , the real part of z , and y , the imaginary part of z , in terms of r and θ .**
- Substitute for x and y in z .** Use Euler's identity¹ $e^{j\theta} = \cos \theta + j \sin \theta$ to **conclude that,**

$$z = re^{j\theta}$$

¹also known as de Moivre's Theorem.

(d) In the complex plane, **draw out all the complex numbers such that $|z| = 1$. What are the z values where the figure intersects the real axis and the imaginary axis?**

(e) If $z = re^{j\theta}$, **prove that $\bar{z} = re^{-j\theta}$** . Recall that the complex conjugate of a complex number $z = x + jy$ is $\bar{z} = x - jy$.

(f) **Show that,**

$$r^2 = z\bar{z}$$

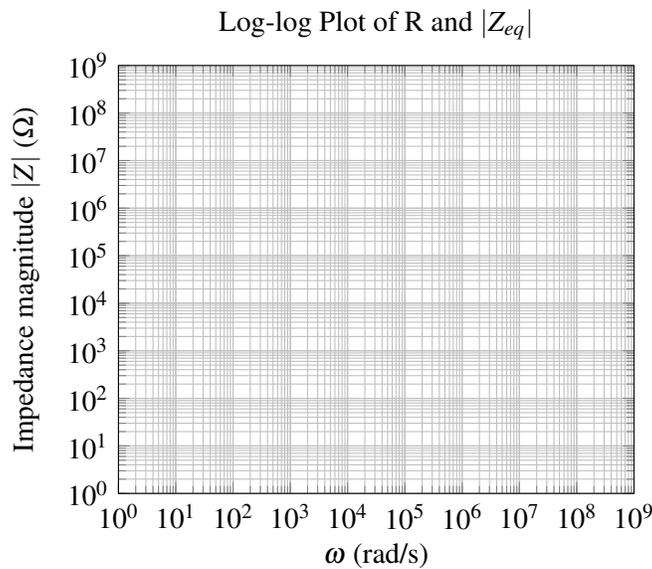
(g) Intuitively **argue that**

$$\sum_{k=0}^2 e^{j\frac{2\pi}{3}k} = 0$$

Do so by drawing out the different values of the sum in the complex plane and making an argument based on the vector sum.

2. Component impedances

Here is a blank log-log plot.



On this log-log plot, trace the magnitudes of the impedances ($|Z|$) of the following components:

(a) Capacitors $C_1 = 1\text{nF}$, $C_2 = 1\text{pF}$, $C_3 = 1\mu\text{F}$

(b) Inductors $L_1 = 1\text{H}$, $L_2 = 1\text{mH}$, $L_3 = 1\mu\text{H}$

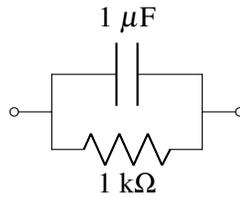
(c) Resistors $R_1 = 1\Omega$, $R_2 = 1\text{k}\Omega$, $R_3 = 1\text{M}\Omega$

3. Approximating an equivalent impedance

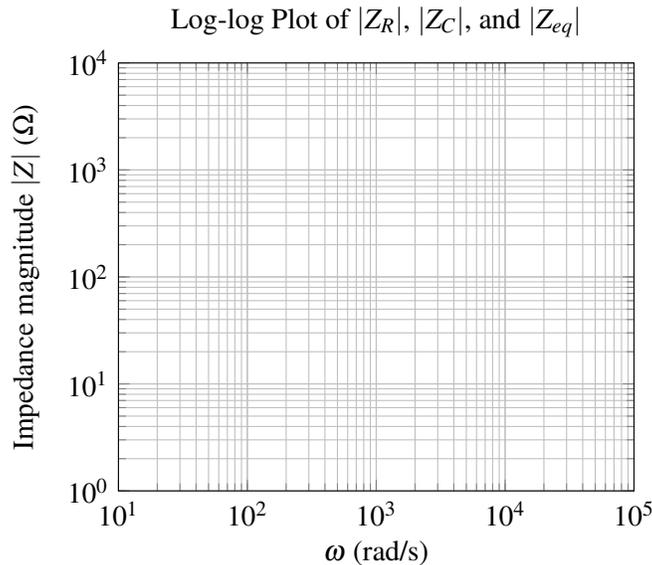
In class, you learned about the some useful approximations for series and parallel impedances. Suppose you have two components, which can be any of selection of resistors, capacitors, and inductors. Say component 1 has complex impedance Z_1 and component 2 has complex impedance Z_2 .

If you place these two components in a series, you can approximate the equivalent impedance magnitude as $|Z_{eq}| \approx \max(|Z_1|, |Z_2|)$. If you place them in parallel, then you can make the approximation that $|Z_{eq}| \approx \min(|Z_1|, |Z_2|)$. As long as the two components aren't a capacitor and an inductor (any other combination is okay), then the approximation will be off by no more than a factor of two.

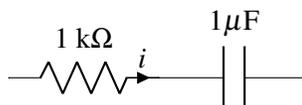
We will use this approximation method to analyze the impedance of the following circuit across different frequencies of $v(t)$:



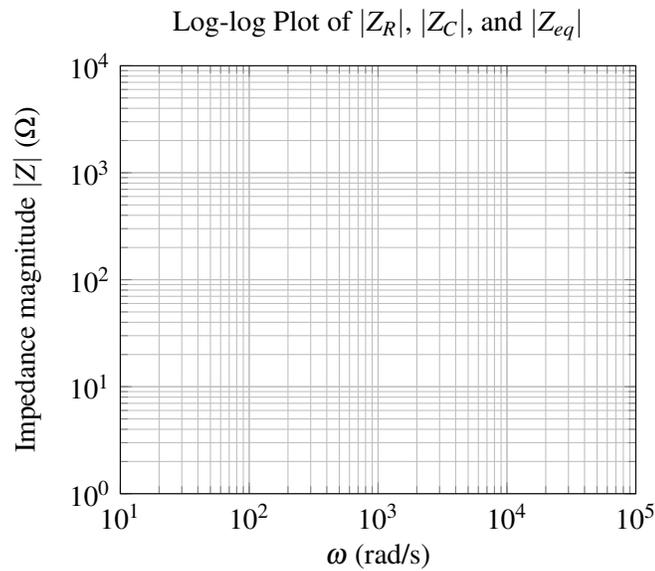
Here is a blank log-log plot, in which you can draw the impedance of this circuit by following the steps below.



- What are the impedances of the capacitor and resistor, Z_C and Z_R ? What are the magnitudes of these impedances, $|Z_C|$ and $|Z_R|$? Draw the impedance of the capacitor, $|Z_C|$, and the impedance of the resistor, $|Z_R|$, on the plot above using dotted lines.
- Using the approximation method for parallel components, draw an approximation of the equivalent impedance $|Z_{eq}|$ of the circuit with a solid line.
- Under this approximation, compute $|Z_{eq}|$ if ω takes the following values: $2 \cdot 10^5 \text{ rad/s}$, $5 \cdot 10^3 \text{ rad/s}$, 10^6 rad/s , $3 \cdot 10^{-4} \text{ rad/s}$.
- Now plot $|Z_{eq}|$ for this series circuit



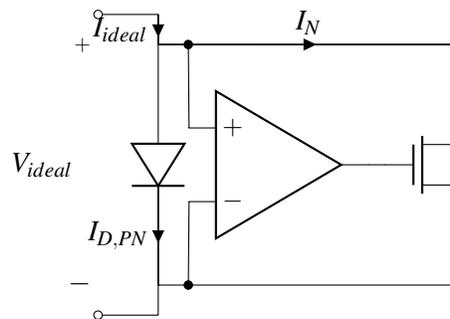
on the following blank log-log plot.



- (e) Again, under this approximation, **compute** $|Z_{eq}|$ **if** ω **takes the following values:** $2 \cdot 10^5 \text{ rad/s}$, $5 \cdot 10^3 \text{ rad/s}$, 10^6 rad/s , $3 \cdot 10^{-4} \text{ rad/s}$.

4. Ideal Diode

Here's an example of an ideal diode circuit. In this circuit, the op-amp is working like a comparator (which you have seen in EE16A), so the "golden rules" don't apply. Rather, when the differential input voltage on the op-amp is positive the output goes to a logic 1 value, turning the transistor on, and when the differential input is negative the output goes to a logic 0 value, turning the transistor off. The diode in the figure is a real PN junction diode as discussed in class. For a typical PN junction diode, the I/V relationship is given by $I_{D,PN} = I_S(e^{V_{D,PN}/V_{th}} - 1)$ where at room temperature $V_{th} = 25 \text{ mV}$. The "saturation current" I_S is a function of the geometry of the junction, the doping, and other things. Assume that this PN diode has $I_S = 1 \text{ pA}$.



- (a) Assuming that the transistor is a switch with no "on-resistance", **sketch the I/V curve of the overall circuit, I_{ideal} vs. V_{ideal} .**
 Now assume that the transistor has an "on-resistance" of $R_{on} = 1 \Omega$.
- (b) **Draw the circuit equivalent to this model in the cases where the "switch" of the NMOS is on and where it is off.**

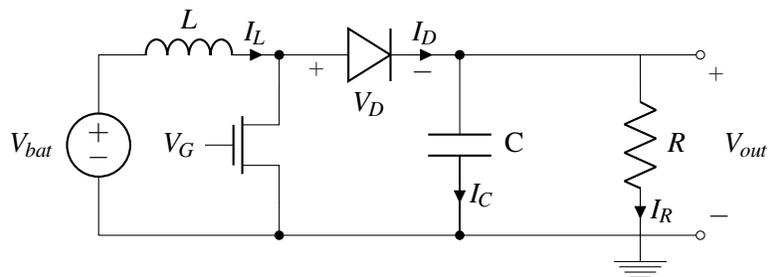
- (c) Sketch the I/V curves of both the diode, $I_{D,PN}$ and the transistor, I_N , for V_{ideal} from -50 mV to +50 mV applied to this circuit.
- (d) Estimate the applied voltage $V_{eq} > 0$ at which the diode current and the transistor current are equal. Sketch both the diode current and the transistor current vs. voltage for voltages within 50 mV of V_{eq} .
- (e) Sketch the I/V curve of the overall circuit for an applied voltage from -1 V to +1 V, with a current axis from -1 to 1 A.

5. Time-Domain: Boost Converter

Remember the boost converter circuit from Lecture (Thursday 01/31), which is shown below. The boost converter is a simple version of a circuit that is very common in battery-operated devices. While a single alkaline battery has a voltage that varies from 1.6 V when it is fresh to 0.8V at end of life, most electronic devices require a constant supply voltage that stays within $\pm 10\%$ of a certain nominal value. That nominal value is often greater than 1.6 V. So how do we power electronics with batteries?

The boost converter is a commonly used circuit to provide that stable supply voltage, even from a battery whose voltage is slowly decreasing as it discharges. With proper choice of components and proper control of the gate voltage V_G of the transistor, power from the battery can be converted to higher voltages with almost perfect efficiency.

Consider the following circuit:



with

$$V_{bat} = 1 \text{ V}$$

$$L = 1 \mu\text{H}$$

$$C = 1 \text{ mF}$$

$$R = 1 \text{ k}\Omega$$

We model the electronic component that is consuming power as a resistor R with voltage V_{out} across it. Assume that the diode is ideal and that the gate voltage at the NMOS is low (and thus the NMOS is an open circuit) for $t < 0$. Remember that an ideal diode allows current to flow when $V_D > 0$ and that no current flows through an ideal diode when $V_D \leq 0$.

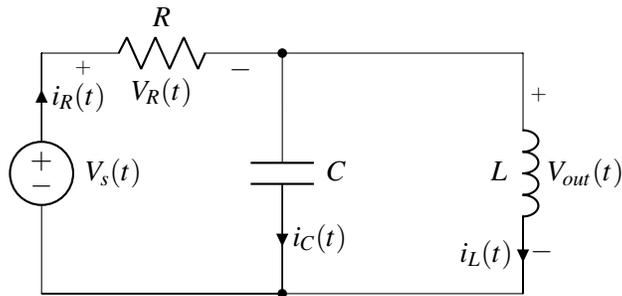
- (a) Assume that there is no current flowing through the diode. Calculate the time constant of the RC circuit. How long will it take for the output voltage to decay by 10% from any non-zero value? If the voltage on the capacitor is 3 V at $t = 0$, what is the current in the resistor at time $t = 0$? What is the voltage in the capacitor for $t > 0$?

- (b) Let's assume that at $t=0$, the voltage on the capacitor is 3 V. At $t=0$, the gate voltage on the NMOS V_G goes high, turning the transistor on (effectively shorting that circuit path). **What is the rate of change of current in the inductor? How long does it take for the current in the inductor to increase to 100mA?**
- (c) Based on your two previous answers, **how much has the capacitor voltage decayed during this time?** Your answer should be such a small number that in the following parts, we will consider the output voltage constant.
- (d) When the current in the inductor hits 100mA, the transistor gate goes low, turning the transistor off. The energy stored in the magnetic field in the inductor quickly causes the voltage on the positive side of the diode to increase to 3 V, turning it on (to understand why, think about what a rapid decrease in current will do to the inductor voltage when the transistor turns off). **What is the rate of change of current in the inductor when the diode is on? How long does it take the current in the inductor to go to zero?**
- (e) **Sketch the following currents vs time: i_L, i_{NMOS}, i_{diode} for a single inductor charge/discharge cycle described above.**
If you like circuits like this, take EE113 Power Electronics.
- (f) To maintain V_{out} at 3V, we need the average current through the diode to bring enough charge to the capacitor. **Calculate the charge that flows through the diode in a single cycle. At what frequency does the transistor need to cycle through the on/off phases you analyzed above in order to supply the current consumed by the resistor, computed in part (a)?**
- (g) **How would the shape of the curves in part (e) change if the battery voltage were 1.6 V? A rough sketch, and some notations like "steeper, shorter, longer" is enough.**

6. Phasor-Domain Circuit Analysis

The analysis techniques you learned previously for resistive circuits are equally applicable for analyzing circuits driven by sinusoidal inputs in the phasor domain. In this problem, we will walk you through the steps with a concrete example.

Consider the following circuit where the input voltage is sinusoidal. The end goal of our analysis is to find an equation for $V_{out}(t)$.



The components in this circuit are given by:

$$V_s(t) = 10\sqrt{2}\cos\left(100t - \frac{\pi}{4}\right)$$

$$R = 5\Omega$$

$$L = 50 \text{ mH}$$

$$C = 2 \text{ mF}$$

- Give the amplitude V_0 , oscillation frequency ω , and phase ϕ of the input voltage V_s .
- Transform the circuit the phasor domain. What are the impedances of the resistor, capacitor, and inductor? What is the phasor \tilde{V}_S of the input voltage $V_s(t)$?
- Use Kirchoff's law(s) to relate the current phasors \tilde{I}_R , \tilde{I}_C , and \tilde{I}_L in the circuit.
- Rewrite the equation you found in part (c) in terms of voltage phasors \tilde{V}_s and \tilde{V}_{out} , and solve for \tilde{V}_{out} .
- Convert the phasor \tilde{V}_{out} back to the time-domain signal $V_{out}(t)$.

7. Frequency domain: Filter

You have a sensor that gives a signal with frequencies in the range of 1 to 10 Hz. There is a noise signal present in your system at 100 kHz. If you use an RC low-pass filter to clean up your signal, **what is the most that you can attenuate the high frequency signal while still keeping at least 99% of the sensor signal? Design an RC filter to do that.**

8. Solar Cells

Modern rooftop solar panels typically contain sixty silicon solar cells wired up in series. (You can learn much more about solar power and how it works in EE134.) Panel manufacturers supply datasheets which often include the current-voltage relationships of the panel at different illuminations. Usually these datasheets don't follow our passive sign convention, and show the current as positive. But we know that the current from a solar cell is actually negative, meaning that the panels are generating power rather than dissipating power. One popular panel made by LG Solar is offered by CivicSolar for \$ 321. The datasheet shows the I/V characteristics under irradiance levels from 200 W/m^2 to 1000 W/m^2 . Note that the datasheet incorrectly labels those curves as W, not W/m^2 . The I/V plot is copied below with correct labels.

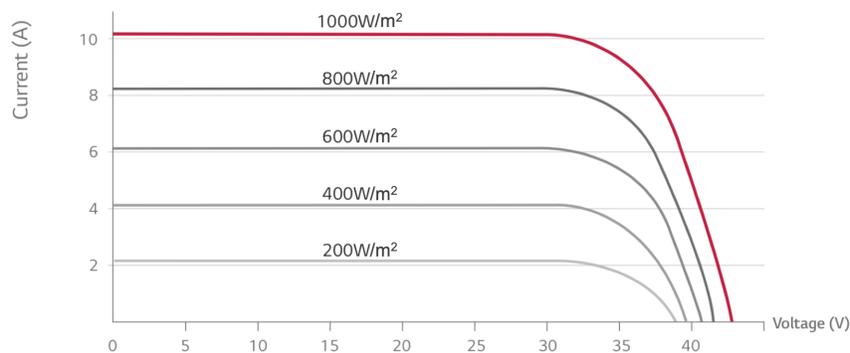


Figure 2: LG Solar CivicSolar Solar Panel I/V characteristics

As we learned in class and can see from the datasheet, solar panels produce a DC voltage. Your house runs on 60 Hz AC, so we need a device to convert DC voltage into AC. This device is called an *inverter* and looks like a variable resistive "load" to the panel. The panel is modeled in parallel with the inverter load as a stack of 60 diodes. To be efficient, the inverter needs to change the amount of current that it pulls from the panel depending on how much sunlight is falling on the panel.

(a) **Load optimization**

Make a table with rows for five different resistive loads: $0\ \Omega$, $1\ \Omega$, $3.5\ \Omega$, $10\ \Omega$, ∞ ; and columns for Voltage, Current, and Power.

For each resistor value, **draw a line on the solar panel IV plot corresponding to each of those resistor values, and label it with the resistance.** (*Hint: Remember $V = IR$ for resistors.*)

For each resistor value, **estimate the voltage and current that would be delivered to the resistor from the solar panel at $1000\ \text{W}/\text{m}^2$ irradiance.**

From your estimates, **calculate the power delivered to the load for each value of resistance.**

Which resistive load value is best? Why?

(b) **Maximum Power Point Tracking**

Let's assume that the inverter is smart enough to be operating at the best resistance value you found in part (a) on a bright sunny day in June when the irradiance is $1000\ \text{W}/\text{m}^2$. Suddenly, though, a cloud blocks out 80% of the sunlight, and the irradiance drops down to $200\ \text{W}/\text{m}^2$.

What is the voltage, current, and power delivered to the load now? Did that power also drop to 20%, or lower? Why? (Use the resistance you chose in part (a) (iv) above.)

If the inverter is smart and can change its resistive load value, what is the new resistance value that it should use to maximize power output? (Assume your choices are **NOT** limited to the resistance values given.)

What is the voltage, current, and power delivered to the load at this new resistance value?

(c) **Efficiency**

The dimensions of the panel is given in the datasheet are $1.686\ \text{m} \times 1.016\ \text{m}$.

What is the area of the panel?

What is the total solar power hitting the panel at $1000\ \text{W}/\text{m}^2$ irradiance?

What is ratio of the best case electrical power out from part (a) to the solar power into the panel at $1000\ \text{W}/\text{m}^2$ irradiance? This is the best case efficiency of the panel.

(d) **Economics**

Most places in California average insolation (total sunlight delivered per year) is equivalent to about 2,000 hours per year of full $1000\ \text{W}/\text{m}^2$ sunlight.

What is the total electrical energy, in kW-hours, produced per year by this solar panel under those conditions? Assume you are using the best load resistance you found in part (a).

If the average cost of electricity on a time-of-use meter is $0.20\ \text{\$/kW-hour}$, **how much money is a year of solar panel output worth? How many years will it take to pay off the cost of the $\$321$ panel?**

9. Write Your Own Question And Provide a Thorough Solution.

Writing your own problems is a very important way to really learn material. The famous "Bloom's Taxonomy" that lists the levels of learning is: Remember, Understand, Apply, Analyze, Evaluate, and Create. Using what you know to create is the top level. We rarely ask you any homework questions about the lowest level of straight-up remembering, expecting you to be able to do that yourself (e.g. making flashcards). But we don't want the same to be true about the highest level. As a practical matter, having some practice at trying to create problems helps you study for exams much better than simply counting on solving existing practice problems. This is because thinking about how to create an interesting problem forces you to really look at the material from the perspective of those who are going to create the exams. Besides, this is fun. If you want to make a boring problem, go ahead. That is your prerogative. But it is more fun to really engage

with the material, discover something interesting, and then come up with a problem that walks others down a journey that lets them share your discovery. You don't have to achieve this every week. But unless you try every week, it probably won't ever happen.

10. Homework Process and Study Group

Citing sources and collaborators are an important part of life, including being a student! We also want to understand what resources you find helpful and how much time homework is taking, so we can change things in the future if possible.

- (a) What sources (if any) did you use as you worked through the homework?
- (b) Who did you work on this homework with? List names and student ID's. (In case of homework party, you can also just describe the group.)
- (c) How did you work on this homework? (For example, *I first worked by myself for 2 hours, but got stuck on problem 3, so I went to office hours. Then I went to homework party for a few hours, where I finished the homework.*)
- (d) Roughly how many total hours did you work on this homework?

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