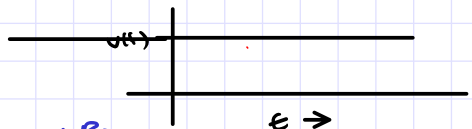
ANNOUNCEMENTS

→ NEW SECTIONS: 1–2, and 2–3 ← SEE PIAZZA FOR LOCATIONS (MOFFITT)

→ DSP ACCOMODATION LETTERS: PLEASE REQUEST THEM FROM THE DSP OFFICE.

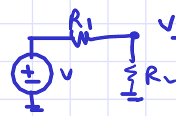
→ RECAP

→ CIRCUIT ANALYSIS



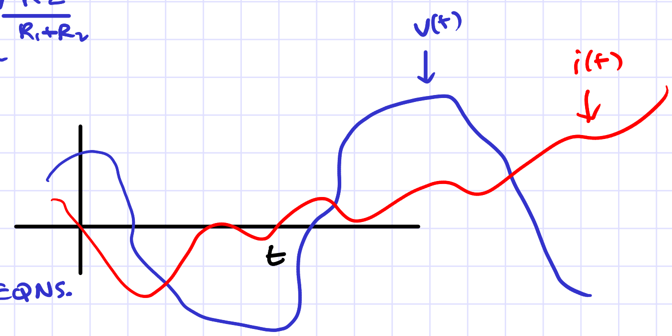
→ DC ANALYSIS ← 16A

→ EQNS: $i = \frac{V}{R}$,



$\frac{v R_2}{R_1 + R_2}$

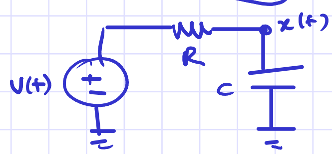
→ TRANSIENT ANALYSIS



→ $i(t)$, $v(t)$

→ EQNS: DIFFERENTIAL EQNS.

→ $i(t) = C \frac{dv(t)}{dt}$,



$$C \frac{dx}{dt} = \frac{v(t) - x(t)}{R}$$

$$\Rightarrow \frac{dx}{dt} = \frac{v(t) - x(t)}{RC}$$

if $v(t) \equiv 0$ $\quad \frac{dx}{dt} = -\frac{x(t)}{RC}$ $\quad$ call: $\lambda = -\frac{1}{RC}$

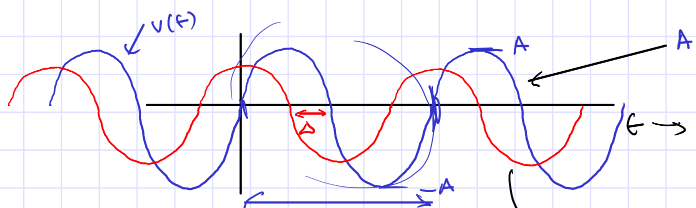
$$\frac{dx}{dt} = \lambda x(t)$$

$$\Rightarrow \boxed{x(t) = X_0 e^{\lambda t}}$$

↳ KRIS: showed that this solves

↳ HW: SHOWED THAT <u>ONLY</u> this solves

→



$T = 1/f$

$A \sin(\omega t)$ — angular freq.

$\omega \to 2\pi f$ — frequency

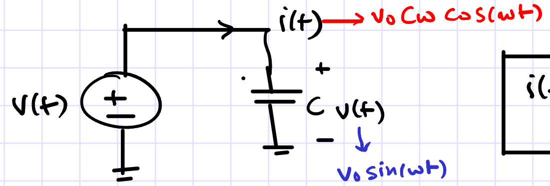
$B \sin(\omega t + \phi)$

$\Delta = \frac{\phi}{\omega}$

→ <u>DEMO</u>: AUDIO CAN BE SPLIT INTO A SUM OF SINUSOIDS
→ OF DIFFERENT FREQS.

→ <u>WITH ALL SINUSOIDS</u>: **MUCH EASIER** TO SOLVE
THE CIRCUIT → NO DIFF. EQN. SOLNS.
NEEDED, INSTEAD: LIKE DC

→ LAST CLASS:

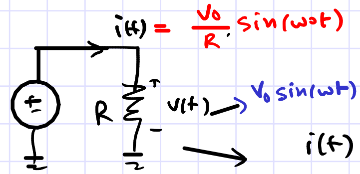


$$\boxed{i(t) = C\frac{dv(t)}{dt}}$$

→ $v(t) = V_0 \sin(\omega t) \Rightarrow i(t) = C\frac{d}{dt}(V_0 \sin(\omega t))$

$= V_0 C \omega \cos(\omega t)$

RES:



→ $i(t) = \frac{V_0 \sin(\omega t)}{R}$

Using Ohm's Law: $\boxed{i(t) = \frac{v(t)}{R}} \Rightarrow R = \frac{v(t)}{i(t)}$; CHECK: $\frac{\cancel{V_0 \sin(\omega t)}}{\frac{\cancel{V_0}}{R}\cancel{\sin(\omega t)}} = R$ ✓

→ WHAT IS $\frac{v(t)}{i(t)}$ FOR THE CAP?

$\frac{\cancel{V_0}\sin(\omega t)}{\cancel{V_0} C\omega\cos(\omega t)} = \frac{1}{\omega C}\tan(\omega t)$ ← ???

∞

STRANGE / ∞/0 VALUES

→ KEY TO A BETTER ANALYSIS: EXPRESS sin()/cos() using **COMPLEX NUMBERS**

→ **QUICK COMPLEX NUMBER TUT:**

→ $j = \sqrt{-1}$, $j^2 = -1$ j = imaginary

→ complex: $a = \underbrace{a_r}_{\text{real part}} + j\underbrace{a_i}_{\text{imag. part}}$

→ same arithmetic rules as real numbers

→ eg: $a + b = (a_r + ja_i) + (b_r + jb_i) = (a_r + b_r) + j(a_i + b_i)$

→ CONJUGATE OF A COMPLEX NUMBER:

→ $a = a_r + ja_i$, $\bar{a} \triangleq a_r - ja_i$ (change sign of imag. part)

→ $a + \bar{a} = (a_r + \cancel{ja_i}) + (a_r - \cancel{ja_i}) = 2a_r$ ← always real ← IMPORTANT

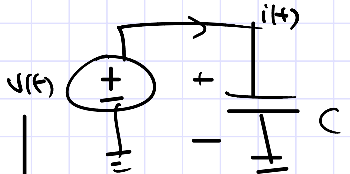
→ $a - \bar{a} = (\cancel{a_r} + ja_i) - (\cancel{a_r} - ja_i) = 2ja_i$ ← " imag.

→ RELATIONSHIP w sin()/cos().

→ de Moivre's Theorem (dMT): $\boxed{e^{j\theta} = \cos(\theta) + j\sin(\theta)}$

$$
\begin{aligned}
&e^{j\theta} = \cos(\theta) + j\sin(\theta)\\
+\;&e^{-j\theta} = \cos(\theta) - j\sin(\theta)\\
\hline
&e^{j\theta} + e^{-j\theta} = 2\cos(\theta) \Longrightarrow \boxed{\cos(\theta) = \frac{e^{j\theta}+e^{-j\theta}}{2}}
\end{aligned}
$$

$$
\begin{aligned}
&e^{j\theta} = \cos(\theta) + j\sin(\theta)\\
-\;&e^{-j\theta} = \cos(\theta) - j\sin(\theta)\\
\hline
&e^{j\theta} - e^{-j\theta} = 2j\sin(\theta)\\
&\Longrightarrow \boxed{\sin(\theta) = \frac{e^{j\theta}-e^{-j\theta}}{2j}}
\end{aligned}
$$



$$i(t) = C\frac{dv}{dt}$$

$$v(t) = V_0\sin(\omega t) = \frac{V_0}{2j}\left[e^{j\omega t} - e^{-j\omega t}\right] = \underbrace{\frac{V_0}{2j}e^{j\omega t} - \frac{V_0}{2j}e^{-j\omega t}}_{\text{COMPLEX CONJ.}}$$

COMPLEX CONJ.

$$\boxed{\begin{aligned} \rightarrow e^{j\omega t} &= \cos(\omega t) + j\sin(\omega t)\\ e^{-j\omega t} &= \cos(\omega t) - j\sin(\omega t)\end{aligned}}$$

$$
\begin{aligned}
i(t) &= C\frac{d}{dt}\left(\frac{V_0}{2j}\left[e^{j\omega t} - e^{-j\omega t}\right]\right) = \frac{V_0 C}{2j}\left[j\omega e^{j\omega t} - (-j\omega)e^{-j\omega t}\right]\\
&= \frac{V_0 C j\omega}{2\cancel{j}}\left[e^{j\omega t} + e^{-j\omega t}\right]\\
&= V_0 C\omega \underbrace{\left(\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right)}_{\cos(\omega t)}
\end{aligned}
$$

$$v(t) = \frac{V_0}{2j}e^{j\omega t} + \underbrace{\left(\overline{\frac{V_0}{2j}}\right)e^{-j\omega t}}_{cc}$$

$$\frac{V_0}{2j} = \tilde{V}$$

$$i(t) = \frac{V_0 C\omega}{2}e^{j\omega t} + \text{c.c term.}$$

$$\frac{V_0 C\omega}{2} = \tilde{I}$$

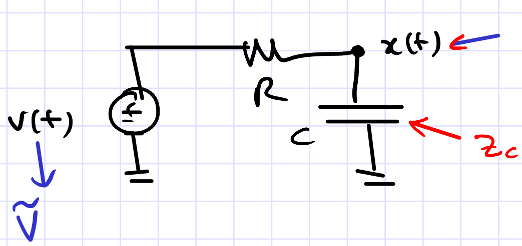
PHASOR = coeff. of $e^{j\omega t}$ term.

→ TRY: $\dfrac{\tilde{V}}{\tilde{I}} = \dfrac{\frac{\cancel{V_0}}{2j}}{\frac{\cancel{V_0}}{\cancel{2}}C\omega} = \boxed{\dfrac{1}{j\omega C}}$

IMPEDANCE

$$\tilde{I} = \frac{\tilde{V}}{\left(\frac{1}{j\omega C}\right)} \leftarrow Z_C$$

→ CIRCUITS CAN BE ANALYZED W PHASORS!



$Z_C = \frac{1}{j\omega C}$

$$\tilde{X} = \tilde{V} \frac{Z_C}{R + Z_C} = \boxed{\tilde{V} \frac{1}{1 + j\omega RC}}$$

$$\rightarrow x(t) = \underline{\tilde{X} e^{j\omega t}} + c.c.$$

→ TODAY

→ DEMO: AUDIO = SUM OF MANY SINUSOIDS

→ SINUSOIDS: WROTE USING COMPLEX NUMBERS

→ PHASOR: coeff. of $e^{j\omega t}$

→ CAP "LOOKS LIKE" RES IN "PHASOR DOMAIN": IMPEDANCE

→ CIRCUITS, including CAPS, CAN BE SOLVED EASILY USING PHASORS.