1 Changing behavior through feedback

In this question, we discuss how feedback control can be used to change the effective behavior of a system.

a) Consider the scalar system:

$$x(t+1) = 0.9x(t) + u(t) + w(t) \tag{1}$$

where $u(t)$ is the control input we get to apply based on the current state and $w(t)$ is the external disturbance.

Is the system stable? If $|w(t)| \leq \epsilon$ and $u(t) = 0$ find a bound on $|x(t)|$ given the initial condition $x(0) = 0$?

b) Suppose that we decide to choose a control law $u(t) = kx(t)$ to apply in feedback. Find the possible values λ we can obtain, such that the system behaves as:

$$x(t+1) = \lambda x(t) + w(t). \tag{2}$$

What choice of k results in such values?

c) For the previous part, find k such that the upper bound on $|x(t)|$ is minimized?

d) Suppose instead of a 0.9, we had a 3 in the original (1). Is the new system stable? Can we still change the eigenvalues by choosing k?

2 Eigenvalues Placement in Discrete Time

Consider the following linear discrete time system

$$\vec{x}(t+1) = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) + \vec{w}(t) \tag{3}$$

a) Is this system controllable from $u(t)$?

b) Is the linear discrete time system stable?

c) Derive a state space representation of the resulting closed loop system using state feedback of the form $u(t) = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \vec{x}(t)$

d) Find the appropriate state feedback constants, k_1, k_2 in order the state space representation of the resulting closed loop system to place the eigenvalues at $\lambda_1 = -\frac{1}{2}, \lambda_2 = \frac{1}{2}$

e) Is the system now stable?

f) Suppose that instead of $\begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$ in (3), we had $\begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$ as the way that the discrete-time control acted on the system. Is this system controllable from $u(t)$?

g) For the part above, suppose we used $[k_1, k_2]$ to try and control the system. What would the eigenvalues be? Can you move all the eigenvalues to where you want? Give an intuitive explanation of what is going on.