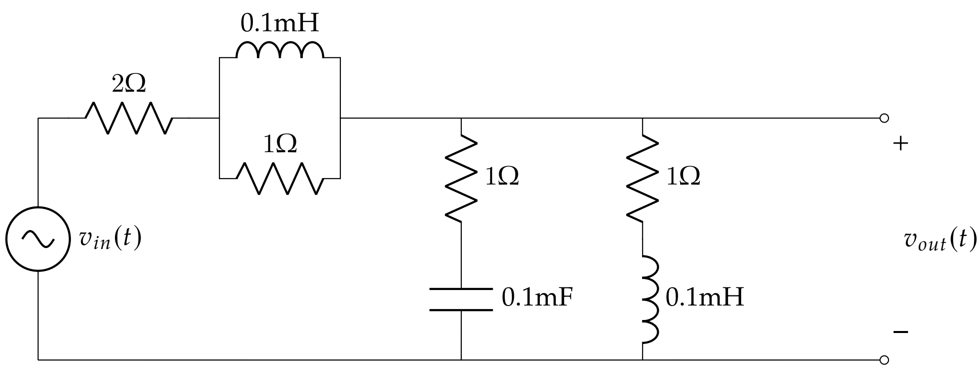
1 Analyzing Circuits in the Phasor Domain

Consider the circuit below:

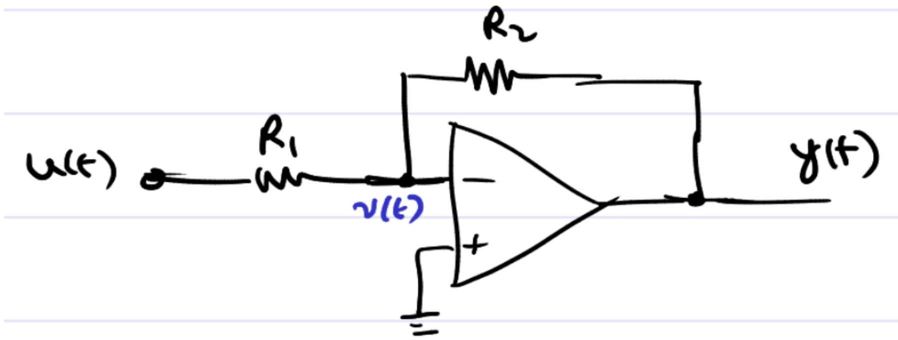


a) Find $v_{out}(t)$ for an input voltage $v_{in}(t) = 10$ V (v_{in} is a DC voltage).

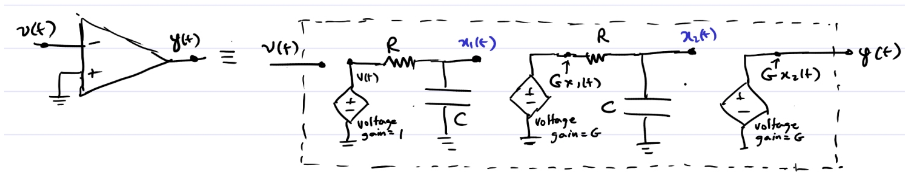
b) Find $v_{out}(t)$ for an input $v_{in}(t) = 10\sin(10^4 t + 30°)$ V. Your final answer should be in the time domain. (Hint: $\text{atan}(\frac{4}{3}) \approx 51°$).

2 Two-capacitor op-amp model

It's common to use an op-amp in the following configuration:



Here, the op-amp is in negative feedback, and it can be concluded from the Golden Rules that $y(t) = -\frac{R_2}{R_1}u(t)$. The Golden Rules idealization of op-amps abstracts away finite gain as well as any internal dynamics, and we've encountered various richer models this semester. The following model represents an op-amp by three voltage-controlled voltage sources with RC connections:



We are using $v(t)$ for the voltage at the terminal of the op-amp IC, and $u(t)$ for the input to the inverting amplifier circuit.

a) Using $\vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, write the state space equations for an op-amp's internal circuitry in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{x}(t) = A\vec{x}(t) + \vec{b}v(t). \tag{1}$$

b) Give necessary and sufficient conditions for the op-amp (without feedback) to be i) stable, ii) marginally stable, and iii) unstable. Note that R, C, and G are positive.

c) For the inverting amplifier, give a formula for "internal input" $v(t)$ in terms of state variables and $u(t)$, the inverting amplifier's "external input."

d) With $v(t)$ constrained by the previous part, write the state space equations for the inverting amplifier in the following form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{x}(t) = A_f\vec{x}(t) + \vec{b}_f u(t). \tag{2}$$

e) Give necessary and sufficient conditions for the op-amp (*with* feedback) to be i) stable, ii) marginally stable (a special case of unstable), and iii) unstable.