1 RC Circuits

In this problem, we will be using differential equations to find the voltage across a capacitor over time in an RC circuit. We set up our problem by first defining three functions over time: $I(t)$ is the current at time t, $V(t)$ is the voltage across the circuit at time t, and $V_o(t)$ is the voltage across the capacitor at time t.

Recall from 16A that the voltage across a resistor is defined as $V_R = RI_R$ where I_R is the current across the resistor. Also, recall that the voltage across a capacitor is defined as $V_o = \frac{Q}{C}$ where Q is the charge across the capacitor.

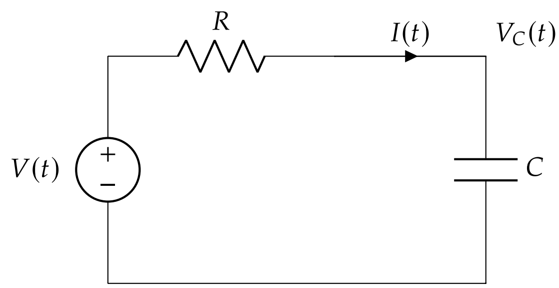

Figure 1: Example Circuit

1. First, find an equation that relates the current across the capacitor $I(t)$ with the voltage across the capacitor $V_C(t)$.
2. Write a system of equations that relates the functions $I(t)$, $V_C(t)$, and $V(t)$.
3. Rewrite the previous equation in part (b) in the form of a differential equation involving only $V_C(t)$ and $V(t)$.

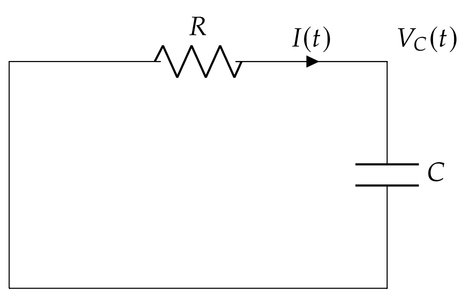

Figure 2: Circuit for part (d)

4. Let's suppose that at $t = 0$, the capacitor is charged to a voltage V_{DD} ($V_C(0) = V_{DD}$). Let's also assume that $V(t) = 0$ for all $t \geq 0$. Solve the differential equation for $V_C(t)$ for $t \geq 0$.

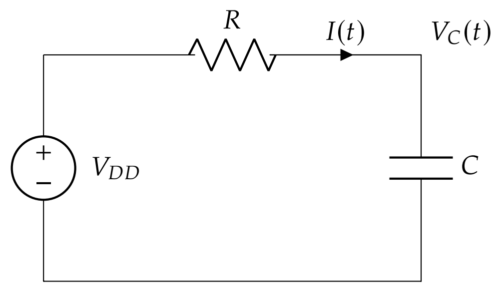

Figure 3: Circuit for part (e)

5. Now, let's suppose that we start with an uncharged capacitor $V_C(0) = 0$. We apply some constant voltage $V(t) = V_{DD}$ across the circuit. Solve the differential equation for $V_C(t)$ for $t \geq 0$.

2 Systems of Differential Equations

Consider the following circuit.

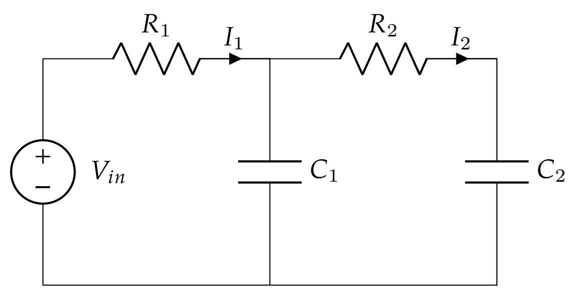

Figure 4: Two dimensional system: a circuit with two capacitors.

1. Write a system of differential equations that governs the voltages V_{C1}, V_{C2} across the capacitors. Use the following values: $C_1 = 1\mu F, C_2 = \frac{1}{3}\mu F, R_2 = \frac{1}{2}M\Omega,\ R_1 = \frac{1}{3}M\Omega$.

2. Suppose also that V_{in} was at 7V for a long time, and then transitioned to be 0V at time $t = 0$. Write the system of differential equations that are valid for $t \geq 0$ in matrix form. What are the initial conditions?

3. Find the eigenvalues $\lambda_1,\ \lambda_2$ and eigenspaces for the matrix corresponding to the differential equation matrix above.