1 Complex Algebra

1. Express the following values in polar forms: -1, j, $-j$, $\sqrt{j}$, and $\sqrt{-j}$. Recall $j^2 = -1$.

2. Represent $\sin\theta$ and $\cos\theta$ using complex exponentials. (*Hint:* Use Euler's identity $e^{j\theta} = \cos\theta + j\sin\theta$.)

3. For complex number $z = x + jy$ show that $|z| = \sqrt{z\bar{z}}$, where $\bar{z}$ is the complex conjugate of z.

For the next two parts, let $A = 1 - j\sqrt{3}$ and $B = \sqrt{3} + j$.

4. Express A and B in polar form.

5. Find AB, $A\bar{B}$, $\frac{A}{B}$, $A + \bar{A}$, $A - \bar{A}$, $\overline{AB}$, $\bar{A}\bar{B}$, and $\sqrt{B}$.

6. Show the number A in complex plane, marking the distance from origin and angle with real axis.

7. Show that multiplying A with j is equivalent to rotating the magnitude of the complex number by $\pi/2$ or 90 degrees in the complex plane.