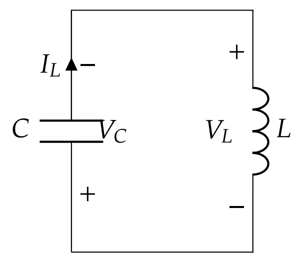
1 LC Tank: Diagonalization with complex eigenvalues

Consider the following circuit like you saw in lecture:



This is sometimes called an LC tank and we will derive its response in this problem. Assume at $t = 0$ we have $V_C(0) = V_S = 1$ V and $\frac{dV_C}{dt}(t = 0) = 0$.

a) **Write the system of differential equations in terms of state variables $x_1(t) = I_L(t)$ and $x_2(t) = V_C(t)$ that describes this circuit for $t \geq 0$. Leave the system symbolic in terms of V_s, L, and C.**

b) **Write the system of equations in vector/matrix form with the vector state variable $\vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$.** This should be in the form $\frac{d}{dt}\vec{x}(t) = A\vec{x}(t)$ with a 2×2 matrix A.

Find the initial conditions $\vec{x}(0)$.

c) **Find the eigenvalues of the A matrix symbolically.**

d) **What are the eigenvectors associated with these eigenvalues?**

e) **Use the eigenvalues and eigenvectors found above to diagonalize A as $A = V\Lambda V^{-1}$ where Λ is a diagonal matrix. Suppose $L = 9\,\mathrm{nH}$ and $C = 1\,\mathrm{nF}$.**

f) Use a change of basis for the state variable $\vec{x}(t)$ into $\vec{z}(t)$ such that $\frac{d}{dt}\vec{z}(t) = \Lambda\vec{z}(t)$, and express the initial conditions $\vec{z}(0)$

Solve the differential equations in $\vec{z}(t)$

g) **Convert your solutions back to $\vec{x}(t)$. Plot $V_C(t)$ and $I_L(t)$.** What do you notice about the solutions? Are they complex functions? HINT: Remember $e^{j\theta} = \cos(\theta) + j\sin(\theta)$.

2 Complex Matrix Inverse

Consider a complex matrix

$$M = M_r + jM_i$$

and its inverse

$$N = N_r + jN_i$$

a) **Show that the inverse of $\overline{M} = M_r - jM_i$ (the complex conjugate of M) is equal to $\overline{N} = N_r - jN_i$ (the complex conjugate of N).**