1 System Identification and Linear Control

A scalar discrete-time system has the following dynamics:

$$x(t+1) = \lambda x(t) + g(u(t)), \tag{1}$$

where $g : \mathbb{R} \to \mathbb{R}$ not necessarily linear.

a) If g is approximated to order 2 around the operating point $u^* = 0$, so that

$$x(t+1) \approx \lambda x(t) + \beta_0 + \beta_1 u(t) + \beta_2 u^2(t), \tag{2}$$

what should β_0, β_1, and β_2 be?

b) Suppose that $x(0) = 0$. We apply a sequence of inputs

$$\vec{u} = \big(u(0), u(1), \ldots, u(N-1)\big) \tag{3}$$

and observe states $x(1), x(2), \ldots, x(N)$. Derive the least-squares estimates of λ, β_0, β_1, and β_2.

2 Scalar feedback control

Suppose that x has the following discrete-time dynamics:

$$x(t+1) = \lambda x(t) + bu(t), \quad x(0) = x_0 \tag{4}$$

a) Assuming that $x_0 = 1$ and $u = 0$, sketch $x(t)$ for a few time steps for $\lambda \in \{-2, -1, 0, 1, 2\}$.

b) What qualifications for λ will result in convergence of x? A scalar system having such a λ is called *stable*.

c) If these system were the discretization from a continuous system at interval T, then $\lambda = e^{\lambda_{\text{cont}} T}$ and $b = b_{\text{cont}} \frac{e^{\lambda_{\text{cont}} T} - 1}{\lambda_{\text{cont}}}$. Use these facts to relate convergence properties of continuous- and discrete-time linear systems. (A stable continuous-time scalar system has an eigenvalue with a strictly negative real part.)

d) If $u(t) = u_0$ and x is stable, what does x converge to? Sketch stable trajectories of x for $\lambda = 0$, $\lambda < 0$, and $\lambda > 0$.

e) If $x(t+1) = \lambda x(t) + bu(t)$ is unstable, describe feedback laws $u(t) = kx(t)$ that stabilize the equilibrium $x = 0$.