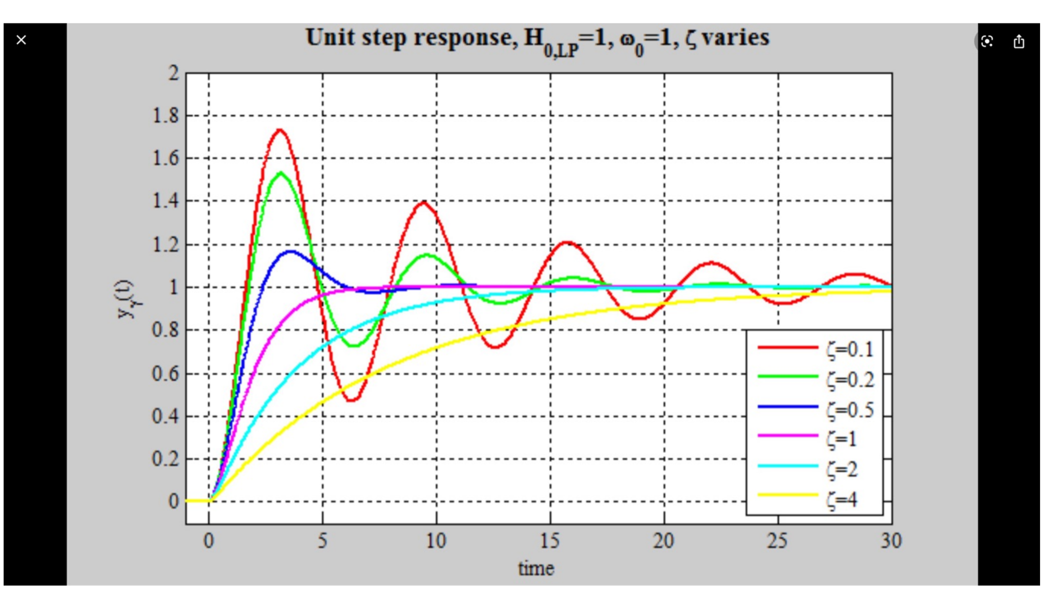
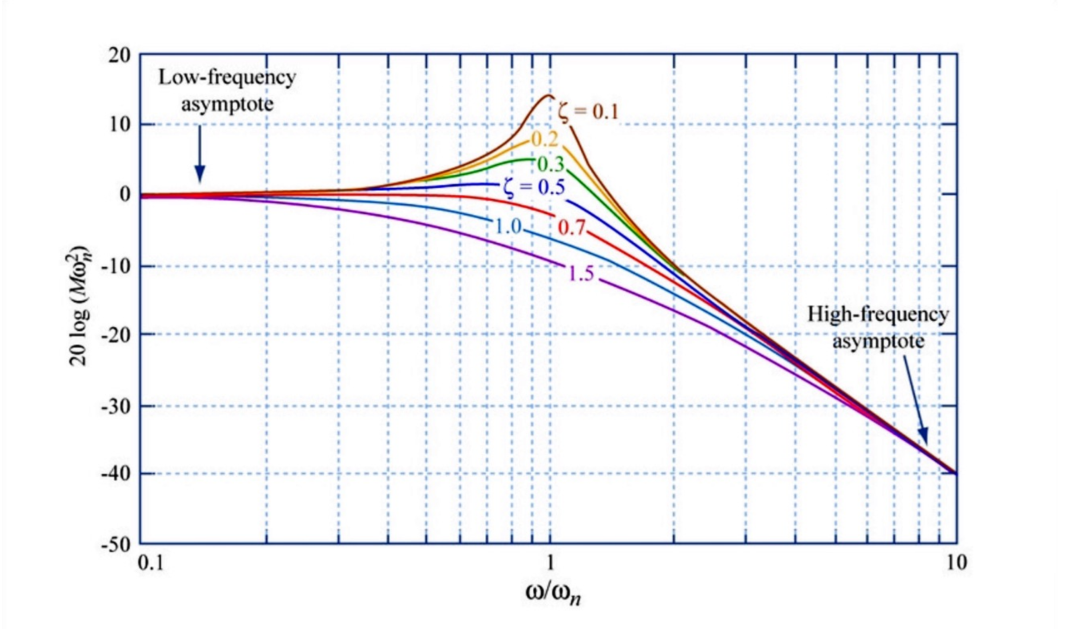
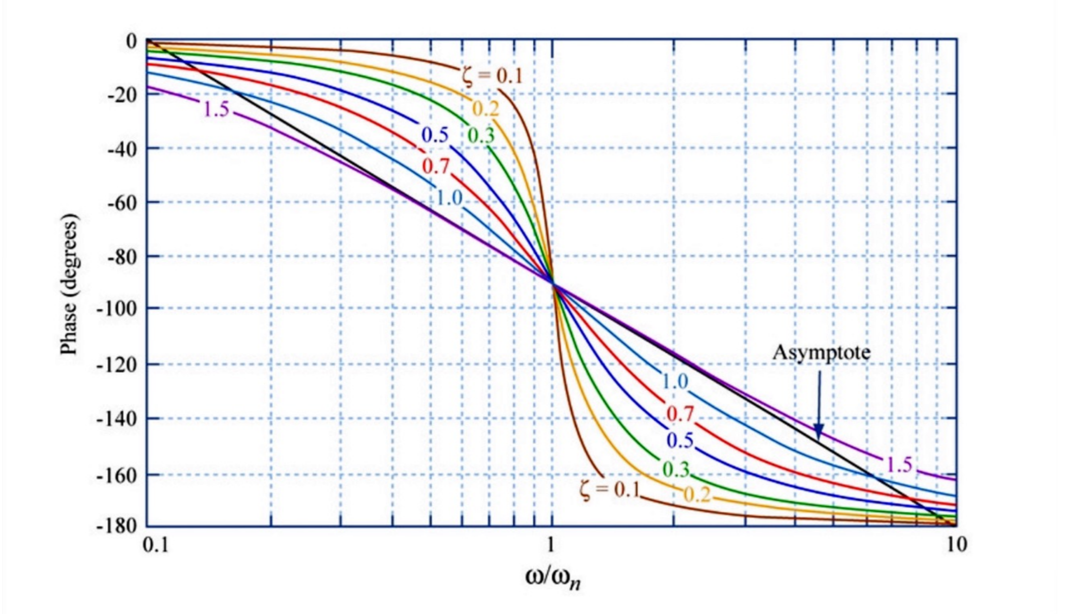
EECS 16B Thursday Feb 20 2020 - Resonance - Bode Plots + examples

Thursday, February 20, 2020 9:56 AM





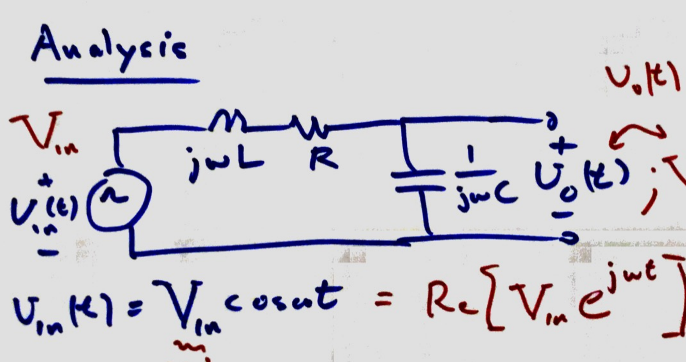


1) EECS 16B Thurs Feb 20

Frequency Response of LRC ckts - "Resonance"

Analysis

$v_o(t) = Re\{V_o e^{j\omega t}\}$ complex



V_{in}, $v_{in}(t)$, $j\omega L$, R, $\frac{1}{j\omega C}$, $v_o(t)$; V_o

$v_{in}(t) = V_{in} \cos\omega t = Re[V_{in} e^{j\omega t}]$ (V_{in} real)

$$\frac{V_o}{V_{in}} = H(j\omega) = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R + j\omega L} = \frac{1/LC}{(j\omega)^2 + j\omega\frac{R}{L} + 1/LC}$$

2) Recall time domain analysis [last week]

$$A = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix}$$

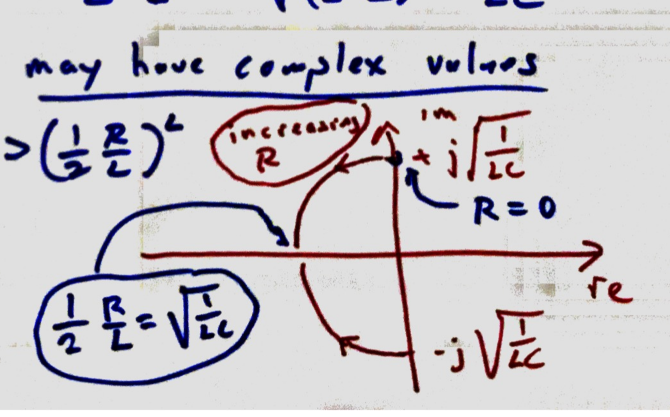
char eq. $\det(\lambda I - A) = \lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC}$

roots: $\lambda_{1,2} = -\frac{1}{2}\frac{R}{L} \pm \sqrt{\left(\frac{1}{2}\frac{R}{L}\right)^2 - \frac{1}{LC}}$

eigenvalues: may have complex values

when $\frac{1}{LC} > \left(\frac{1}{2}\frac{R}{L}\right)^2$

Plot with R as parameter



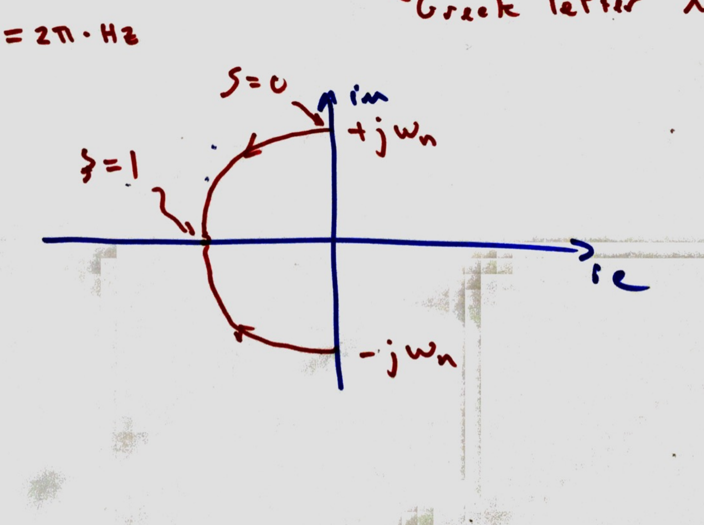
increasing R; $+j\sqrt{\frac{1}{LC}}$; $R=0$; im; re; $\frac{1}{2}\frac{R}{L} = \sqrt{\frac{1}{LC}}$; $-j\sqrt{\frac{1}{LC}}$

3) For 2^{nd} order systems, like our LRC ckt

$\omega_n = \sqrt{\frac{1}{LC}}$; $\zeta = \frac{1}{2}\frac{R}{\sqrt{L/C}}$

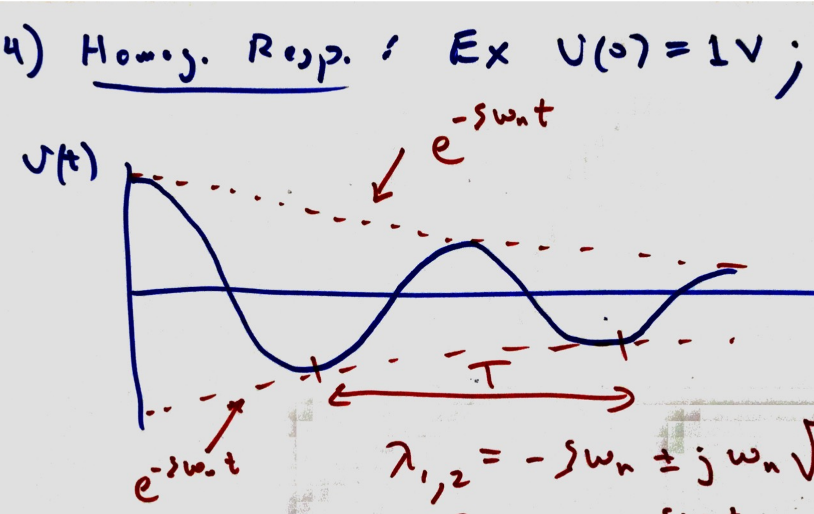
omega rad/s $= 2\pi \cdot Hz$

Greek letter Xi



$\zeta=0$; $\zeta=1$; im; $+j\omega_n$; re; $-j\omega_n$

4) Homog. Resp.: Ex $v(0)=1V$; $i(0)=0A$



$v(t)$; $e^{-\zeta\omega_n t}$; T; $e^{-\zeta\omega_n t}$

$\lambda_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

time waveform: $e^{\lambda_1 t} = e^{-\zeta\omega_n t} Re\{e^{j\omega_n\sqrt{1-\zeta^2}t}\}$

$= e^{-\zeta\omega_n t}\cos(\omega_n\sqrt{1-\zeta^2}\,t)$

$T = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$

5) Frequency Domain: single frequency ω
- use phasor analysis
- can think of ω as a parameter

"Simplified" Parameters ω_n, ζ

$$\frac{V_o}{V_{in}} = H(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + (j\omega)2\zeta\omega_n + \omega_n^2}$$

* Construct Bode Plot

1) Low frequencies: $\omega \ll \omega_n$

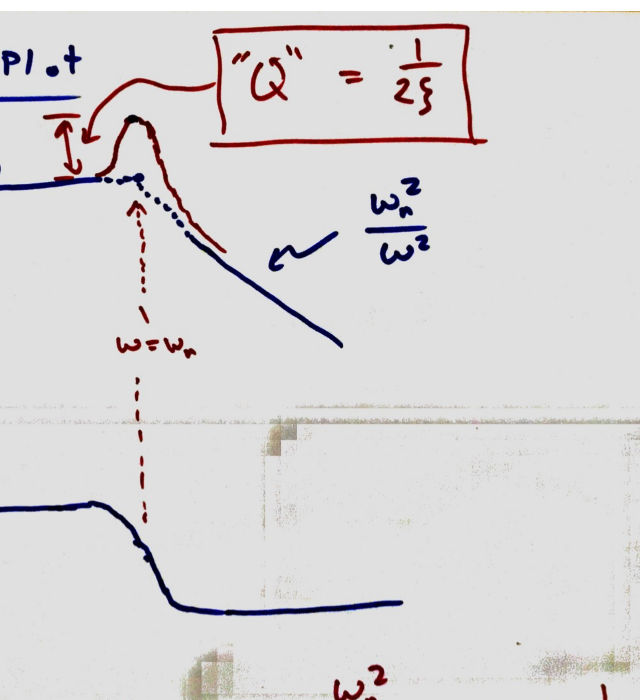
$H(j\omega) \approx 1$

2) High frequencies: $\omega \gg \omega_n$

$H(j\omega) \approx -\omega_n^2/\omega^2$

6) $H(j\omega)$ Bode Plot

$\boxed{\text{"Q"} = \frac{1}{2\zeta}}$

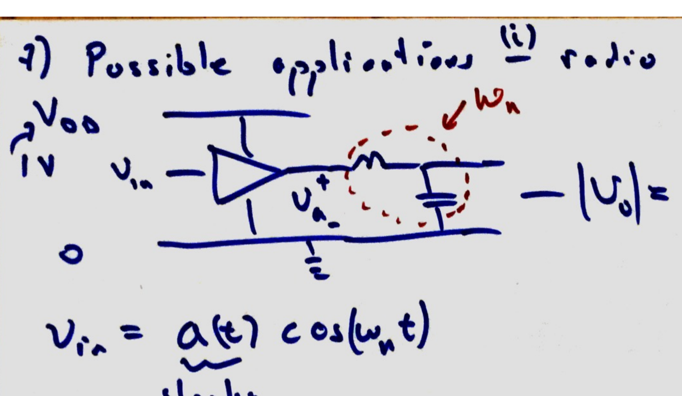


$|H(j\omega)|$; ≈ 1; $\frac{\omega_n^2}{\omega^2}$; $\boxed{\zeta = 0.1}$; $\omega=\omega_n$; $\measuredangle H(j\omega)$ $0°$, $-90°$, $-180°$

3) "Middle freq's":

Try $\omega=\omega_n \Rightarrow H(j\omega)\big|_{\omega=\omega_n} = \frac{\omega_n^2}{j2\zeta\omega_n^2} = -j\frac{1}{2\zeta}$

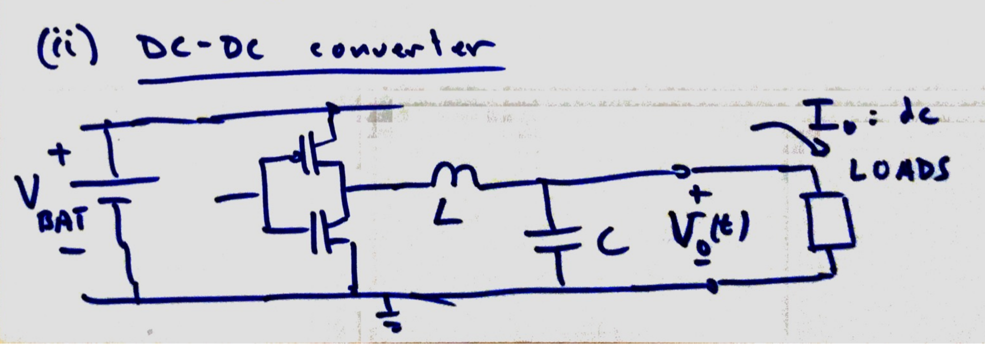
7) Possible applications (i) radio - "matching" network



V_{DD} 1V; 0; v_{in}; v_a; ω_n; $|v_o| = \frac{1}{2\zeta}\cdot|v_a|$

$v_{in} = a(t)\cos(\omega_n t)$ ($a(t)$ slowly varying)

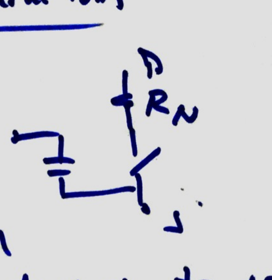
(ii) DC-DC converter



V_{BAT}; L; C; $V_o(t)$; I_o: dc; LOADS

8) DC-DC considerations

1) NMOS



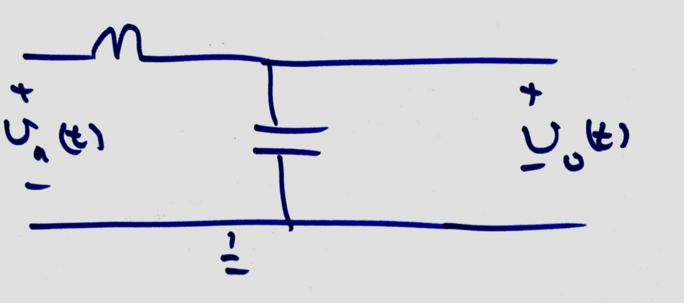
G; D; R_N; S

maybe parallel many NMOS devices to reduce resistance to keep loss low [Low resistance ⇒ low loss]

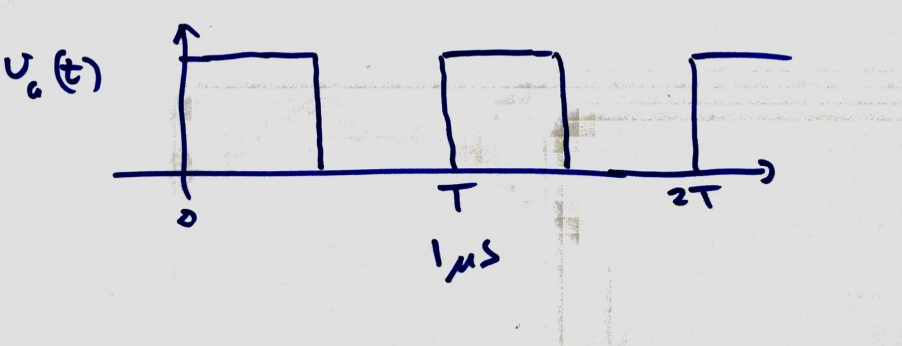
2) PMOS - analogous idea - parallel many

3) Inductor - try to make series resistance low: R small

9) Look at filter:

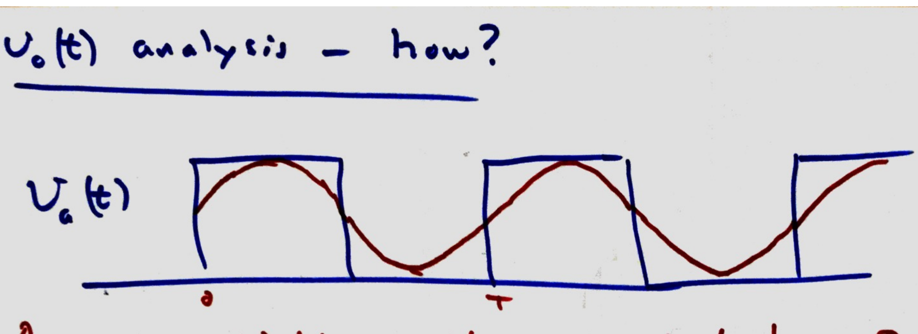


$v_a(t)$; $v_o(t)$



$v_a(t)$; 0; T; 2T; 1μs

$v_o(t)$ analysis - how?



$v_a(t)$; 0; T

Approx $v_a(t)$ as sinewave raised above 0.

$= \underbrace{\frac{1}{2}V_{BAT}\sin(\omega t)}_{ac} + \underbrace{\frac{1}{2}V_{BAT}}_{dc}$

two voltage components into L-C filter:

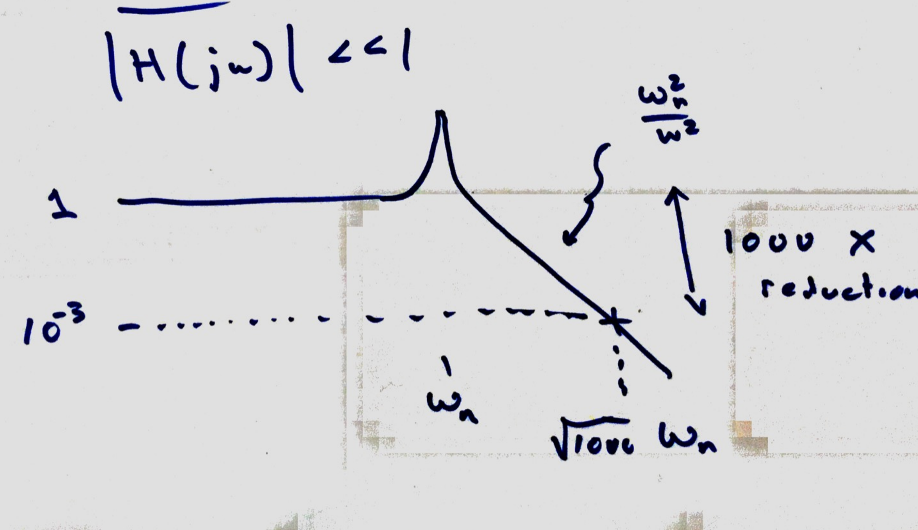
- use superposition: $v_o(t) = \underbrace{\overbrace{\frac{1}{2}V_{BAT}}^{dc}}_{} + V_o\cos(\omega t + \phi)$ (analysis of LRC at $\omega=\omega_o$)

11) How to coordinate ω_o vs ω_n?

$\omega = 2\pi \cdot 1\,MHz$

- choice: make $\omega \gg \omega_n$

$|H(j\omega)| \ll 1$



1; 10^{-3}; $\frac{\omega_n^2}{\omega^2}$; 1000 x reduction; ω_n; $\sqrt{1000}\,\omega_n$