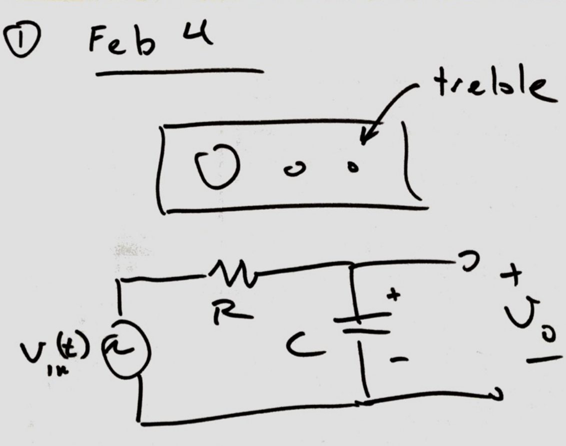
EECS16B Vector Differential Equations Feb 4 2020

Tuesday, February 4, 2020 7:05 PM

① Feb 4



$$\frac{d}{dt} v_o(t) = -\frac{1}{RC} v_o(t) + \frac{1}{RC} v_{in}(t) \; ; \; v_o\Big|_{t_0} = \checkmark$$

Last time: gen. sol'n composed of homog. part + part. sol'n

$$v_{in}(t) = V_{in} \cos(\omega t) \qquad \omega = \text{freq in } \frac{\text{rad}}{s} = 2\pi \cdot f \leftarrow \text{Hz}$$

② Favorite Sol'n Method

- plug in and do hard calculus
- or, guess

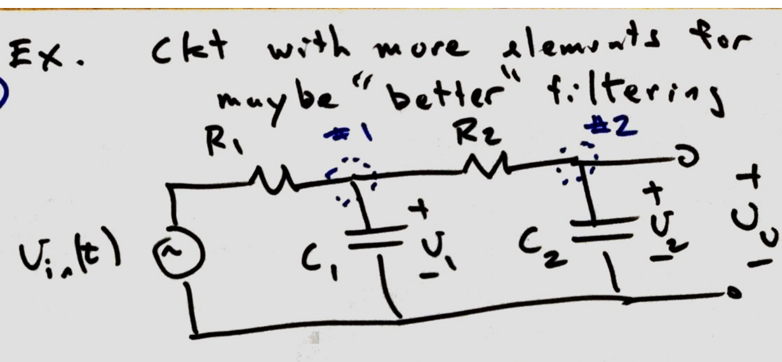
$$v_o(t) = A\cos(\omega t + \phi)$$

$$v_o(t) = \frac{V_{in}}{\sqrt{\omega^2 (RC)^2 + 1}} \cos(\omega t + \phi) + B e^{-\frac{1}{RC}(t - t_0)}$$

$$\phi = -\tan^{-1}(\omega RC)$$

what's B? : $\frac{V_{in}}{\sqrt{\omega^2 (RC)^2 + 1}} \cos(\omega t_0 + \phi) + B = v_o\Big|_{t_0}$

③ Ex. ckt with more elements for maybe "better" filtering



"2nd order ckt" or "2nd order filter"

$C_1 = C_2 = 1\mu F \; ; \; R_1 = \frac{1}{3} M\Omega \; ; \; R_2 = \frac{1}{2} M\Omega$

Analysis

① $C_1 \frac{d}{dt} v_1 + \frac{v_1 - v_{in}(t)}{R_1} + \frac{v_1 - v_2}{R_2} = 0$

② $C_2 \frac{d}{dt} v_2 + \frac{v_2 - v_1}{R_2} = 0$

④ Algebra step(s)

$$\frac{d}{dt} v_1 = -v_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C_1} + v_2 \frac{1}{R_2 C_1} + \frac{v_{in}(t)}{R_1 C_1}$$

$$\frac{d}{dt} v_2 = v_1 \frac{1}{R_2 C_2} - v_2 \frac{1}{R_2 C_2}$$

Notation: $\frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2} = (R_1 // R_2)^{-1}$

$G_1 + G_2 = G_1 + G_2$

With #'s

$$\frac{d}{dt}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} v_{in}(t)$$

"Vector Diff Eqn" $\Leftrightarrow$ "State-Space Eqn"

⑤ General Structure

$$\frac{d}{dt}\underbrace{\vec{x}}_{\text{vector}} = \underbrace{A}_{\substack{\text{matrix} \\ 2\times 2}}\vec{x} + \vec{b}\, u(t)$$

Guide guessing with eigenvector - eigenvalue

$$A\vec{v} = \cancel{\lambda} \lambda\vec{v}$$

guess: some $\vec{x}(t)$ to homogeneous problem $\frac{d}{dt}\vec{x} = A\vec{x}$ looks like

$$\vec{x}(t) = \vec{v} e^{\lambda t}$$

⑥ Check

LHS $\frac{d}{dt}\left(\vec{v} e^{\lambda t}\right) = \lambda \vec{v} e^{\lambda t}$

$\Updownarrow$

RHS $A\left(\vec{v} e^{\lambda t}\right) = \lambda \vec{v} e^{\lambda t}$

$\begin{cases} A\vec{v}_1 = \lambda_1 \vec{v}_1 \\ A\vec{v}_2 = \lambda_2 \vec{v}_2 \end{cases}$ | Presume have $\vec{v}_1, \vec{v}_2$ linearily indep.

$$A\underbrace{\left[\vec{v}_1 \middle| \vec{v}_2\right]}_{V} = \left[\vec{v}_1 \middle| \vec{v}_2\right]\underbrace{\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}}_{\Lambda}$$

⑦ $\vec{v}_1$ & $\vec{v}_2$ lin. indep.

V has inverse V^{-1}

$A = V \Lambda V^{-1} \Leftrightarrow$ eig value eig vector decomp of A

general $\vec{x}(0) = \tilde{x}_1 \vec{v}_1 + \tilde{x}_2 \vec{v}_2$

$$\vec{x}(0) = \left[\vec{v}_1 \middle| \vec{v}_2\right]\begin{bmatrix} \tilde{x}_1(0) \\ \tilde{x}_2(0) \end{bmatrix}$$

Can set $\begin{bmatrix} \tilde{x}_1(0) \\ \tilde{x}_2(0) \end{bmatrix} = V^{-1}\vec{x}(0)$

⑧ Homogeneous Sol'n

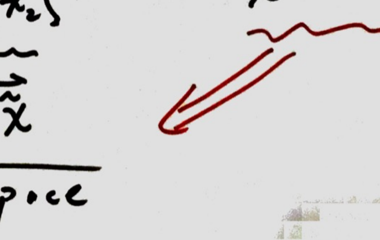
$$\vec{x}(t) = \vec{v}_1 e^{\lambda_1 t}\tilde{x}_1(0) + \vec{v}_2 e^{\lambda_2 t}\tilde{x}_2(0)$$

$$= \underbrace{\left[\vec{v}_1 \middle| \vec{v}_2\right]}_{V}\begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}\begin{bmatrix} \tilde{x}_1(0) \\ \tilde{x}_2(0) \end{bmatrix}$$

Check with initial cond.:

$$\vec{x}(0) = \left[\vec{v}_1 \middle| \vec{v}_2\right]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \tilde{x}_1(0) \\ \tilde{x}_2(0) \end{bmatrix}$$

⑨ Would like to decompose vector $\vec{x}$



$$\vec{x} = \underbrace{\left[\vec{v}_1 \middle| \vec{v}_2\right]}_{V}\underbrace{\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}}_{\tilde{\vec{x}}} \leftrightarrow \tilde{\vec{x}} = V^{-1}\vec{x}$$

In transformed space

$$\frac{d}{dt}\tilde{\vec{x}} = V^{-1}\frac{d}{dt}\vec{x} = V^{-1}\left(A\vec{x} + \vec{b}u\right)$$

$$= V^{-1}AV\tilde{\vec{x}} + V^{-1}\vec{b} \cdot u$$

⑩

$$\frac{d}{dt}\tilde{\vec{x}} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}\tilde{\vec{x}} + \underbrace{\tilde{\vec{b}}}_{V^{-1}b} \cdot u$$

Know how to solve since reduced to problems from prev. classes

"Modal Decomposition"