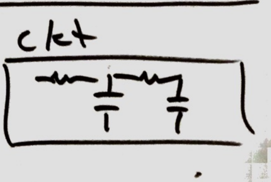
EECS16B Thursday Feb 6 2020

Thursday, February 6, 2020 7:54 PM

①

EECS 16 B Thurs Feb 6

Last Class: ckt



cap voltages

$$\frac{d}{dt}\vec{x} = A\vec{x} + \vec{b}u \; ; \quad \vec{x} = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$$

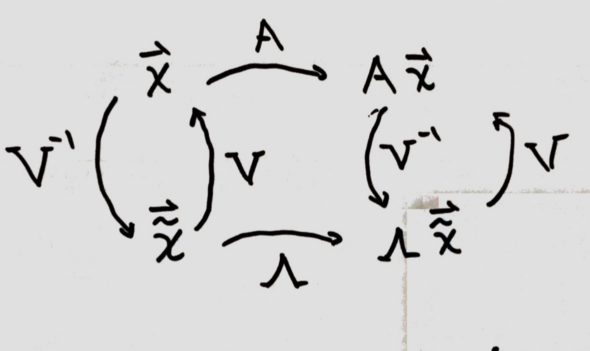
$\vec{x}(0)$ or $\vec{x}(t_0)$ specified

Plan: represent $\vec{x} = \vec{v}_1 \tilde{x}_1 + \vec{v}_2 \tilde{x}_2$ ($\vec{v}_1 \leftrightarrow \lambda_1$, $\vec{v}_2 \leftrightarrow \lambda_2$)

$$\vec{x} = \underbrace{\left[\vec{v}_1 \,|\, \vec{v}_2\right]}_{V} \underbrace{\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}}_{\tilde{\vec{x}}}$$

$\lambda_1 \neq \lambda_2 \Rightarrow V$ has inverse V^{-1}

②



$\underbrace{V^{-1}AV}_{\Lambda}\tilde{\vec{x}}$

Last Time: $A\underbrace{\left[\vec{v}_1 \,|\, \vec{v}_2\right]}_{V} = \left[\vec{v}_1 \,|\, \vec{v}_2\right]\underbrace{\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}}_{\Lambda}$

Solution Plan: $\frac{d}{dt}\vec{x} = A\vec{x} + \vec{b}u$; $\vec{x}(t_0)$

1) Compute λ_1, λ_2 & $\vec{v}_1, \vec{v}_2$
2) Use $\tilde{\vec{x}} = V^{-1}\vec{x}$

③

3) Solve easier problem

$$\frac{d}{dt}\tilde{\vec{x}} = \Lambda\tilde{\vec{x}} + \underbrace{V^{-1}\vec{b}}_{\tilde{b}} \cdot u \qquad \tilde{x}(t_0) = V^{-1}\vec{x}(t_0)$$

4) $\vec{x} = V\tilde{\vec{x}}$ ⇐ recovers sol'n

Book keeping

$$\begin{aligned} \vec{x} &= V\tilde{\vec{x}} \\ \tilde{\vec{x}} &= V^{-1}\vec{x} \\ \Lambda &= V^{-1}AV \\ A &= V\Lambda V^{-1} \\ \tilde{\vec{b}} &= V^{-1}b \end{aligned}$$

④ Numerical Ex. from Ckt

$$A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

(i) eigenstructure of A

char. poly = $\det(\lambda I - A)$

$$\det\begin{bmatrix} \lambda+5 & -2 \\ -2 & \lambda+2 \end{bmatrix} = \lambda^2 + 7\lambda + 10 - 4$$

$\lambda^2 + 7\lambda + 6 = 0$ char. eq.

$\lambda_1 = -1$; $\lambda_2 = -6$

⑤. Find eigen vectors

- Plug in λ_1

$$\lambda_1 I - A = \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$$

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\lambda_2 I - A = \begin{bmatrix} -1 & -2 \\ -2 & -4 \end{bmatrix}$$

$$\vec{v}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

⑥ Have $V = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

$$V^{-1} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$$

Next step: solve using simplified var's; $t_0 = 0$

$$\frac{d}{dt}\tilde{\vec{x}} = \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & -6 \end{bmatrix}}_{\Lambda}\tilde{\vec{x}} + \underbrace{V^{-1}\vec{b}}_{\begin{bmatrix} 3/5 \\ 6/5 \end{bmatrix}} \cdot u$$

$$\Rightarrow \tilde{\vec{x}}(t) = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}\tilde{\vec{x}}(0) +$$

⑦ cont'd

$$+ \underbrace{\int_0^t \begin{bmatrix} e^{\lambda_1(t-\tau)} & 0 \\ 0 & e^{\lambda_2(t-\tau)} \end{bmatrix}\tilde{\vec{b}} \cdot u(\tau)\, d\tau}_{\text{particular sol'n}}$$

Component wise

$$\tilde{x}_1(t) = e^{\lambda_1 t}\tilde{x}_1(0) + \int_0^t e^{\lambda_1(t-\tau)}\tilde{b}_1 u(\tau)\, d\tau$$

$$\tilde{x}_2(t) = e^{\lambda_2 t}\tilde{x}_2(0) + \int_0^t e^{\lambda_2(t-\tau)}\tilde{b}_2 u(\tau)\, d\tau$$

(iv) $\vec{x}(t) = \left[\vec{v}_1 \,|\, \vec{v}_2\right]\begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \end{bmatrix}$

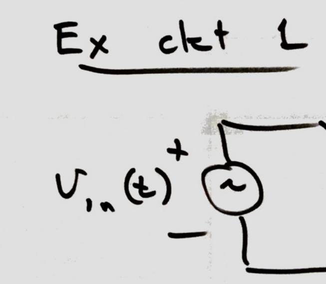
⑧ Another Key ckt element: **Inductor**

Capacitor

q, v across C – Farads, current i

$\frac{d}{dt}q = i$

$v = \frac{q}{C}$



electric field

$q \leftrightarrow$ Coulombs

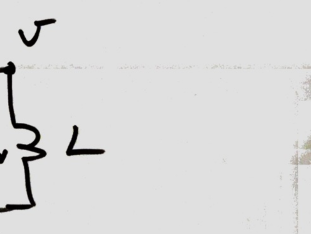
Inductor

λ, v across L – Henry, current i

λ = flux

$\frac{d}{dt}\lambda = v$

$i = \frac{\lambda}{L}$



Magnetic field

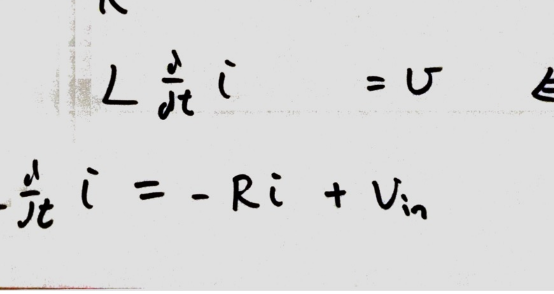
Webers = Volt-sec

$q \leftrightarrow \lambda$

$i \leftrightarrow v$

$v \leftrightarrow i$

⑨ Ex ckt L



Node eqn(s)

① $\frac{v - v_{in}}{R} + i = 0$

② $L\frac{d}{dt}i = v$ ⇐ extra eqn

$L\frac{d}{dt}i = -Ri + v_{in}$

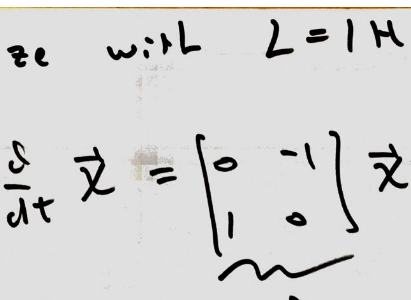
⑩

$$\frac{d}{dt}i = -\frac{R}{L}i + \frac{v_{in}}{L}$$

very similar structure to R-C ckt already studied

- use analogous solution method

More Fun: Ex. ckt 2 with L



① $C\frac{d}{dt}v = -i$

② $L\frac{d}{dt}i = v$

$$\Rightarrow \frac{d}{dt}\begin{bmatrix} v \\ i \end{bmatrix} = \begin{bmatrix} 0 & -1/C \\ 1/L & 0 \end{bmatrix}\begin{bmatrix} v \\ i \end{bmatrix}$$

⑪ Analyze with $L = 1H$ $C = 1F$

$$\frac{d}{dt}\vec{x} = \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{A}\vec{x}$$