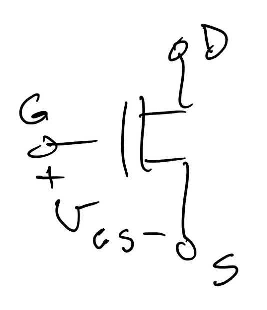
EECS16B Lecture Jan 28 2020: intro to transient analysis

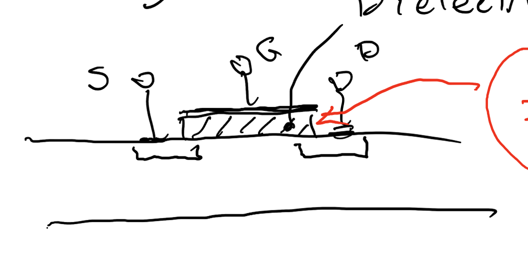
Tuesday, January 28, 2020 11:06 AM



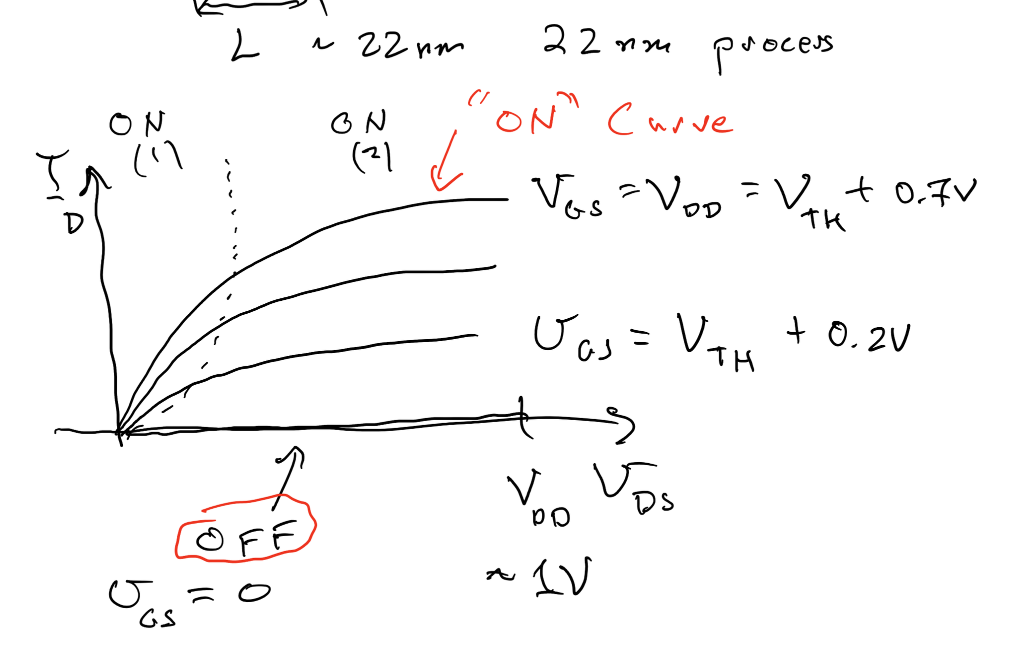
NMOS

$I_G = 0$

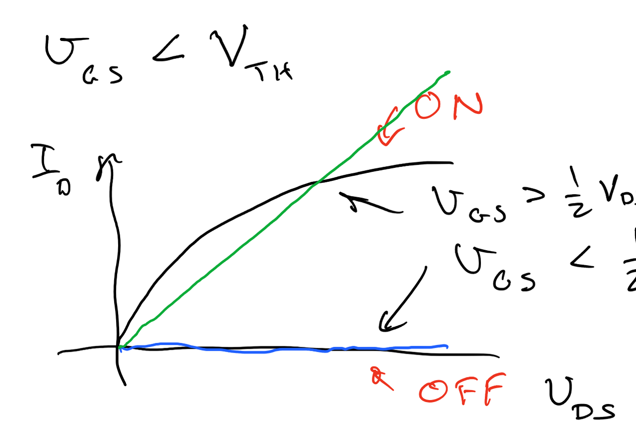
Dielectric [non-conducting]



L ~ 22nm   22nm process

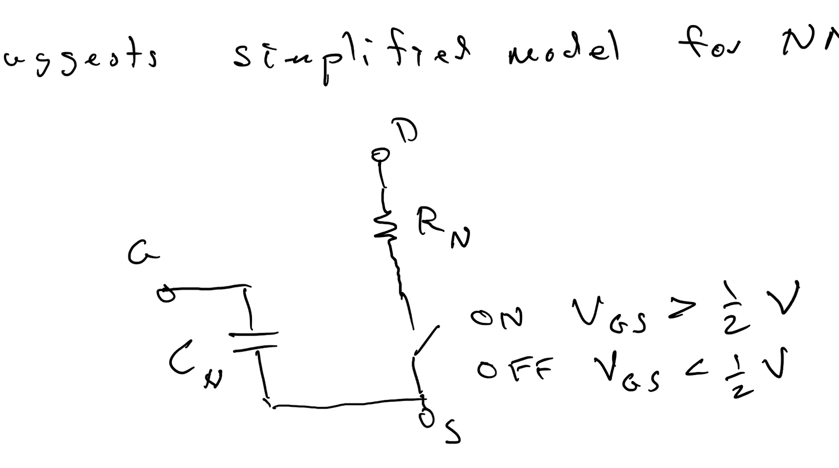


$v_{GS} < V_{TH}$

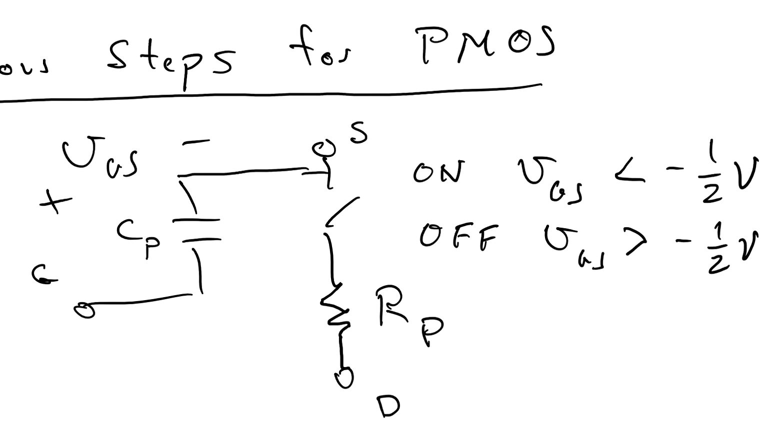


Today: model transitions between ON & OFF as occurring at $v_{GS} = \frac{1}{2} V_{DD}$

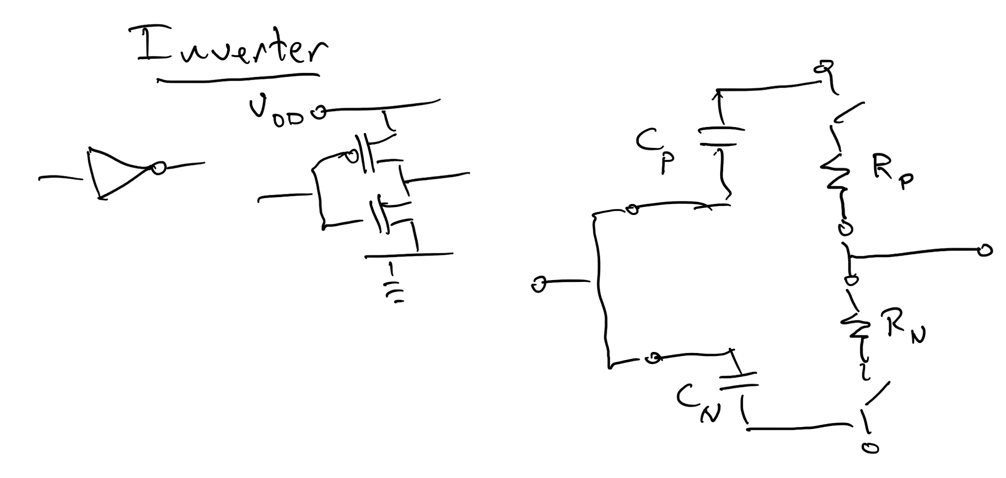
Suggests simplified model for NMOS

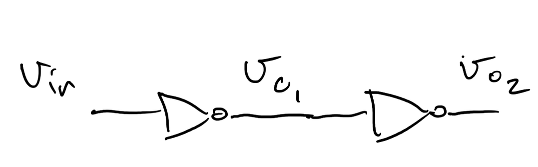


Analogous Steps for PMOS

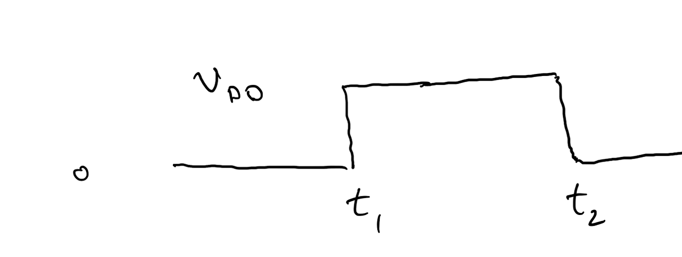


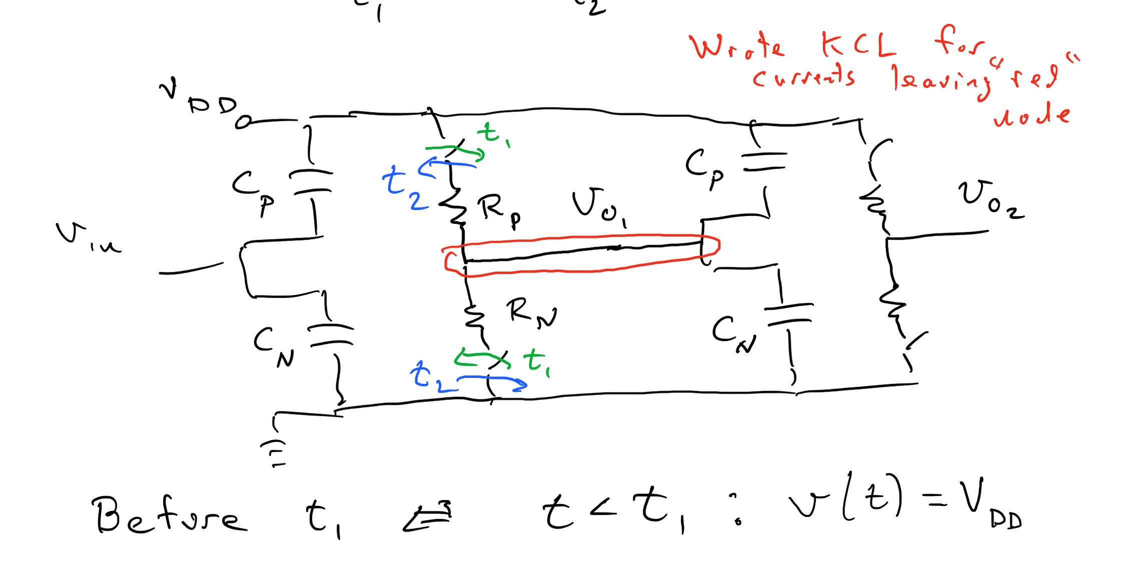
Inverter





"chain of inverters" only 2





Before t_1 ⇔ $t < t_1$: $v(t) = V_{DD}$

Key Node eqn after t_1:
node potential v_{o_1}

$$(*) \quad \frac{v_{o_1}}{R_N} + C_N \frac{d}{dt} v_{o_1} + C_P \frac{d}{dt}\left(v_{o_1} - V_{DD}\right) = 0$$

($\frac{d}{dt} V_{DD} \to 0$)

$$\frac{d}{dt} v_{o_1} + \frac{1}{R_N(C_N + C_P)} v_{o_1} = 0$$

$$v_{o_1}(t_1) = V_{DD}$$

$R_N(C_N + C_P)$ ⇐ time constant = τ "Tau"

$\Omega \cdot F = s$ (seconds)

$M\Omega$ fF

$10^6 \Omega$ $10^{-15} F = 10^{-9} s = nS$

○ state of art time constant ≈ 10pS

$$\frac{d}{dt} v_{o_1} = -\frac{1}{\tau} v_{o_1} \; ; \; v_{o_1}(t_1) = V_{DD}$$

$-\frac{1}{\tau} = \lambda$ "lambda"

$$\boxed{\frac{d}{dt} v_{o_1} = \lambda v_{o_1}} \; ; \; \boxed{v_{o_1}(t_1) = V_{DD}}$$

○ separation of variables

$$\int \frac{dv_{o_1}}{v_{o_1}} = \int \lambda \, dt \qquad \text{def. integral}$$

$v_{o_1} = e^{\lambda t}$ ✓

$\frac{d}{dt} v_{o_1} = \lambda e^{\lambda t} = \lambda v_{o_1}$

Need $v_{o_1}(t_1) = V_{DD}$

$v_{o_1} = A e^{\lambda t}$

$\frac{d}{dt} v_{o_1} = \lambda A e^{\lambda t} = \lambda v_{o_1}$ ✓

$A = ?$ ⇐ $v_{o_1}(t_1) = V_{DD}$

$$A e^{\lambda t}\Big|_{t_1} = A e^{\lambda t_1} = V_{DD}$$

$A = V_{DD} e^{-\lambda t_1}$

$v_{o_1} = V_{DD} e^{-\lambda t_1} \cdot e^{\lambda t}$

Just solved a differential eqn
- universal and ubiquitous in science & engineering

Broad theorem [rule] says a big class of diff eqns has unique solution

$$\frac{d}{dt} x = f(x, t)$$

(1) { say $f(x,t)$ is differentiable w.r.t x
and $\left|\frac{\partial f}{\partial x}(x,t)\right| < M$ ← real #

and

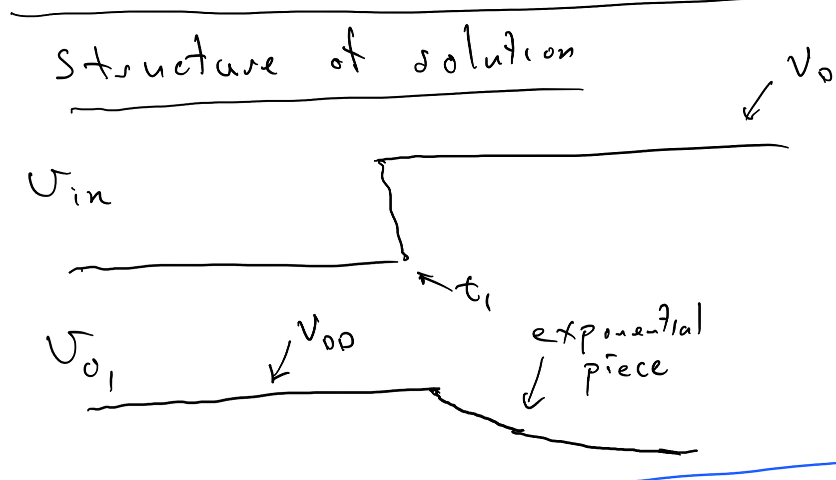
(2) { $f(x,t)$ has only finite # of discontinuities in t in any unit interval [1 s]

If (1) and (2) hold, then our diff eqn has unique solution

$\frac{d}{dt} v_{o_1} = \lambda v_{o_1}$ = "$f(x)$" $x = v_{o_1}$

$\frac{\partial f}{\partial x} = \lambda$

Structure of solution



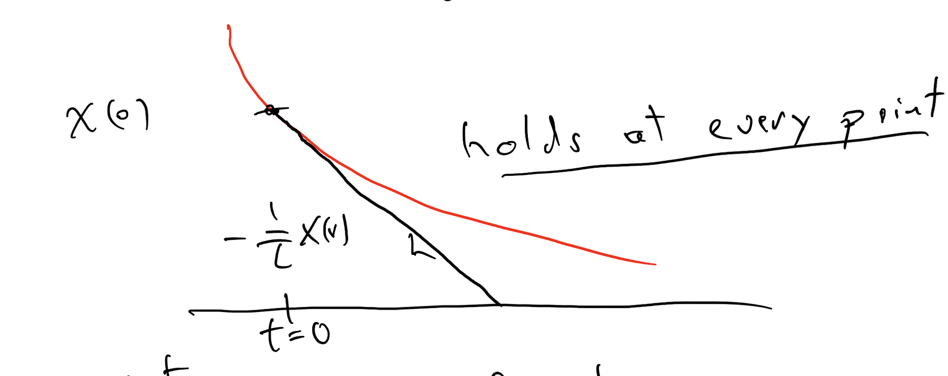
Determined that solution was: $v_{o_1} = V_{DD} e^{-\lambda t_1} \cdot e^{\lambda t}$

$\lambda = -\frac{1}{\tau}$ $\tau = R_N(C_P + C_N)$

$$v_{o_1} = V_{DD} \, e^{-\left(\frac{t - t_1}{\tau}\right)}$$

· Sketching exponentials $e^{-\frac{t}{\tau}}$

$$\frac{d}{dt} e^{-\frac{t}{\tau}} = -\frac{1}{\tau} e^{-\frac{t}{\tau}}$$



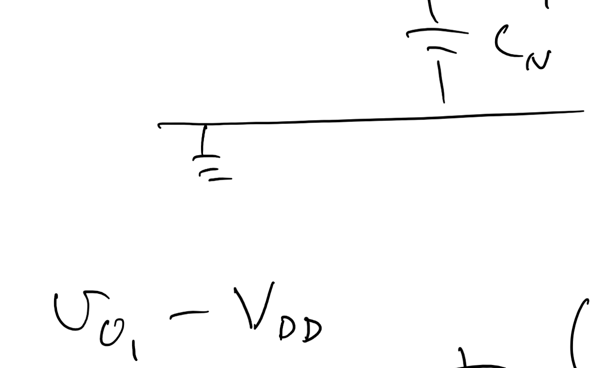
$e^{\lambda t}$ ≡ eigenfunction

---

extend to $t = t_2$ and beyond

$$v_{o_1}(t_2) = V_{DD} \, e^{-\left(\frac{t_2 - t_1}{\tau}\right)}$$

· What is relevant diff eqn for $t > t_2$



$$\frac{v_{o_1} - V_{DD}}{R_P} + (C_P + C_N) \frac{d}{dt} v_{o_1} = 0$$

$$\frac{d}{dt} v_{o_1} + \frac{1}{R_P(C_P + C_N)} v_{o_1} = \frac{V_D}{R_P(C_P + C_N)}$$

time constant: "τ_P" $= R_P(C_P + C_N)$

Solution ?

$$v_{o_1} = V_{DD} + \left(v_{o_1}(t_2) - V_{DD}\right) e^{-\frac{(t - t_2)}{\tau_P}}$$