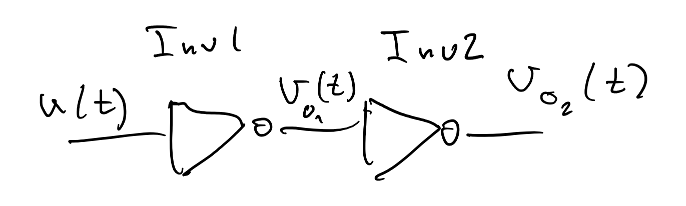
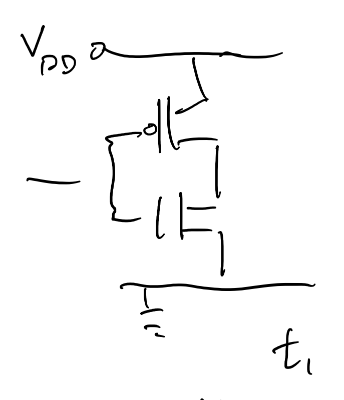
EECS 16B Inverter Delay Analysis cont'd + Diff Eqns

Thursday, January 30, 2020 11:09 AM

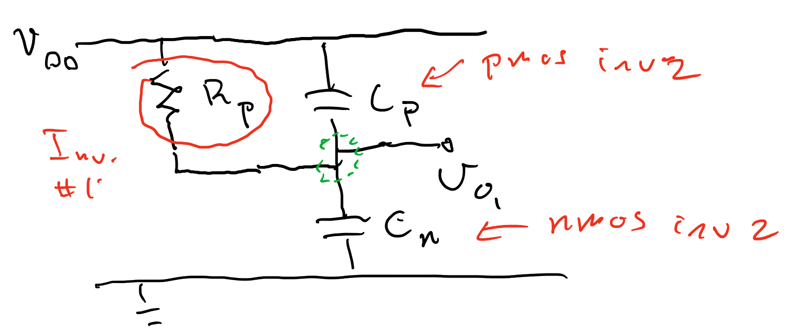




$t < t_1$

$V_{o_1} = V_{DD}$

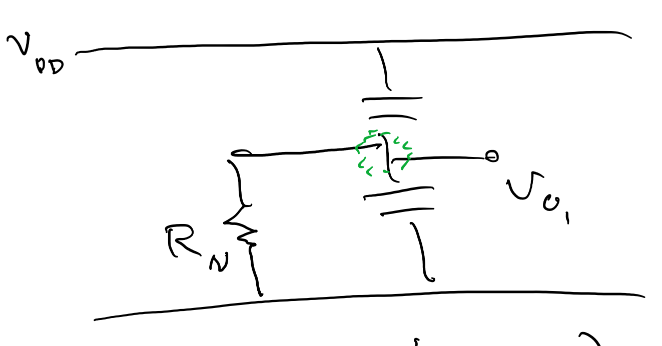
$t > t_2$



time constant

$$\tau_p = R_p (C_p + C_N)$$

$t_1 < t < t_2$



$$\tau_N = R_N (C_p + C_N)$$

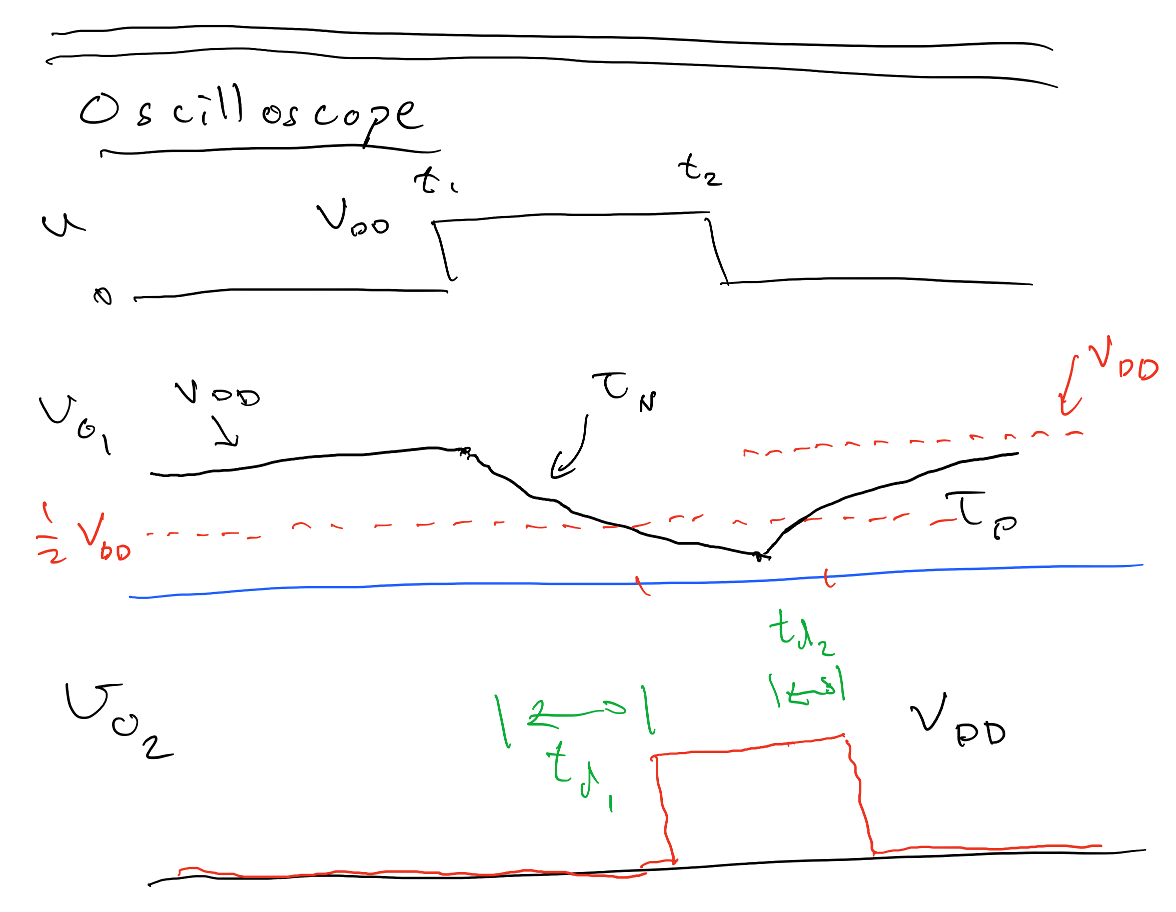
$t < t_1$ and $t > t_2$

$$\frac{d}{dt} v_{o_1} = -\frac{1}{R_p(C_N + C_p)} v_{o_1} + \frac{V_{DD}}{R_p(C_N + C_p)}$$

$t_1 < t < t_2$

$$\frac{d}{dt} v_{o_1} = -\frac{1}{R_N(C_N + C_p)} v_{o_1}$$

Oscilloscope



$$\frac{d}{dt} v = -\frac{1}{\tau} v + \frac{1}{\tau} V_{DD}$$

external input

$$\lambda = -\frac{1}{\tau}$$

Rule:

1st order – only variable in play, and only 1st derivative

– RHS has external input that is const.

$v(t_0)$ = defined

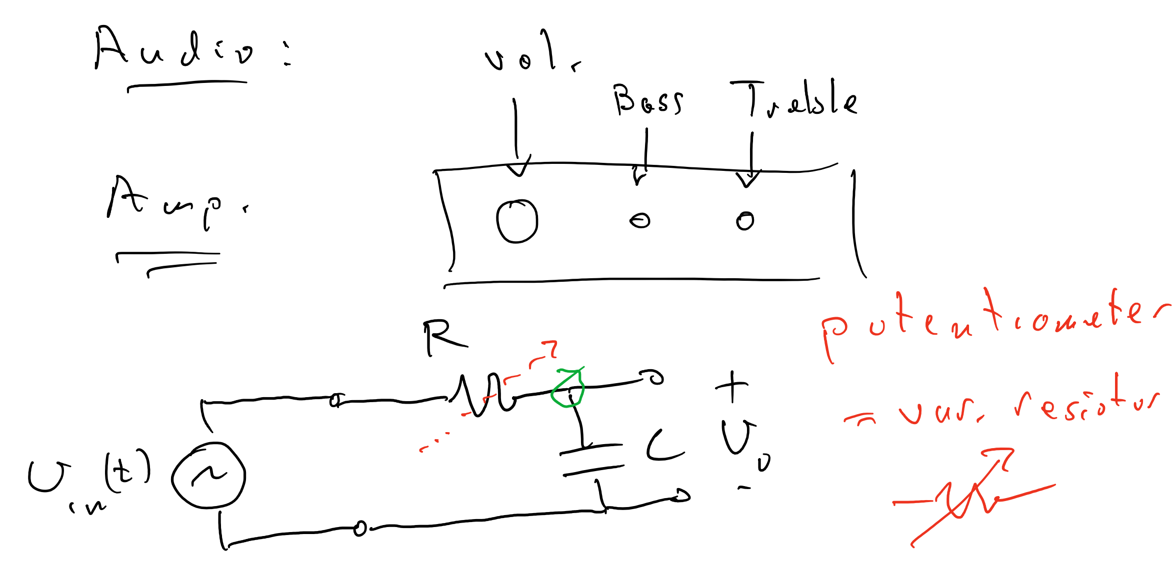
$$v = v_\infty + (v(t_0) - v_\infty) e^{-\frac{t - t_0}{\tau}}$$

evaluate: $t = t_0$

$$v(t_0) = v_\infty + [v(t_0) - v_\infty] \cdot 1$$ ✓

Audio:

Amp.



$$\frac{d}{dt} v_o = -\frac{1}{RC} v_o + \frac{1}{RC} V_{in}(t)$$

$$v_{in} = V_{in} e^{st}$$

- e^{st} is eigenfunction for input→output behavior
- $v_o(t) = V_o e^{st}$

check by plugging in:

LHS: $\frac{d}{dt} v_o = \frac{d}{dt} V_o e^{st} = s V_o e^{st}$

$$s V_o e^{st} = -\frac{1}{RC} V_o e^{st} + \frac{1}{RC} V_{in} e^{st}$$

$$s V_o + \frac{1}{RC} V_o = \frac{1}{RC} V_{in}$$

$$V_o = \frac{1}{RC} \cdot \frac{1}{s + \frac{1}{RC}} \cdot V_{in}$$

$\boxed{\lambda = -\frac{1}{RC}}$ plug in

$$V_o = \frac{1}{1 - \frac{s}{\lambda}} V_{in}$$

$$v_o(t) = V_o e^{st} = \frac{1}{1 - \frac{s}{\lambda}} V_{in} e^{st}$$

Solution for initial condition at time $t = 0$

$$v_o \Big|_{t=0} = V_1$$

Total actual solution

$$v_o(t) = A e^{-\frac{t}{RC}} + \frac{1}{RC} \frac{V_{in} e^{st}}{s + \frac{1}{RC}}$$

evaluate at $t = 0$

$$V_1 = A + \frac{1}{RC} \frac{V_{in}}{s + \frac{1}{RC}}$$

$$A = V_1 - \frac{1}{RC} \frac{V_{in}}{s + \frac{1}{RC}}$$

General Structure of Solution to Diff Eqn.

$$v(t) = v_{homogeneous}(t) + v_{particular}(t)$$

For exponential input, particular solution is also exponential with same e^{st}

"Most General" u(t)

$$\frac{d}{dt} x(t) = \lambda x(t) + u(t), \quad x(t_0) = x_0$$

Solution "guess"

$$x(t) = e^{\lambda(t - t_0)} x_0 + \int_{t_0}^{t} e^{\lambda(t - \tau)} u(\tau) \, d\tau$$

← homogeneous (first term) ← part. (integral)

Check: At $t = t_0$, what is value

1st term ⇒ x_0

2nd term ⇒ 0 ✓

$$\frac{d}{dt} x(t) = \lambda e^{\lambda(t - t_0)} x_0 + u(t) + \int_{t_0}^{t} \lambda e^{(t - \tau)} u(\tau) \, d\tau$$

input: u(t) — b.c. of integral

$\lambda x(t)$ ✓

$$v_{in}(t) = V_{in} \cos(\omega t)$$

$$v_o = V_o \cos(\omega t + \phi)$$