EECS 16B Spring 2021 — Designing Information Devices and Systems II — Discussion Worksheet — Discussion 4A

The relevant notes for this discussion is Note 3A.

1. Changing Coordinates and Systems of Differential Equations, II

In the previous discussion we analyzed and solved a pair of differential equations where the variables of interest were coupled.

$$\frac{d}{dt}z_1(t) = -5z_1(t) + 2z_2(t)$$
$$\frac{d}{dt}z_2(t) = 6z_1(t) - 6z_2(t).$$

We solved this system by using a coordinate transformation that gave us a decoupled system of equations. In the last discussion we were simply handed this transformation, but in this discussion we will construct the transformation for ourselves.

We will focus our explorations on the voltages across the capacitors in the following circuit.

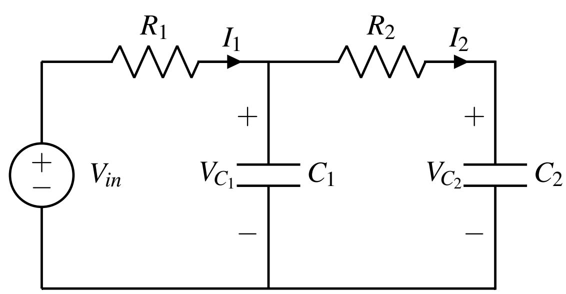

Figure 1: Two dimensional system: a circuit with two capacitors, like the one in lecture.

(a) Write the system of differential equations governing the voltages across the capacitors V_{C_1}, V_{C_2}. Use the following values: $C_1 = 1\mu\text{F}, C_2 = \frac{1}{3}\mu\text{F}, R_1 = \frac{1}{3}\text{M}\Omega, R_2 = \frac{1}{2}\text{M}\Omega$.

(b) Suppose also that V_{in} was at 7 Volts for a long time, and then transitioned to be 0 Volts at time $t = 0$. This "new" system of differential equations (valid for $t \geq 0$)

$$\frac{d}{dt} y_1(t) = -5y_1(t) + 2y_2(t) \tag{1}$$

$$\frac{d}{dt} y_2(t) = 6y_1(t) - 6y_2(t) \tag{2}$$

with initial conditions $y_1(0) = 7$ and $y_2(0) = 7$.
Write out the differential equations and initial conditions in matrix/vector form.

(c) Find the eigenvalues $\lambda_1,\ \lambda_2$ and eigenspaces for the matrix corresponding to the differential equation matrix above.

(d) Change coordinates into the eigenbasis to re-express the differential equations in terms of new variables $z_{\lambda_1}(t),\ z_{\lambda_2}(t)$. (These variables represent eigenbasis-aligned coordinates.)

(e) Solve the differential equation for $z_{\lambda_i}(t)$ in the eigenbasis.

(f) Convert your solution back into the original coordinates to find $y_i(t)$.

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