



## 2. Minimum Energy Control

In this question, we build up an understanding for how to get the minimum energy control signal to go from one state to another

(a) Consider the scalar system:

$$x[k + 1] = ax[k] + bu[k] \quad (6)$$

where  $x[0] = 0$  is the initial condition and  $u[k]$  is the control input we get to apply based on the current state. Consider if we want to reach a certain state, at a certain time, namely  $x[K]$ . Write a matrix equation for how a choice of values of  $u[k]$  for  $k \in \{0, 1, \dots, K - 1\}$  will determine the output at time  $K$ .

[Hint: write out all the inputs as a vector  $[u[0] \ u[1] \ \dots \ u[K - 2] \ u[K - 1]]^T$  and figure out the combination of  $a$  and  $b$  that gives you the state at time  $K$ .]

(b) Consider the scalar system:

$$x[k + 1] = 1.0x[k] + 0.7u[k] \quad (7)$$

where  $x[0] = 0$  is the initial condition and  $u[k]$  is the control input we get to apply based on the current state. Suppose if we want to reach a certain state, at a certain time, namely  $x[K] = 14$ . With our dynamics  $a = 1$ , solve for the best way to get to a specific state  $x[K] = 14$ , when  $K = 10$ . When we say **best way** to control a system, we want the sum squared of the inputs to be minimized

$$\operatorname{argmin}_{u[k]} \sum_{k=0}^K u[k]^2.$$

Hint: recall the Cauchy-Schwarz inequality  $\langle \vec{a}, \vec{b} \rangle \leq \|\vec{a}\| \|\vec{b}\|$  where equality holds if  $\vec{a}$  and  $\vec{b}$  are linearly dependent.

(c) Consider the scalar system:

$$x[t + 1] = 0.5x[t] + 0.7u[t] \quad (8)$$

where  $x[0] = 0$  is the initial condition and  $u[t]$  is the control input we get to apply based on the current state. Consider if we want to reach a certain state, at a certain time, namely  $x[K] = 14$ , when  $K = 10$ . Explain in words the trend of the control input that will be used to solve this problem.

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