EECS 16B Spring 2021 | Designing Information Devices and Systems II UC Berkeley

Homework 7

This homework is due on Friday, March 5, 2021, at 11:00PM. Self-grades and HW Resubmission are due on Tuesday, March 9, 2021, at 11:00PM.

1. **Reading Lecture Notes**

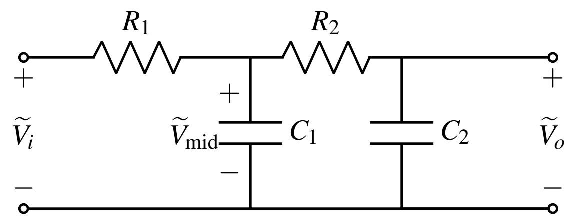
   Staying up to date with lectures is an important part of the learning process in this course. Here are links to the notes that you need to read for this week: Note 5, Note 6

   (a) In the magnitude Bode plot for a first-order-low-pass filter drawn on a log-log graph (i.e. $|H(j\omega)|$ is plotted on a log scale), what is the slope of the straight line approximation at frequencies higher than the cut-off frequency? What is the slope for a third-order-low-pass filter (three identical first-order-low-pass filters cascaded with unity gain buffers)?

   (b) In the phase Bode plot for a first-order-low-pass filter, what is the approximation for the phase at frequencies higher than 10 times the cut-off frequency? What is the approximation for the phase for a third-order-low-pass filter?

2. Transfer functions and why loading is annoying

Consider the circuit below.



The circuit has an input phasor voltage $\widetilde{V}_i$ at frequency ω rad/sec applied at the input terminals shown in the illustration above, causing an output phasor voltage $\widetilde{V}_o$ at output terminals.

(a) We are going to construct the transfer function $H(\omega) = \frac{\widetilde{V}_o}{\widetilde{V}_i}$ in two steps. We will compute two intermediate transfer functions, $H_1(\omega) = \frac{\widetilde{V}_{\text{mid}}}{\widetilde{V}_i}$ and $H_2(\omega) = \frac{\widetilde{V}_o}{\widetilde{V}_{\text{mid}}}$. Then, we will find the overall transfer function as the product of these two intermediate transfer functions, i.e. $H(\omega) = \frac{\widetilde{V}_o}{\widetilde{V}_i} = H_1(\omega)H_2(\omega)$.

This approach is valid since the $\widetilde{V}_{\text{mid}}$ cancel.

For the first step, **find the intermediate transfer function $H_2(\omega) = \frac{\widetilde{V}_o}{\widetilde{V}_{\text{mid}}}$.** Have your expression be in terms of Z_{R2} and Z_{C2}, which are the impedances of R_2 and C_2 respectively.

(b) Now, **compute the other intermediate transfer function $H_1(\omega) = \frac{\widetilde{V}_{\text{mid}}}{\widetilde{V}_i}$**. Have your expression be in terms of Z_{R1}, Z_{R2}, Z_{C1}, and Z_{C2}. (i.e. Don't forget to consider the impact of loading by R_2 and C_2 in this transfer function.) *Hint: You should try to find an expression for H_1 that has factors that H_2 can cancel out.*

(c) Then, **use these two intermediate transfer functions to calculate the overall transfer function as $H(\omega) = \frac{\widetilde{V}_o}{\widetilde{V}_i} = H_1(\omega)H_2(\omega)$.**

(d) Sometimes it is useful to collect all the frequency dependence into one place and to figure out how to think about what scale might be somewhat natural for the frequency. **Obtain an expression for $H(\omega) = \frac{\widetilde{V}_o}{\widetilde{V}_i}$ in the form**

$$H(\omega) = \frac{\widetilde{V}_o}{\widetilde{V}_i} = \frac{1}{1 + 2\xi\frac{j\omega}{\omega_c} + \frac{(j\omega)^2}{\omega_c^2}},$$

given that $R_1 = 100\,\Omega$, $R_2 = 1\,\Omega$, $C_1 = 2\,\text{F}$, and $C_2 = 2\,\text{F}$. What are the values of ξ and ω_c?

(e) **For the previous case, what is the magnitude of the transfer function at the $\omega = \omega_c$ you calculated?**

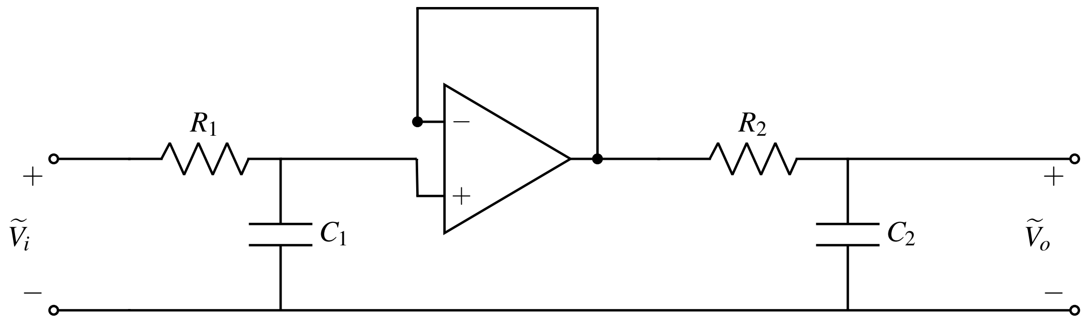
This is here so that you can see that just because we called it ω_c doesn't mean that the amplitude here is $\frac{1}{\sqrt{2}}$.

(f) We can express the transfer function $H(\omega)$ in the polar form. That is,

$$H(\omega) = M(\omega)e^{j\phi(\omega)}$$

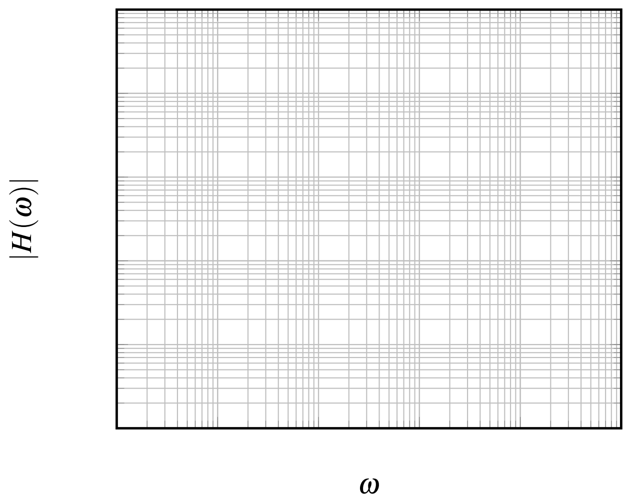
The functions $M(\omega)$ and $\phi(\omega)$ are the magnitude and the phase of $H(\omega)$, respectively. **Write down $M(\omega)$ and $\phi(\omega)$ using the transfer function you derived in part (d).**

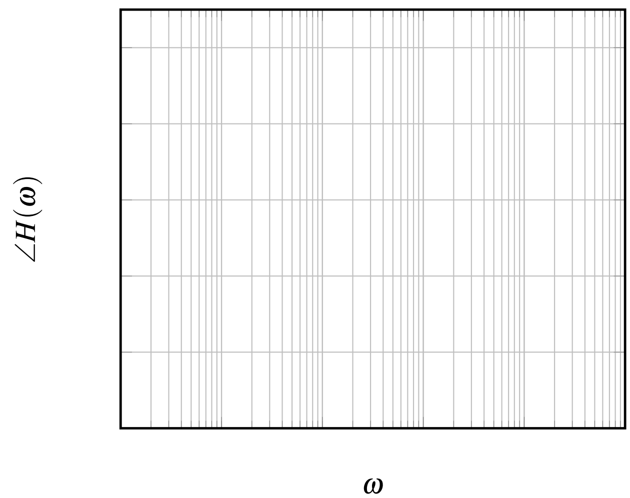
(g) Now consider the same circuit but with a unity gain buffer to avoid the loading effect, as shown below.



Derive the new transfer function $H_{\mathbf{wbuf}}(\omega) = \frac{\widetilde{V}_o}{\widetilde{V}_i}$ of the filter, using the same values of R_1, R_2, C_1 and C_2 from part (d).

(h) Use a computer and then **superpose the plots of $|H(\omega)|$ and $|H_{\mathbf{wbuf}}(\omega)|$, and the plots of $\angle H(\omega)$ and $\angle H_{\mathbf{wbuf}}(\omega)$.** Blank plots are provided here for you to use. Feel free to use the **tf_ gen.ipynb** from HW6.





3. Bandpass Filter: Lowpass and Highpass Cascade

In lecture, you heard about how you can go through the design of a bandpass filter by cascading lowpass and highpass filters via buffers (Op-Amps in unity-gain negative feedback to prevent loading effects). In this problem, you will do this for yourself.

Consider an input signal that is composed of the superposition of:

- 20mV level pure tone at 60Hz corresponding to power line noise.
- 1mV level pure tone at 600Hz corresponding to a voice signal.
- 10mV level pure tone at 60kHz corresponding to fluorescent light noise.

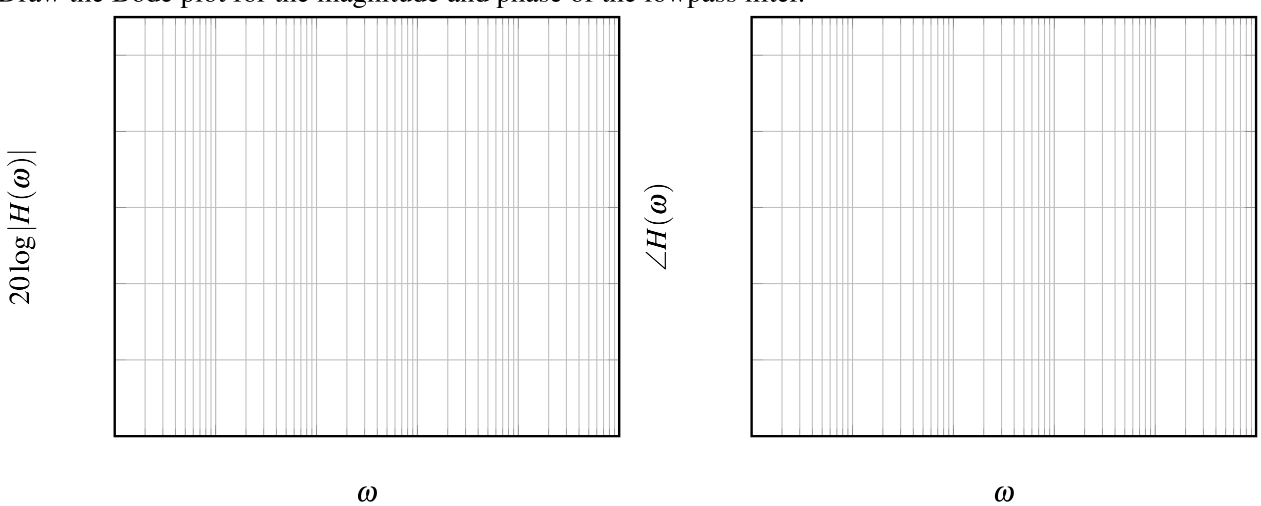
(a) We would like to keep the 600Hz tone, which could correspond to a voice signal.
Ignoring any phase offset for each signal (i.e. set the phases to zero), **write the $V_{\text{in}}(t)$ that describes the above input in time domain, in the following format:**.

$$V_{\text{in}}(t) = A_p \cos\left(2\pi \cdot f_{\text{p}} t\right) + A_v \cos(2\pi \cdot f_{\text{v}} t) + A_f \cos(2\pi \cdot f_{\text{f}} t)$$

(b) **What are the angular frequencies (i.e. $\omega_{\text{p}}, \omega_{\text{v}}, \omega_{\text{f}}$) involved and the phasors associated with each tone?** Remember that the frequencies of the tones were provided in Hz. To convert this to angular frequency we use $\omega = 2\pi f$

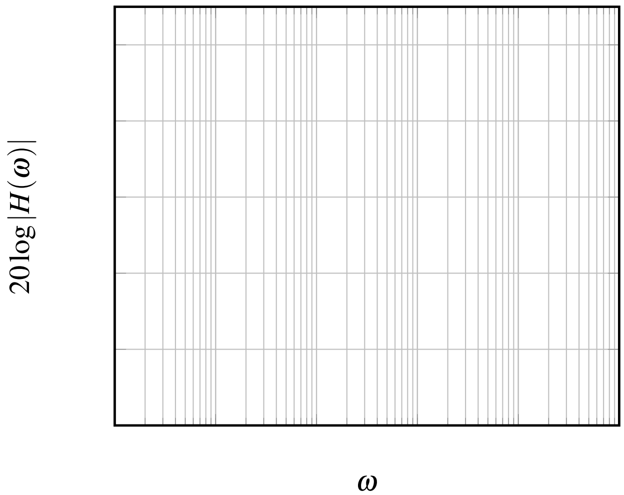
(c) To achieve your goal of keeping the voice tone but rejecting the noise from the power-lines and fluorescent lights, **at what frequency do you want to have the knee or cutoff-frequency for the lowpass filters**? *Hint: To arrive at a unique solution consider computing the the geometric mean of the two frequencies of interest.*

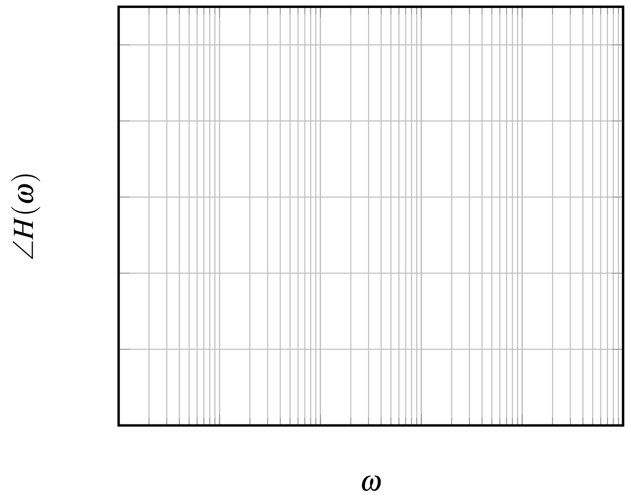
(d) Draw the Bode plot for the magnitude and phase of the lowpass filter.



(e) To achieve your goal of keeping the voice tone but rejecting the noise from the power-lines and fluorescent lights, **at what frequency do you want to have the knee or cutoff-frequency for the highpass filters**? *Hint: To arrive at a unique solution consider computing the the geometric mean of the two frequencies of interest.*

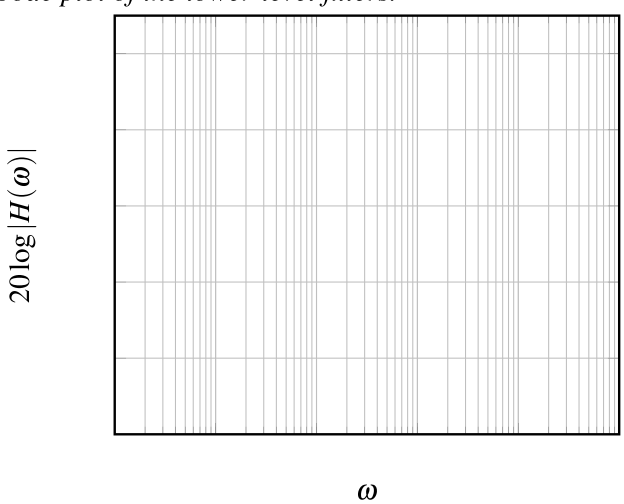
(f) Draw the Bode plot for the magnitude and phase of the highpass filter.

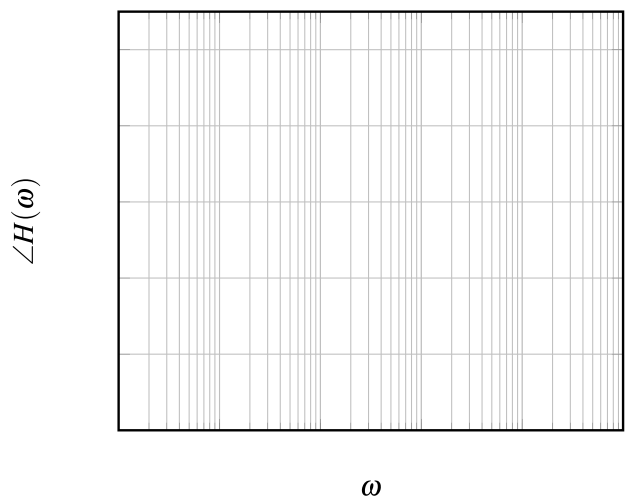




(g) For the following questions assume your cut-off frequency for lowpass and highpass are $6KHz$ and $189Hz$, respectively. Suppose that you only had 1μF capacitors to use. **What resistance values would you choose for your highpass and lowpass filters so that they have the desired cutoff frequencies?**

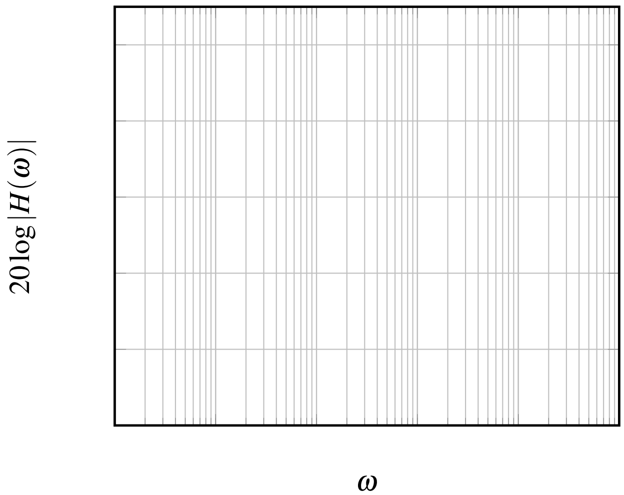
(h) The overall bandpass filter that is created by cascading the lowpass and highpass with ideal buffers in between. Draw the Bode plot for the magnitude and phase of the overall bandpass transfer function. *Hint: You should think about how the bode plot of cascade of two filters can be derived based on the bode plot of the lower-level filters.*

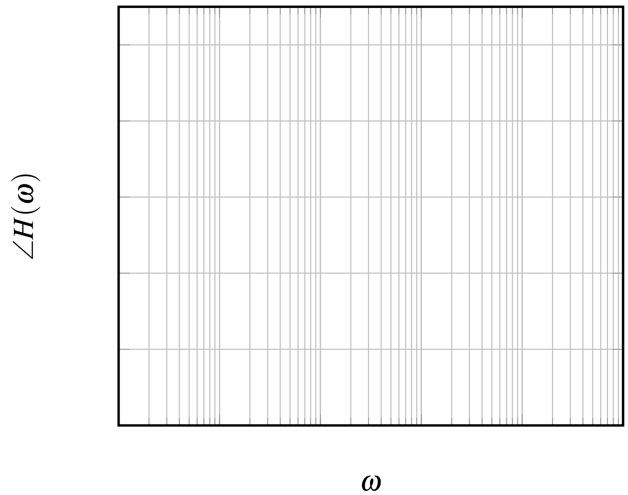




(i) Suppose that the band-pass filter does not have enough suppression at 60Hz and 60kHz and you decide to use cascade of three band-pass filters (with unity-gain buffers in between) to mitigate the issue. **What are the phasors for each of the frequency tones after all three bandpass filters?** Feel free to use a computer to help you evaluate both the magnitudes and the phases here.

(j) **Draw the Bode plot for the magnitude and phase of the 3rd order bandpass filter.** To highlight the difference between the 3rd and 1st order filters, please draw both Bode plots on a single figure.





(k) **Write the final time domain voltage waveform that would be present after the filter.**

(l) The included jupyter notebook sets up the same problem described above. In the notebook, you can use the slider bars to play around with:

- highpass cutoff frequency (i.e. the knee frequency of the highpass filters)
- lowpass cutoff frequency (i.e. the knee frequency of the lowpass filters)
- Filter order N. Filter order means the number of lowpass filters and highpass filters that are used in a row. Here, N means that there are N lowpass filters and N highpass filters, so the overall order of the entire filter is actually $2N$.

The notebook will plot the magnitude and phase, the input voltage waveform, and the output waveform at the end of the filter.

Play around with the values for the highpass and lowpass cutoff frequencies, and N.

Observe the waveforms at the output of the filter. **Comment on the limits of f_{lowpass}, f_{highpass}, and N that you can use to successfully isolate the desired 600Hz tone. What happens if you keep f_{lowpass} and f_{highpass} constant, and just increase N?**

4. Circuit Design

In this problem, you will find a circuit where several components have been left *blank* for you to fill in.

Assume that the op-amp is *ideal*. *A special note on op amps in frequency domain analysis: The op-amps you learned about in 16A can be used in exactly the same way for setting up differential equations and even Phasor analysis in 16B. Treat them as ideal op-amps and invoke the Golden Rules.*

You have at your disposal *only one of each* of the following components (not including R_1):

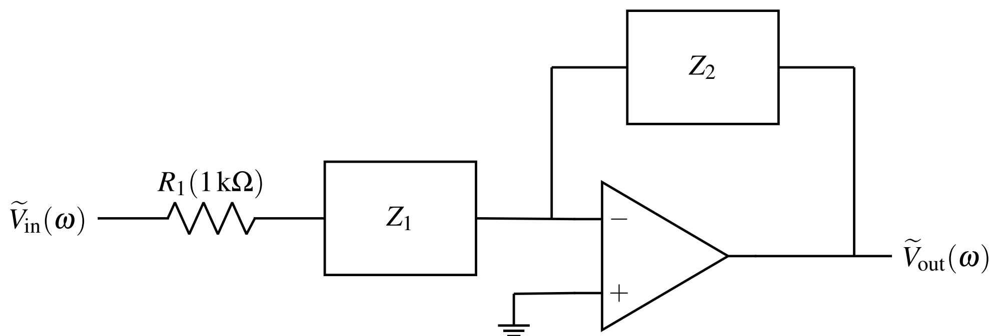
(a) an open circuit

(b) a short circuit

(c) a resistor (you choose from the values $R = 1\,\text{k}\Omega, 10\,\text{k}\Omega, 20\,\text{k}\Omega$)

(d) a capacitor (you choose from the values $C = 0.5\,\mu\text{F}, 1\,\mu\text{F}, 2\,\mu\text{F}$)

Consider the circuit below. The voltage source $V_{\text{in}}(t)$ has the form $V_{\text{in}}(t) = v_0 \cos(\omega t + \phi)$. The labeled voltages $\widetilde{V}_{\text{in}}(\omega)$ and $\widetilde{V}_{\text{out}}(\omega)$ are the phasor representations of $v_{\text{in}}(t)$ and $v_{\text{out}}(t)$. The transfer function $H(\omega)$ is defined as $H(\omega) = \frac{\widetilde{V}_{\text{out}}(\omega)}{\widetilde{V}_{\text{in}}(\omega)}$.



(a) Let the left box be Z_1 and the right box be Z_2. **Write the expression of the transfer function $H(\omega)$.**

(b) Let R_1 be $1\,\text{k}\Omega$. We have to find Z_1 and Z_2, such that the circuit's transfer function $H(\omega)$ has the following properties:

- It is a high-pass filter.
- $|H(\infty)| = 10$.
- $|H(10^3)| = \sqrt{50}$.

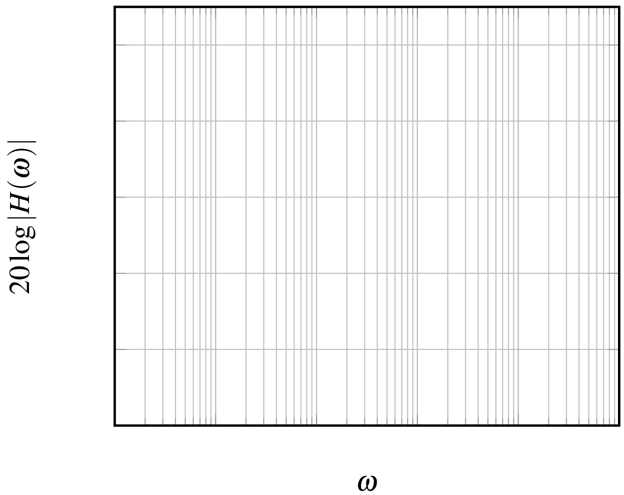
Using the fact that the circuit is a high pass filter, infer the components (we will find values later) of Z_1 and Z_2. Write the transfer function $H(\omega)$ using these components.

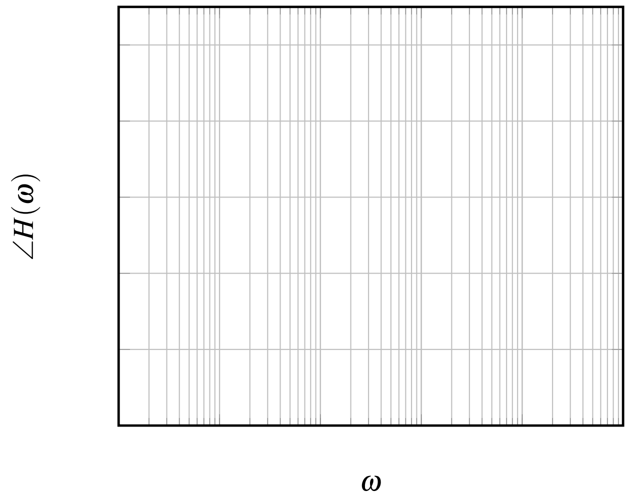
Hint: Try method of elimination: figure out what Z_2 cannot be. Once you find what Z_2 is, what does Z_1 have to be for the circuit to be a filter?

(c) Now use the facts that $|H(\infty)| = 10$ and $R_1 = 1\,\text{k}\Omega$ to find the component value of Z_2.

(d) Finally use the fact that $|H(10^3)| = \sqrt{50}$ and the values of R_1 and Z_2 to find the component value of Z_1.

(e) **Draw the magnitude and phase Bode plots of this transfer function.** Blank plots are provided here for you to use.





5. Homework Process and Study Group

Citing sources and collaborators are an important part of life, including being a student!
We also want to understand what resources you find helpful and how much time homework is taking, so we can change things in the future if possible.

(a) **What sources (if any) did you use as you worked through the homework?**

(b) **If you worked with someone on this homework, who did you work with?**
List names and student ID's. (In case of homework party, you can also just describe the group.)

Contributors:

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