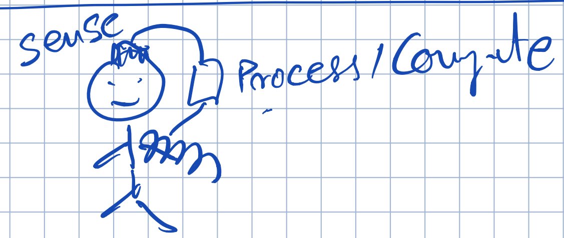
EECS 16A Review
5pm Mon
Prof. Ranade

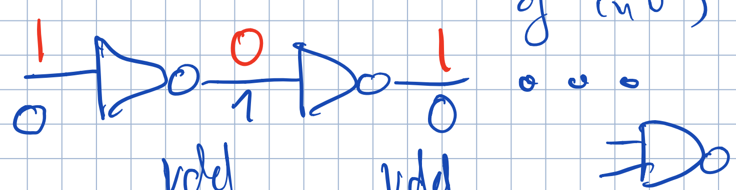
Lecture 4

EECS 16B

* Recap
* Filter response to $\cos \omega t$
* Start 2^{nd} order diff eqns (L-C filters)

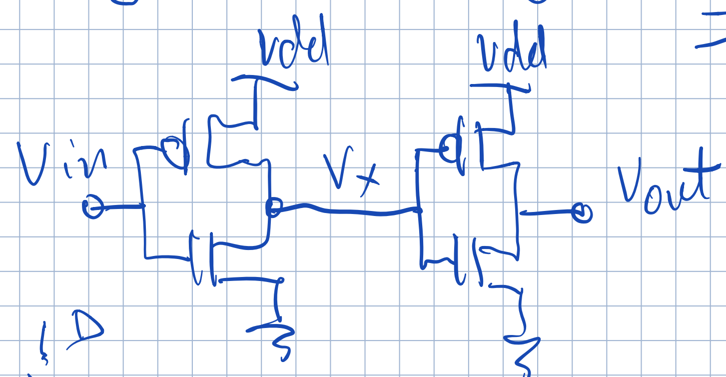


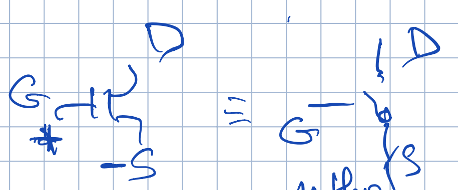
Computing: Simple model (Cascade of inv)



"1" = Vdd

"0" = 0V

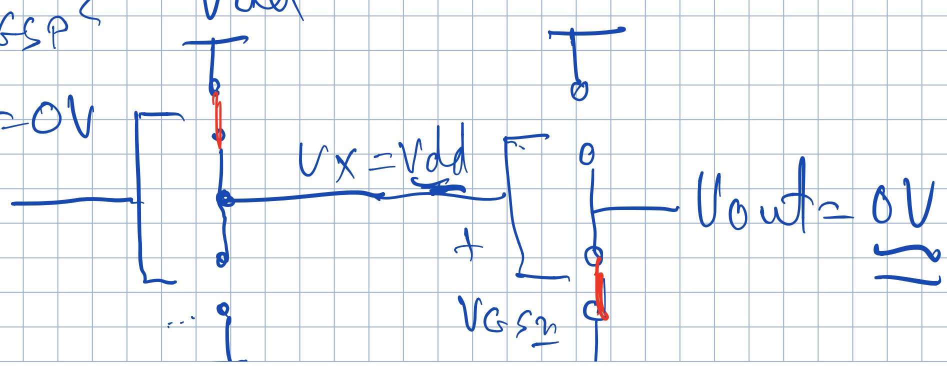


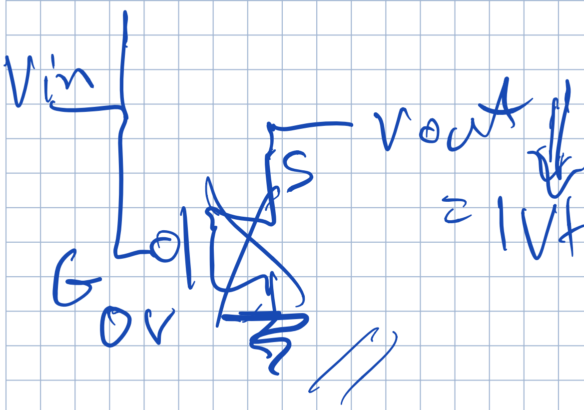


$V_{GS} \geq V_{thn}$ → "on"

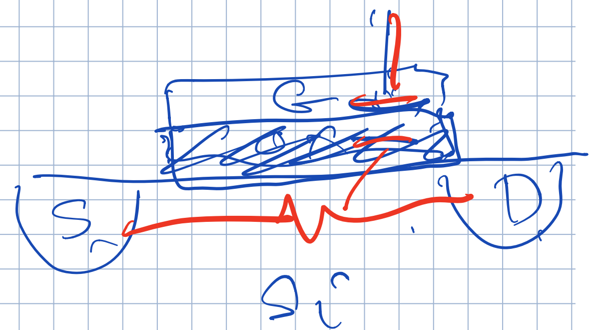
$V_{GSP} < -|V_{thp}|$

Vin = 0V

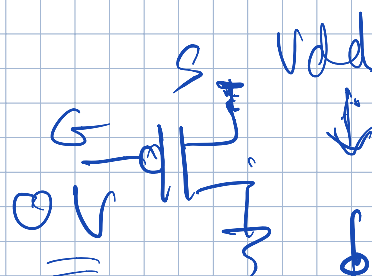




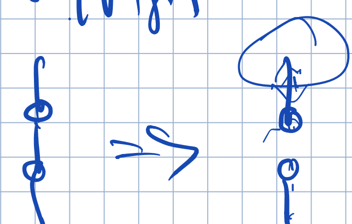
$\approx |V_{tp}|$



$$\underbrace{V_{GSp} \le -|V_{tp}|}$$



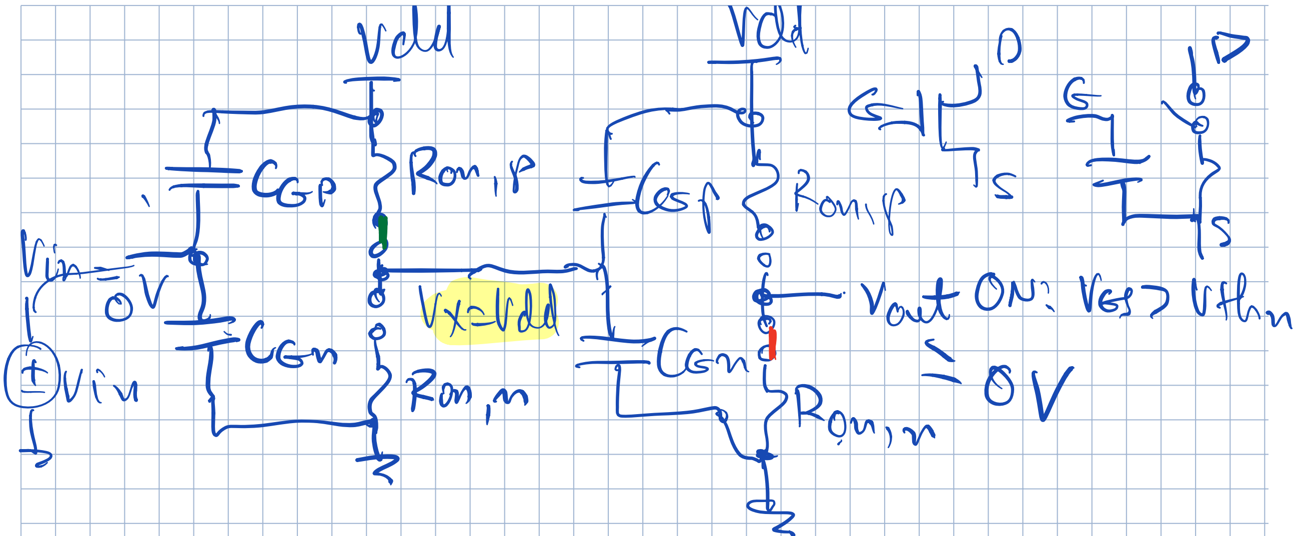
$V_{GSp} = -V_{dd} \le -|V_{tp}|$

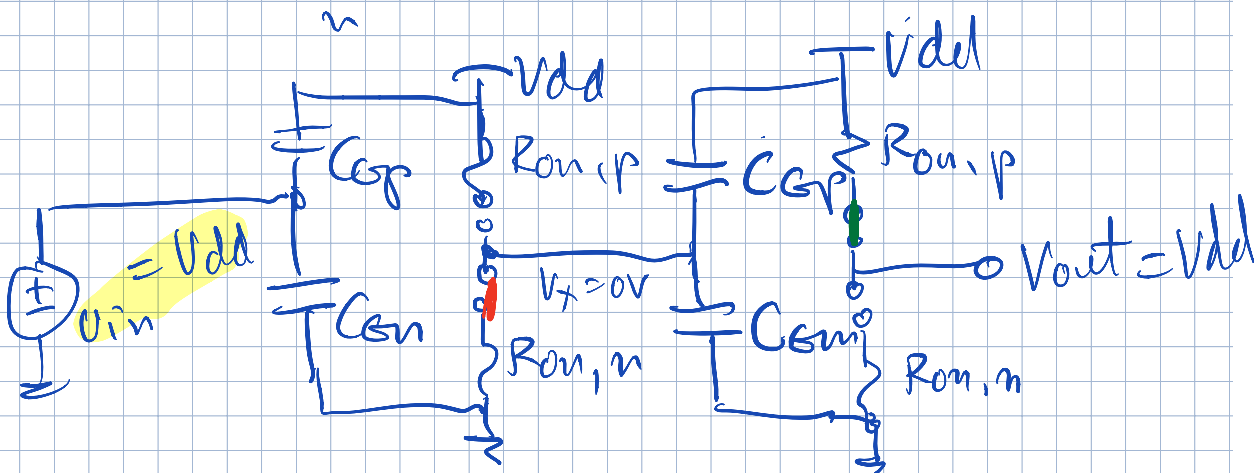


$0 < V_{tn}, |V_{tp}| < V_{dd}$

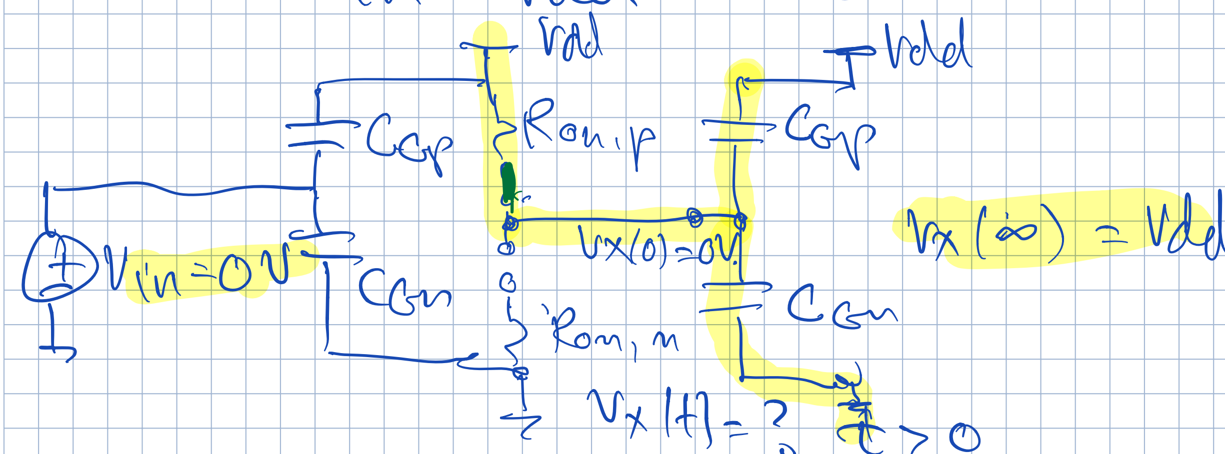
$\underline{0.2\text{-}0.3\,V_{dd}}$

How fast can compute work?
Need a better model.





t=0 Vin : Vdd → 0 V

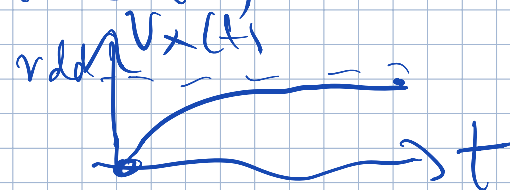


solve to get V(x)

Saw:
$$\boxed{\frac{dV_x}{dt} = -\frac{V_x}{\underbrace{R_{on,p}(C_{Gn}+C_{Gp})}_{\tau}} + \frac{V_{dd}}{R_{on,p}(C_{Gn}+C_{Gp})}}$$

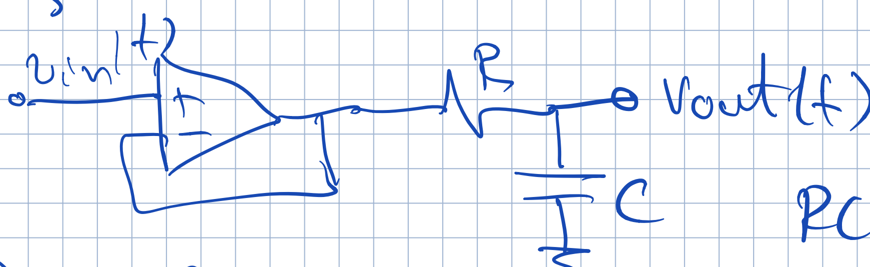
used a change of variables to solve: $\tilde{V}_x = V_x - V_{dd}$ (b/c $\frac{d}{dt}V_{dd}=0$)

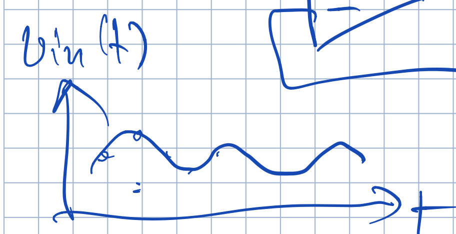
$$V_x(t) = V_{dd}\left(1-e^{-\frac{t}{\tau}}\right)$$



Any RC circuit can be solved like this — not just logic.

Want to design a ckts filter to remove unwanted signals.

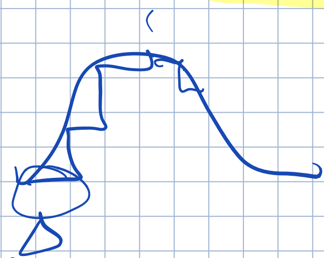




RC circuit again!

solving: $\boxed{\frac{d}{dt} V_{out}(t) = \frac{-V_{out}(t)}{RC} + \frac{V_{in}(t)}{RC}}$

contin.



solution:

$$V_{out}(t) = \underbrace{V_{in}(0) \cdot e^{-\frac{t}{RC}}}_{\text{homogeneous (response to init. condition)}} + \underbrace{\frac{1}{RC} \int_0^t V_{in}(\theta) \cdot e^{-\frac{1}{RC}(t-\theta)} \, d\theta}_{\text{response to input}}$$

Let's see how this responds to different input signals:

e.g. $V_{in}(t) = RC \cos(\omega t)$?

To simplify: $u(t) = \cos \omega t$

$$\boxed{\frac{d}{dt} x(t) = \lambda x(t) + u(t)}$$

why pick a $\cos \omega t$?

AC power, RF (cell-phone $\sim$ $\cos \omega t$

$$x(t) = x(0)\cdot e^{\lambda t} + \int_0^t \underbrace{\cos(\omega\theta)}_{u(\theta)} \cdot e^{\lambda(t-\theta)} d\theta$$

$$= x(0)e^{\lambda t} + e^{\lambda t} \cdot \int_0^t \cos(\omega\theta) \cdot e^{-\lambda\theta} d\theta$$

$$\boxed{\int \cos bx \, e^{ax} dx = \frac{e^{ax}}{a^2+b^2}(b \sin bx + a \cos bx)}$$

$$x(t) = x(0)\cdot e^{\lambda t} + \frac{\lambda e^{\lambda t}}{\lambda^2+\omega^2} + \frac{\omega \sin(\omega t) - \lambda\cos(\omega t)}{\lambda^2+\omega^2}$$

① ② ③

$t \to \infty$ (steady-state)

① $\to 0$ b/c $\lambda = -\frac{1}{RC}$

② $\to 0$ — " —

3 $\quad \dfrac{\omega \sin(\omega t) - \lambda \cos(\omega t)}{\lambda^2 + \omega^2}$

$\lambda = -\dfrac{1}{RC}$

$$x(t) = \frac{\omega \sin(\omega t) + \frac{1}{RC}\cos(\omega t)}{\left(\frac{1}{RC}\right)^2 + \omega^2} =$$

$$= \boxed{\frac{RC\,(RC\omega \sin(\omega t) + \cos(\omega t))}{1 + (RC)^2\omega^2}}$$

1 $\quad \omega \gg \dfrac{1}{RC} \Rightarrow \omega RC \gg 1$

$x(t) \approx 0$

2 $\quad \omega \ll \dfrac{1}{RC} \Rightarrow \omega RC \ll 1$

$x(t) \simeq RC \cos(\omega t + \theta)$

$$\theta = -\tan^{-1}(\omega RC)$$

From 1 and 2

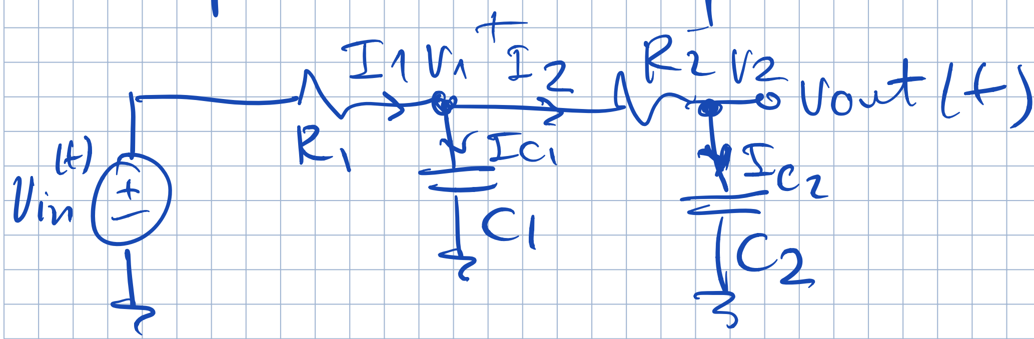
we see that the ckt is

a "low-pass" filter

since it passes frequencies

lower than $\frac{1}{RC}$.

Simplest two-capacitor example:



$\Rightarrow V_{out} = V_1 - I_2 \cdot R_2$

$I_{C_2} = C\dfrac{dV_2}{dt}$, $I_2 = I_{C_2}$

$I_1 = I_2 + I_{C_1} = C_2\dfrac{dV_2}{dt} + C_1\dfrac{dV_1}{dt}$

$V_{in} - I_1 R_1 = V_1$

① $\dfrac{V_{in} - V_1}{R_1} = C_2\dfrac{dV_2}{dt} + C_1\dfrac{dV_1}{dt}$

② $\dfrac{V_1 - V_2}{R_2} = C_2\dfrac{dV_2}{dt} \Rightarrow$

$\Rightarrow V_1 = V_2 + R_2C_2\dfrac{dV_2}{dt}$

$$R_1 C_1 R_2 C_2 \frac{d^2 V_2}{dt} + (R_1 C_1 + R_1 C_2 + R_2 C_2) \frac{dV_2}{dt}$$

$$+ V_2 - V_{in} = 0$$

(2nd order diff. eqn).

$$\frac{V_{in} - V_1}{R_1} = C_1 \frac{dV_1}{dt} + \frac{V_1 - V_2}{R_2}$$

$$\frac{V_1 - V_2}{R_2} = C_2 \frac{dV_2}{dt}$$

$$\frac{dV_1}{dt} = -\left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1}\right) V_1 + \frac{V_2}{R_2 C_1} + \frac{V_{in}}{R_1 C_1}$$

$$\frac{dV_2}{dt} = \frac{1}{R_2 C_2} V_1 - \frac{1}{R_2 C_2} V_2$$