EECS 16B Designing Information Devices and Systems II
Spring 2020 UC Berkeley Midterm 1

PRINT your student ID: ____

PRINT AND SIGN your name: __________________, __________________ ______________________

(last) (first) (signature)

PRINT your discussion section and GSI (the one you attend): ____

Exam location: ____

Row Number (front row is 1): __________ Seat Number (left most is 1): __________

Name and SID of the person to your left: ____

Name and SID of the person to your right: ____

Name and SID of the person in front of you: ____

Name and SID of the person behind you: ____

- You have 120 minutes to complete the exam.
- There are seven problems, scored out of a total of 360 points.
- Closed book, except for ONE double-sided 8.5" x 11" cheat sheet.
- No electronics of any form.
- Show all of your work to receive full or partial credit on each problem.
- **No collaboration is allowed at all, and you are not allowed to look at another person's work.**
- **Please write your name and SID on every page of the exam.**
- **There are 14 pages in this exam, printed double-sided. Notify a proctor immediately if a page is missing.**

Do not turn this page until the proctor tells you to do so.

PRINT your name and student ID: ____

1. CMOS Circuits (40 pts)

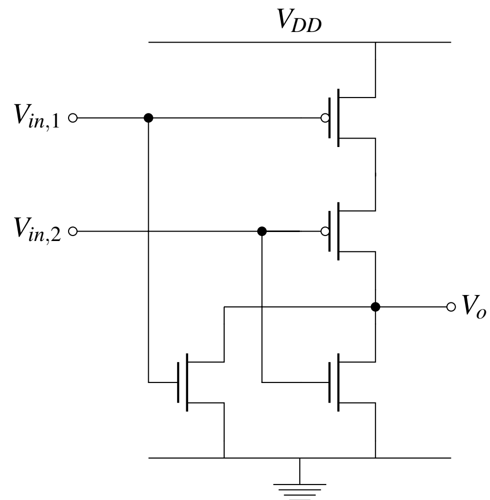


Figure 1: CMOS circuit

Consider the CMOS circuit of Figure 1. For each of the sets of $V_{in,1}$ and $V_{in,2}$ in the table below, fill in the corresponding voltage of the output V_o. You may assume that the threshold voltages for the transistors are $0 < V_{tn} < V_{DD}$ and $0 < |V_{tp}| < V_{DD}$.

$V_{in,1}$	$V_{in,2}$	V_o
0 V	0 V	
V_{DD}	0 V	
0 V	V_{DD}	
V_{DD}	V_{DD}	

PRINT your name and student ID: ______________________________

2. Differential equations (40 pts)

Consider a certain radioactive isotope sample with an initial mass, M_0. It is observed that the mass of radioactive sample decays with time. The rate of mass decay at time t is proportional to the mass $M(t)$ at that moment. The constant of proportionality is the decay rate constant, $r > 0$. The rate of mass decay is given by

$$\frac{dM(t)}{dt} = -rM(t). \tag{1}$$

(a) (20 pts) **Solve the differential equation (1) to determine the mass of the sample at a given time.**

$M(t) =$

(b) (20 pts) **Find the half life $t_{\frac{1}{2}}$ of the sample as a function of r. Note that half life is the time when half of the initial mass has decayed.**

$t_{\frac{1}{2}} =$

PRINT your name and student ID: ____________________

3. Complex Numbers (60 pts)

(a) (20 pts) **Write the following numbers in polar form.** That is, write the numbers as $Ae^{j\theta}$ for some real numbers A and θ with $A \geq 0$ and $-\pi \leq \theta \leq \pi$.

i. $1+j$

$1+j =$

ii. $\sqrt{j}$

$\sqrt{j} =$

(b) (20 pts) **Write the following numbers in rectangular form.** That is, write each number as $a+bj$ where a and b are real numbers. HINT: $e^{j\theta} = \cos\theta + j\sin\theta$.

i. $3e^{j\frac{\pi}{3}}$

$3e^{j\frac{\pi}{3}} =$

ii. $-\sqrt{7}e^{\pi j}$

$-\sqrt{7}e^{\pi j} =$

PRINT your name and student ID: ______________________________

(c) (20 pts) **Prove the following identities.**

i. $\frac{1}{j} = -j$

ii. $\sin(2x) = 2\cos x\ \sin x$.

PRINT your name and student ID: ____________________

4. Vector differential equations (60 pts)

For this problem, x satisfies the following differential equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{x} = A\vec{x}; \quad A = \begin{bmatrix} \alpha & -\omega \\ \omega & \alpha \end{bmatrix},$$

where α and ω are real. Take initial condition $\vec{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

(a) (30 pts) Using the following two eigenvectors of A:

$$v_1 = \begin{bmatrix} j \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -j \\ 1 \end{bmatrix},$$

determine a matrix T and a diagonal matrix D such that

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{z} = Dz, \quad \text{where } \vec{z} = T\vec{x}.$$

You may use the fact that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Write your answers in the boxes below.

$T =$

$D =$

PRINT your name and student ID: ____

(*extra space for (a)*)

(b) (15 pts) Solve for $\vec{z}$ as a function of t. Your solution may not include unknowns other than α and ω.

PRINT your name and student ID: ____

(c) (15 pts) Solve for x_1, the first component of $\vec{x}$, as a function of t. If possible, state your solution without complex numbers.

PRINT your name and student ID: ____

5. Transient identification (40 pts)

For each of following scalar functions of time,

- If the function may describe a voltage transient response in one of the following circuit types we studied in class: RC, RL, RCRC, overdamped RLC, underdamped RLC; write all that apply.
- Otherwise, write "None."

Assume $V_0 \neq 0$, $V_1 \neq 0$, $V_2 \neq 0$ and $\omega > 0$.

(a) (10 pts) $v(t) = V_0 e^{\lambda t}$, $\lambda < 0$

(b) (10 pts) $v(t) = V_0 e^{jt}$

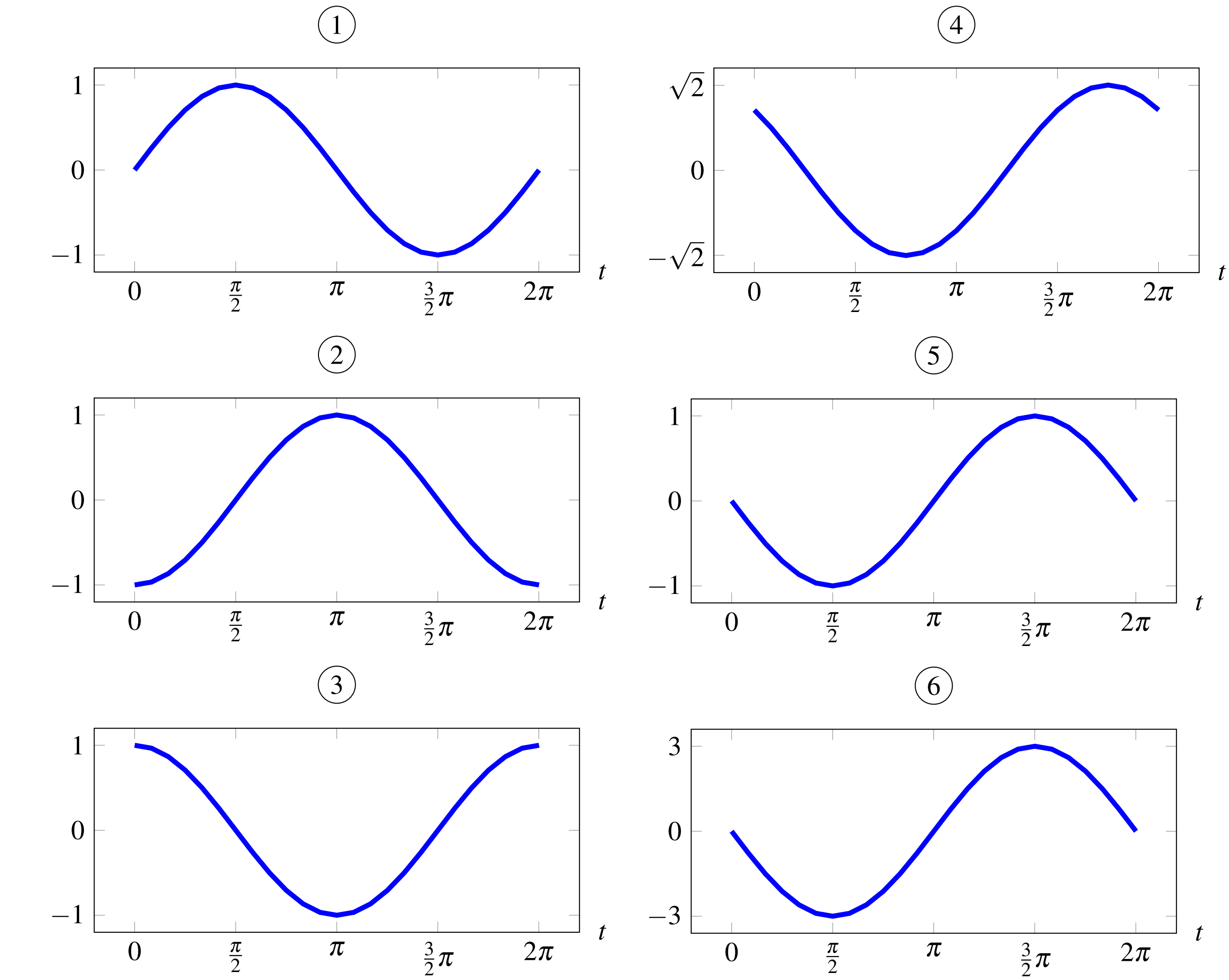
(c) (10 pts) $v(t) = V_0 e^{\alpha t} \cos \omega t$, $\alpha < 0$

(d) (10 pts) $v(t) = V_1 e^{\alpha t} + V_2 e^{\beta t}$, $\alpha < 0$, $\beta < 0$

PRINT your name and student ID: ______________________________

6. Phasors (60 pts)

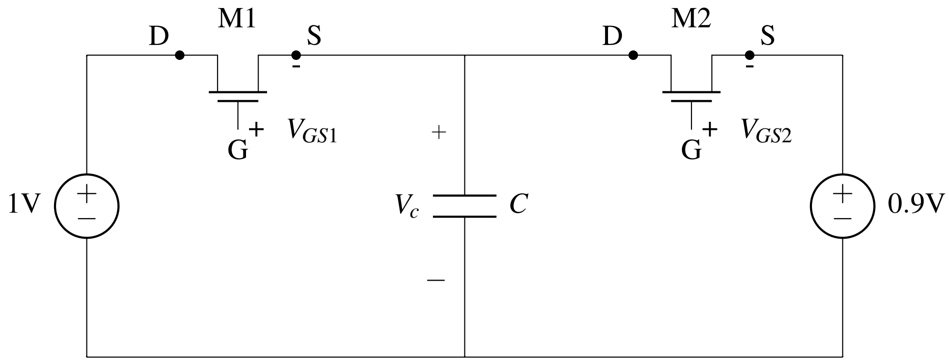
Match each phasor with its corresponding time domain waveform that is shown below. For this problem $\omega = 1$.

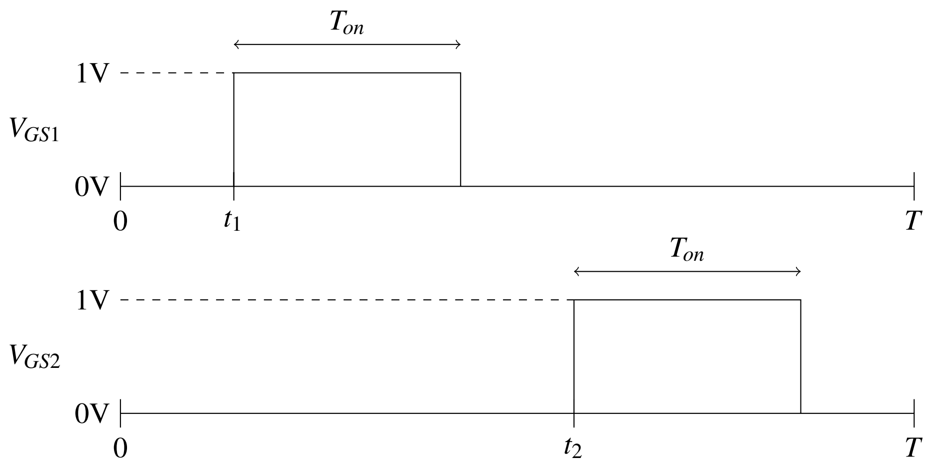


	Phasor	Waveform
(a)	$e^{j\frac{\pi}{2}}$	
(b)	$3e^{j\frac{\pi}{2}}$	
(c)	1	
(d)	$e^{-j\frac{\pi}{2}}$	
(e)	$e^{-j\pi}$	
(f)	$e^{j0} + e^{j\frac{\pi}{2}}$	

PRINT your name and student ID: ____

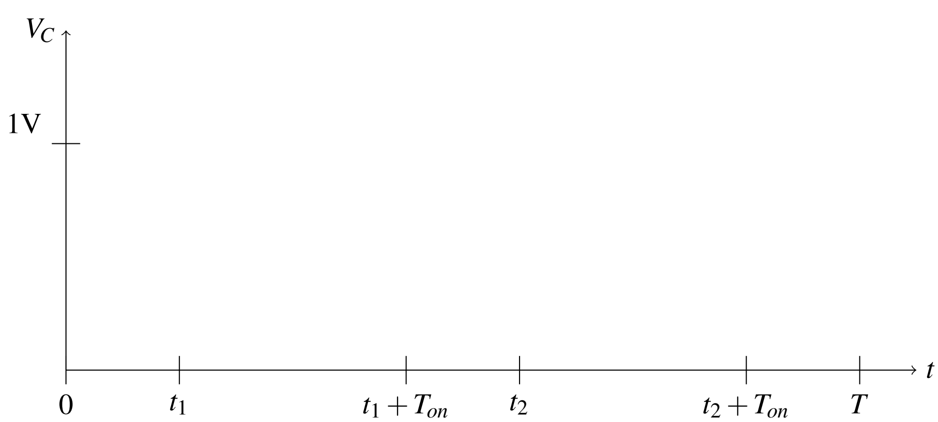
7. Energy (60 pts)





In the figure above, NMOS devices M1 and M2 have threshold voltages of 0.2V, negligible gate-source capacitances, and ON state resistances of value R. Capacitor C has capacitance such that $RC << T_{on}$. The associated timing diagram shows that V_{GS1} rises at t_1, and V_{GS2} rises at t_2.

(a) (20 pts) Take $V_C(0) = 0.9\text{V}$. On the axes below, sketch the waveform corresponding to $V_C(t)$ for $0 < t < T$. You may make approximations as informed by learnings in class to date.



PRINT your name and student ID: ____

(b) (20 pts) Determine how much charge flows out of the 1.0V source over the period from 0 to T. You may make reasonable approximations to simplify the analysis, as done in the making of the sketch of part (a).

(c) (20 pts) Determine how much energy is dissipated in the circuit over the period from 0 to T. Again, you may make reasonable approximations to simplify your analysis.

PRINT your name and student ID: ____

[Extra page. If you want the work on this page to be graded, make sure you tell us on the problem's main page.]

PRINT your name and student ID: ____

[Doodle page! Draw us something if you want or give us suggestions or complaints. You can also use this page to report anything suspicious that you might have noticed.]

