__To do__ : Differential Equations & Complex #s Email : nareauphol.lin @

① Intro to diff. eqs

② Complex #s

__Recall last time__ : $\frac{dV_c(t)}{dt} = \frac{V(t) - V_c(t)}{RC}$

① $\frac{d}{dt}x(t) = \lambda x(t) \rightarrow$ solution $x(t) = \underline{x(0)}e^{\lambda t}$

initial condition

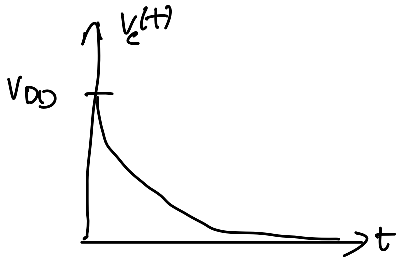
e.g. $\frac{d}{dt}x(t) = 5x(t) \rightarrow x(t) = x(0)e^{5t}$

__Q1__

(a) $V(t) = 0$, $\frac{dV_c(t)}{dt} = \frac{-V_c(t)}{RC}$ $\lambda = \frac{-1}{RC}$

$V_c(t) = V_c(0)e^{-t/RC}$

$\therefore V_c(t) = V_{DD}e^{-t/RC}$



(b) $V(t) = V_{DD}$, $V_c(0) = 0$

extra constant

$\frac{dV_c(t)}{dt} = \frac{V_{DD} - V_c(t)}{RC} = \frac{V_{DD}}{RC} - \frac{V_c(t)}{RC}$

__Goal__ : Reduce into a form that looks like $\frac{d}{dt}x(t) = \lambda x(t)$

__Method__ : Change of variables

let $q(t) = V_{DD} - V_c(t)$, $V_c(t) = V_{DD} - q(t)$

$\frac{d}{dt}V_c(t) = \frac{q(t)}{RC}$

$\frac{d}{dt}[V_{DD} - q(t)] = \frac{q(t)}{RC} \rightarrow \overset{0}{\cancel{\frac{d}{dt}V_{DD}}} - \frac{d}{dt}q(t) = \frac{q(t)}{RC}$

$\frac{d}{dt}q(t) = \frac{-q(t)}{RC}$

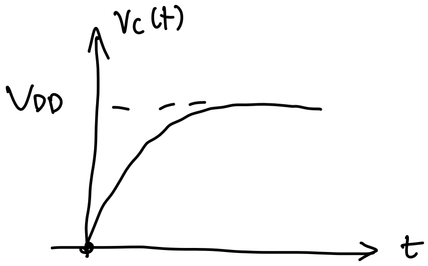
$q(0) = V_{DD} - V_c(0)$
$= V_{DD} - 0 = V_{DD}$

$$\therefore q(t) = \underline{q(0)}\, e^{-t/RC}$$

$$\therefore q(t) = V_{DD}\, e^{-t/RC} \longrightarrow V_{DD}\, e^{-t/RC} = V_{DD} - V_c(t)$$

$$\therefore \underline{V_c(t) = V_{DD} - V_{DD}\, e^{-t/RC}}$$

$$= V_{DD}(1 - e^{-t/RC})$$



constant

② $\frac{d}{dt}x(t) = \lambda x(t) + u$

Practice: Solve ↑ for $x(t)$ using change of variables

Hint: Factor out the λ

$$\therefore \frac{d}{dt}x(t) = \lambda\left(x(t) + \frac{u}{\lambda}\right)$$

let $q(t) = x(t) + \frac{u}{\lambda} \longrightarrow x(t) = q(t) - \frac{u}{\lambda}$

$q(0) = x(0) + \frac{u}{\lambda}$

$$\therefore \frac{d}{dt}x(t) = \lambda q(t)$$

$$\frac{d}{dt}\left[q(t) - \frac{u}{\lambda}\right] = \lambda q(t)$$

$$\frac{d}{dt}q(t) = \lambda q(t)$$

$$\therefore q(t) = \underline{q(0)}\, e^{\lambda t}$$

$$q(t) = \left[x(0) + \frac{u}{\lambda}\right] e^{\lambda t}$$

$$\therefore \left[x(0) + \frac{u}{\lambda}\right] e^{\lambda t} = x(t) + \frac{u}{\lambda}$$

$$\therefore x(t) = \left[x(0) + \frac{u}{\lambda}\right] e^{\lambda t} - \frac{u}{\lambda}$$

$$\therefore \frac{d}{dt}x(t) = \lambda x(t) + u \Rightarrow x(t) = \left(x(0) + \frac{u}{\lambda}\right) e^{\lambda t} - \frac{u}{\lambda}$$

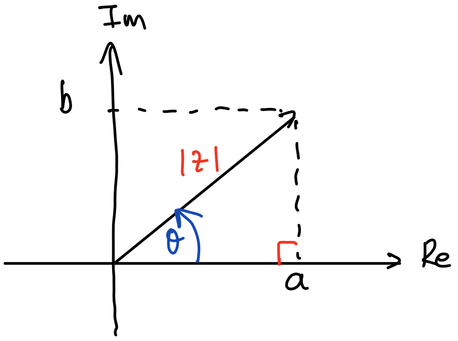
$$\frac{d}{dt}V_c(t) = \underbrace{\frac{V_{DD}}{RC}}_{u} - \underbrace{\boxed{\frac{V_c(t)}{RC}}}_{\lambda = -\frac{1}{RC}}$$

(C) If time I'll get back to it

Complex #s : We define a complex # to be $z = a + bj$ [Rectangular Form]

real part

imaginary part



· Alternatively, we can represent complex #s in polar form

↳ Magnitude : magnitude of the vector $|z| = \sqrt{a^2+b^2}$

↳ Phase : $\tan(\theta) = \frac{b}{a}$

$\theta = \arctan\left(\frac{b}{a}\right)$

Tip : If you're struggling, draw out the vector

$z = |z| \cdot e^{j\theta}$ [polar form]

Q2

(a) $j, -j, (j)^{1/2}, (-j)^{1/2}$

↓

$z = a + bj$

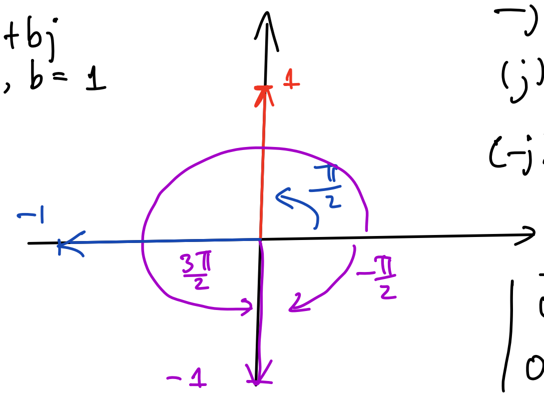
$a = 0, \; b = 1$

$j \rightarrow 1 \cdot e^{j\frac{\pi}{2}} = e^{j\frac{\pi}{2}}$

$-j \rightarrow 1 \cdot e^{-j\frac{\pi}{2}} = e^{-j\frac{\pi}{2}}$

$(j)^{1/2} \rightarrow [e^{j\frac{\pi}{2}}]^{1/2} = e^{j\frac{\pi}{4}}$

$(-j)^{1/2} \rightarrow (e^{-j\frac{\pi}{2}})^{1/2} = e^{-j\frac{\pi}{4}}$



0, 30, 45, 60, 90

$0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$

$-1 : 1 \cdot e^{j\pi} = e^{j\pi}$

(b) $e^{j\theta} = \cos\theta + j\sin\theta$

$(j)^{1/2} = e^{j\frac{\pi}{4}} \rightarrow \cos\frac{\pi}{4} + j\sin\frac{\pi}{4}$

$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j$

$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$, $\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$