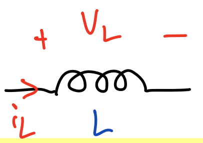
To do : Wrap up diff eqs

1. Inductors
2. Complex #s diff eqs

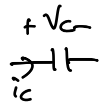
Email : nareauphol.lin @

Inductors



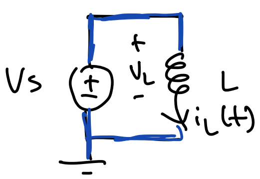
$$\boxed{L\frac{dI_L}{dt} = V_L}$$

Duals ⟷



$$i_C = C\frac{dV_C}{dt}$$

1 (a)



inductor is in parallel with V_s

$$\therefore \quad V_s = V_L(t)$$

$$V_s = L\cdot\frac{dI_L(t)}{dt}$$

$$\frac{dI_L(t)}{dt} = \frac{V_s}{L}$$

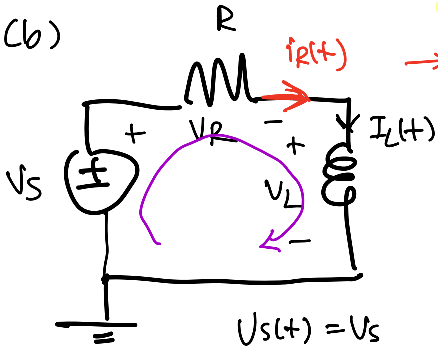
$$I_L(t) = \frac{V_s t}{L} + \cancel{I_L(0)} \rightarrow 0$$

$$I_L(6) = \frac{5}{3}(6) = 10\,A$$

★ Generally, KVL lends itself better than KCL when dealing with inductors

→ KVL: Sum of voltages in a loop = 0

(b)



KVL: $V_s - V_R - V_L = 0$

$$V_s - V_R(t) = V_L(t)$$

$$V_s - V_R(t) = L\frac{dI_L(t)}{dt}$$

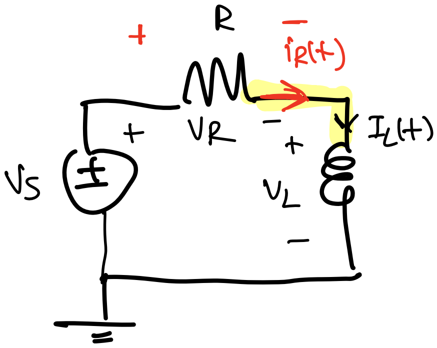
$$\frac{V_s - V_R(t)}{L} = \frac{dI_L(t)}{dt}$$

$$\frac{V_s - i_R(t)\cdot R}{L} = \frac{dI_L(t)}{dt}$$

$$\frac{V_s - i_L(t)\cdot R}{L} = \frac{dI_L(t)}{dt}$$

$$\frac{d}{dt}x(t) = \lambda x(t) + u$$

Alternate way : KCL



$$i_R(t) = I_L(t)$$

$$\frac{V_R(t)}{R} = I_L(t)$$

$$\frac{V_s - V_L(t)}{R} = I_L(t)$$

$$\frac{V_s}{R} - \frac{1}{R}\left[L\frac{d}{dt}I_L(t)\right] = I_L(t)$$

$$\frac{V_s}{R} - \frac{L}{R}\frac{d}{dt}I_L(t) = I_L(t)$$

$$V_s - L\frac{d}{dt}I_L(t) = I_L(t)\cdot R$$

$$\frac{V_s - I_L(t)\cdot R}{L} = \frac{d}{dt}I_L(t)$$

same!

$$\frac{d}{dt}I_L(t) = \frac{V_s - I_L(t)\cdot R}{L} \Rightarrow \frac{d}{dt}x(t) = \lambda x(t) + u$$

Let $q(t) = V_s - I_L(t)\cdot R \longrightarrow \frac{V_s - q(t)}{R} = I_L(t)$

$$\therefore \frac{d}{dt}I_L(t) = \frac{q(t)}{L}$$

$$q(0) = V_s - \cancel{I_L(0)}\cdot R \quad (0)$$

$$\frac{d}{dt}\left[\frac{V_s - q(t)}{R}\right] = \frac{q(t)}{L}$$

$$\cancel{\frac{d}{dt}\frac{V_s}{R}} - \frac{d}{dt}\frac{q(t)}{R} \quad (0)$$

$$-\frac{1}{R}\frac{d}{dt}q(t) = \frac{q(t)}{L}$$

$$\frac{d}{dt}q(t) = -\frac{R}{L}q(t) \quad \checkmark\checkmark \Rightarrow q(t) = q(0)\,e^{-\frac{R}{L}t}$$

$$q(t) = V_s e^{-\frac{R}{L}t}$$

$$\therefore V_s e^{-\frac{R}{L}t} = V_s - I_L(t)\cdot R$$

$$\boxed{I_L(t) = \frac{V_s - V_s e^{-\frac{R}{L}t}}{R}}$$

- $V_L(t) = V_s - V_R(t) = V_s - I_L(t)\cdot R = V_s - \left(V_s - V_s e^{-\frac{R}{L}t}\right)$
  $= V_s e^{-\frac{R}{L}t}$

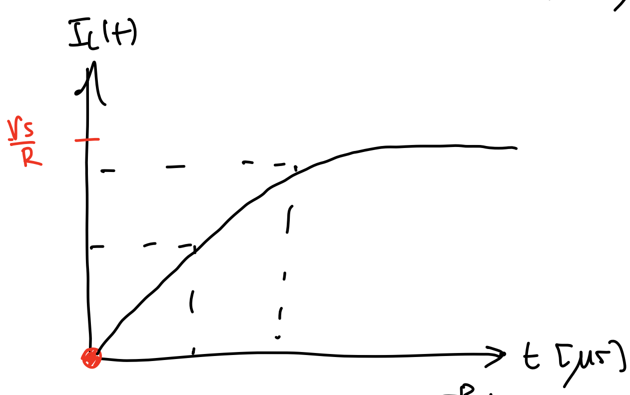
Alternatively: $V_L(t) = L\cdot \frac{dI_L(t)}{dt}$

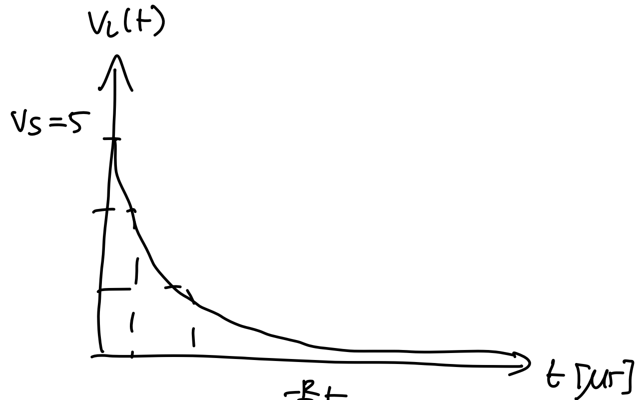
$$= L\cdot \frac{d}{dt}\left[\frac{\cancel{V_s}^{\,0} - V_s e^{-\frac{R}{L}t}}{R}\right]$$

$$= L\left[\frac{\left(-\frac{R}{L}\right)\cdot\left(-V_s e^{-\frac{R}{L}t}\right)}{R}\right]$$

$$= \cancel{L}\left[\frac{\cancel{-R}\cdot\left(\cancel{-}V_s e^{-\frac{R}{L}t}\right)}{\cancel{L}\,\cancel{R}}\right] = V_s e^{-\frac{R}{L}t}$$

(c)





$$I_L(t) = \frac{V_s - V_s e^{-\frac{R}{L}t}}{R}$$

$$V_L(t) = V_s e^{-\frac{R}{L}t}$$

$t \to \infty$, $V_L(\infty) \to 0$

$t \to \infty$

2(a) $\frac{d}{dt}\vec{z}(t) = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}\vec{z}(t)$

$$V = \begin{bmatrix} j & -j \\ 1 & 1 \end{bmatrix} \rightarrow V^{-1} = \begin{bmatrix} j & -j \\ 1 & 1 \end{bmatrix}^{-1} = \frac{1}{2}\begin{bmatrix} -j & 1 \\ j & 1 \end{bmatrix}$$

$$\frac{d}{dt}\vec{z}(t) = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}\vec{z}(t)$$

$$\downarrow$$

$$\frac{d}{dt}\vec{y}(t) = \begin{bmatrix} \cancel{\lambda_1}^{\,2j} & 0 \\ 0 & \cancel{\lambda_2}_{\,-2j} \end{bmatrix}\vec{y}(t)$$

Alternate way to find λ

$A\vec{v} = \lambda\vec{v}$

$$\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} j \\ 1 \end{bmatrix} = \lambda_1\begin{bmatrix} j \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 2j \end{bmatrix} = \lambda_1\begin{bmatrix} j \\ 1 \end{bmatrix}$$

$\lambda_1 = 2j \longrightarrow \lambda_2 = -2j$

★ Complex eigenvalues exists in conjugate pairs

$\lambda_1 = 1+j, \quad \lambda_2 = 1-j$

$\vec{z}(t) = V\vec{y}(t)$

$\vec{y}(0) = V^{-1}\vec{z}(0)$

$= \frac{1}{2}\begin{bmatrix} -j & 1 \\ j & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\therefore y_1(t) = y_1(0)e^{2jt}$

$y_2(t) = y_2(0)e^{-2jt}$

$\therefore y_1(t) = e^{2jt}$

$y_2(t) = e^{-2jt}$

$\therefore \vec{z}(t) = \begin{bmatrix} j & -j \\ 1 & 1 \end{bmatrix}\begin{bmatrix} e^{2jt} \\ e^{-2jt} \end{bmatrix} = \begin{bmatrix} je^{2jt} - je^{-2jt} \\ e^{2jt} + e^{-2jt} \end{bmatrix} = \begin{bmatrix} -2\sin(2t) \\ 2\cos(2t) \end{bmatrix}$

$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$

$\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}, \quad \frac{1}{j} = -j$

$j(e^{2jt}-e^{-2jt}) = \frac{e^{2jt}-e^{-2jt}}{-j}$

$= -2\sin(2t)$

- Final solution is purely real despite complex eigenvalues

2(b) $\vec{z}(t) = \begin{bmatrix} a & \bar{a} \\ b & \bar{b} \end{bmatrix}\vec{y}(t), \quad \vec{y}(t) = \begin{bmatrix} c_0 e^{\lambda t} \\ \bar{c_0} e^{\bar{\lambda} t} \end{bmatrix}$

Show $\vec{z}(t)$ is real.

- $x + \bar{x} = 2a = 2\cdot$(real part of complex #) $\in \mathbb{R}$

  $x \to a+bj, \quad \bar{x} \to a-bj$

- $\overline{(x\cdot y)} = \bar{x}\cdot\bar{y}$ ⇐

$\overline{e^{\lambda t}} = e^{\bar{\lambda} t}$

$\vec{z}(t) = \begin{bmatrix} ac_0e^{\lambda t} + \bar{a}\bar{c_0}e^{\bar{\lambda}t} \\ bc_0e^{\lambda t} + \bar{b}\bar{c_0}e^{\bar{\lambda}t} \end{bmatrix} = \begin{bmatrix} ac_0e^{\lambda t} + \overline{ac_0e^{\lambda t}} \\ bc_0e^{\lambda t} + \overline{bc_0e^{\lambda t}} \end{bmatrix} = \begin{bmatrix} x+\bar{x} \\ \bar{y}+y \end{bmatrix}$

$\therefore \vec{z}(t) = \begin{bmatrix} \in \mathbb{R} \\ \in \mathbb{R} \end{bmatrix} \quad \therefore \vec{z}(t) \in \mathbb{R} \quad \because \bar{x} + x = 2a \in \mathbb{R}$

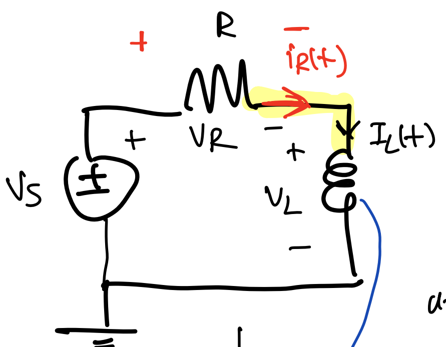
Additional Qs Post Dis [Unrecorded]

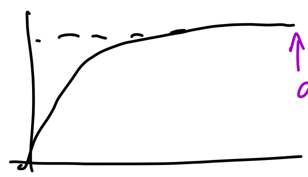
$\lambda^2 + 1 = 0$ $\qquad A\vec{v} = \lambda\vec{v}$

$\lambda = \pm j$

$\det(A - \lambda I) = 0$

$L \frac{dI_L(t)}{dt} = V_L(t)$





$\frac{dI_L(t)}{dt} = 0 \rightarrow V_L(t) = 0$

at steady state $(t \to \infty)$

$I_L(t) \rightarrow \frac{V_s}{R}$