To do : Filters and Transfer Functions

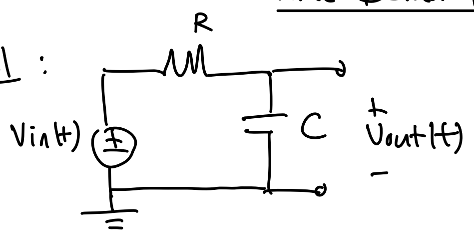
① Recap Phasors

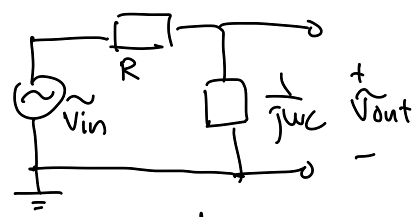
② Transfer Functions

③ Filters

**Time Domain** | **Phasor Domain**

Recall :




$Vin(t) = 12\sin(\omega t - \frac{\pi}{4})$

$\tilde{V}out = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} \cdot \tilde{V}in$

↓

$Vin(t) = V_o\cos(\omega t + \phi)$ [general form] $\Longleftrightarrow$ ① $\tilde{V}in = \frac{V_o}{2} e^{j\phi}$

Transfer Function
$H(j\omega)$ (the gain of the circuit)

$Vout(t) = \boxed{|H(j\omega)|} \cdot V_o\cos(\omega t + \phi + \boxed{\measuredangle H(j\omega)})$

scaled by $(|H(j\omega)|)$

shift by $\measuredangle H(j\omega)$

$\tilde{V}out = \boxed{\frac{1}{1 + j\omega RC}} \cdot \tilde{V}in$

express it in polar form

$\tilde{V}out = H(j\omega) \cdot \tilde{V}in$

$= |H(j\omega)| e^{j\measuredangle H(j\omega)} \cdot \cancel{\tilde{V}in}\ \frac{V_o}{2} e^{j\phi}$

$= |H(j\omega)| \cdot \frac{V_o}{2} e^{j(\phi + \measuredangle H(j\omega))}$

$H(j\omega) = \frac{\tilde{V}out}{\tilde{V}in}$

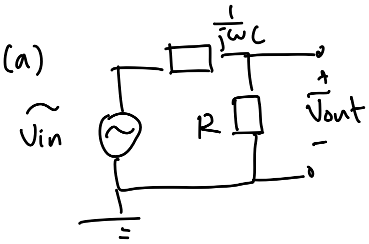
from example above : $H(j\omega) = \frac{1}{1 + j\omega RC}$

$\omega = 0,\ |H(j\omega)| = 1$ } at low freqs → output gets "preserved"

$\omega \to \infty,\ |H(j\omega)| \to 0$ } at high freqs → output dies (becomes 0)

Low-Pass Filter : Low frequencies pass through

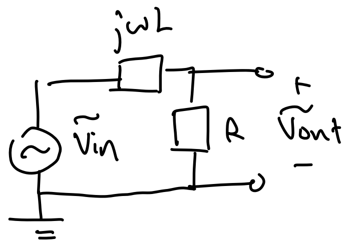
1 (a)




$$H(j\omega) = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}$$

$\omega \to 0, \ |H(j\omega)| = 0$
$\omega \to \infty, \ |H(j\omega)| \to 1$ } High-Pass Filter

(b)

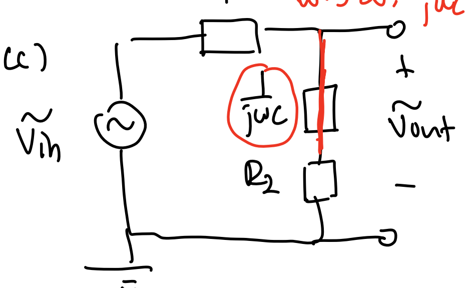



$$H(j\omega) = \frac{R}{R + j\omega L}$$

$\omega \to 0, \ |H(j\omega)| = 1$
$\omega \to \infty, \ |H(j\omega)| \to 0$ } Low-Pass Filter

(c)

$\omega \to \infty, \ \frac{1}{j\omega C} \to 0$




$$H(j\omega) = \frac{R_2 + \frac{1}{j\omega C}}{R_1 + R_2 + \frac{1}{j\omega C}} = \frac{1 + j\omega R_2 C}{1 + j\omega (R_1 + R_2) C}$$

$\omega \to 0, \ |H(j\omega)| \to 1$
$\omega \to \infty, \ |H(j\omega)| \to \frac{R_2}{R_1 + R_2}$ ] No name for this filter

(d) $V_{in}(t) = 12 \sin(100t)$, $H(j\omega) = \frac{j\omega RC}{1 + j\omega RC}$

$\downarrow$

$V_0 \cos(\omega t + \phi) \iff \frac{V_0}{2} e^{j\phi}$

$\tilde{V}_{out} = H(j\omega) \tilde{V}_{in} \longrightarrow V_{out}(t) = \boxed{|H(j\omega)|} \cdot V_0 \cos(\omega t + \phi + \boxed{\angle H(j\omega)})$

scaled by $|H(j\omega)|$ shifted by $\angle H(j\omega)$

$V_{in}(t) = 12 \sin(100t) = 12 \cos(100t - \frac{\pi}{2})$

$\downarrow$

$\tilde{V}_{in} = 6 e^{j \frac{-\pi}{2}}$ $\sin(\theta) = \cos(\theta - \frac{\pi}{2})$

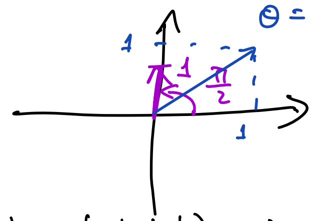
$$\therefore\ \tilde{V}_{out} = \frac{j\omega RC}{1+j\omega RC}\cdot 6e^{j\frac{-\pi}{2}} = \frac{j}{1+j}\cdot 6e^{-j\frac{\pi}{2}}$$

Not in polar

$$\bullet\ \left|\frac{j}{1+j}\right| = \frac{|j|}{|1+j|} = \frac{1}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}$$

$\tan\theta = 1$

$\theta = \frac{\pi}{4}$




$$\bullet\ \angle\left(\frac{j}{1+j}\right) = \angle j - \angle(1+j) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$V_0\cos(\omega t+\phi) \iff \frac{V_0}{2}e^{j\phi}$$

$$\therefore\ \tilde{V}_{out} = \frac{1}{\sqrt{2}}e^{j\frac{\pi}{4}}\cdot 6e^{-j\frac{\pi}{2}} = \frac{6}{\sqrt{2}}e^{-j\frac{\pi}{4}}$$

$\left[0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right]$

$$V_{out}(t) = 2\cdot\frac{6}{\sqrt{2}}\cos\left(100t - \frac{\pi}{4}\right) = \frac{12}{\sqrt{2}}\cos\left(100t - \frac{\pi}{4}\right)$$

$$V_{in}(t) = 12\cos\left(100t - \frac{\pi}{2}\right)$$

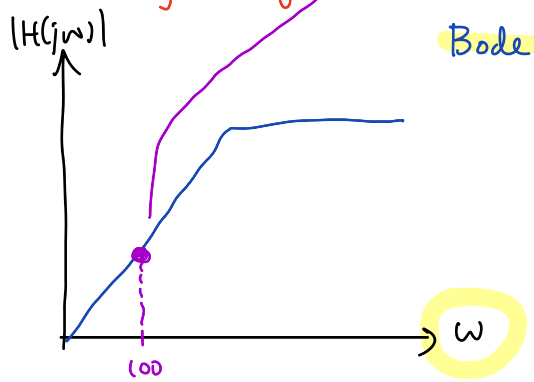
$$V_{out}(t) = \boxed{|H(j\omega)|}\cdot V_0\cos(\omega t + \phi + \boxed{\angle H(j\omega)})$$

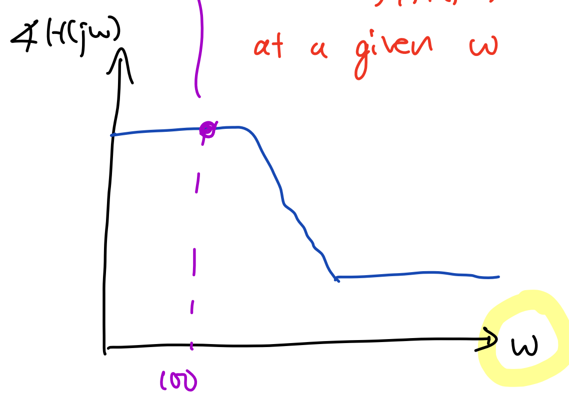
scaled by ($|H(j\omega)|$)

shifted by $\angle H(j\omega)$

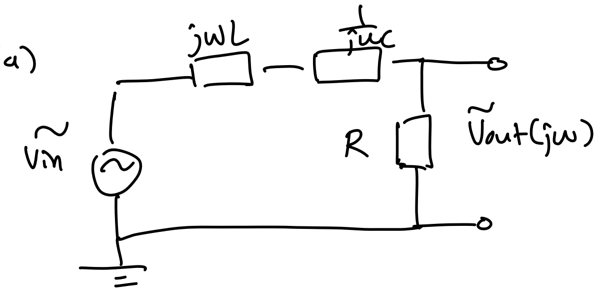
Conceptually all you need is the transfer function $H(j\omega)$ → • $|H(j\omega)|$, • $\angle H(j\omega)$ at a given $\omega$

Bode Plot



2 (a)




$$H(j\omega) = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

(b) $Z_{RLC} = Z_C + Z_L + Z_R$

$$= R + j\omega L + \frac{1}{j\omega C} = \underbrace{R}_{A(\omega)} + j\underbrace{\left(\omega L - \frac{1}{\omega C}\right)}_{X(\omega)}$$

$$X(\omega_n) = 0 \rightarrow \therefore \omega L - \frac{1}{\omega C} = 0$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 LC = 1$$

$$\omega = \sqrt{\frac{1}{LC}} = \frac{1}{\sqrt{LC}}$$

$$\boxed{\therefore \omega_n = \frac{1}{\sqrt{LC}}}$$