To do : Bode Plots

① Transfer Function Review

② Cutoff Freq

③ Bode Plot

Recall :

Time | Phasor

sinusoidal input

$V_{in}(t) \longrightarrow \boxed{\text{Filter}} \longrightarrow V_{out}(t) \longleftrightarrow \tilde{V}_{in} \longrightarrow \boxed{H(j\omega)} \longrightarrow \tilde{V}_{out}$

$V_{in}(t) = V_0 \cos(\omega t + \phi)$

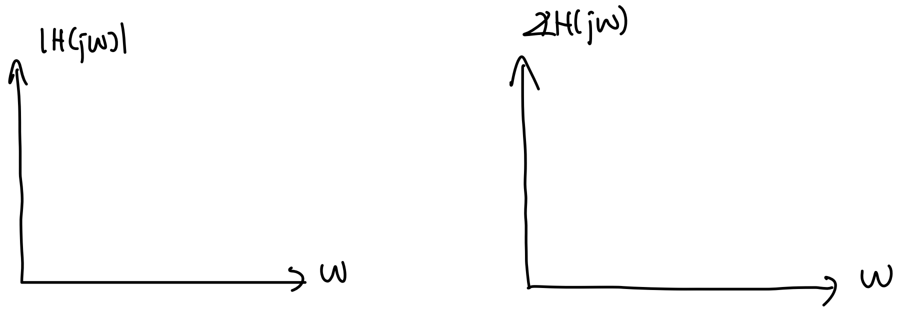
$H(j\omega) = \dfrac{\tilde{V}_{out}}{\tilde{V}_{in}}$

$V_{out}(t) = |H(j\omega)| V_0 \cos(\omega t + \phi + \angle H(j\omega))$

$\boxed{V_0 \cos(\omega t + \phi) \Leftrightarrow \frac{V_0}{2} e^{j\phi}}$

we only care about this

Bode Plot :



Cutoff Frequency : Frequency in which the magnitude is attenuated by $\frac{1}{\sqrt{2}}$

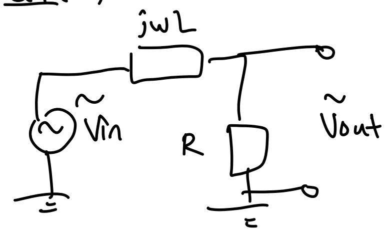
$|H(j\omega_c)| = \frac{1}{\sqrt{2}}$

constant value

· Inspection : $H(j\omega) = \dfrac{K}{1 + \frac{j\omega}{\omega_c}}$ e.g. $H(j\omega) = \dfrac{1}{1 + j\omega RC}$

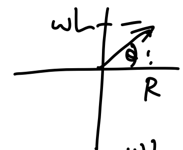
$\boxed{\omega_c = \frac{1}{RC}}$

Q1 (a)



$$H(j\omega) = \frac{R}{R + j\omega L}$$

$$|H(j\omega)| = \left| \frac{R}{R + j\omega L} \right| = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$



$$\measuredangle H(j\omega) = \measuredangle R - \measuredangle (R + j\omega L)$$
$$= 0 - \text{atan2}(\omega L, R)$$

$\tan\theta = \frac{\omega L}{R}$

$\theta = \arctan\left(\frac{\omega L}{R}\right)$

atan2 $(\omega L, R)$

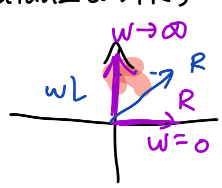
(b) $H(j\omega) = \frac{R}{R + j\omega L} = \frac{1}{1 + \frac{j\omega L}{R}}$

$\therefore$ $\boxed{\omega_c = \frac{R}{L}}$ cutoff freq. for RL circuit

(c) $H(j\omega) = \frac{R}{R + j\omega L}$

$$|H(j\omega)| = \left| \frac{R}{R + j\omega L} \right| = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

$$\measuredangle H(j\omega) = \measuredangle R - \measuredangle (R + j\omega L)$$
$$= 0 - \text{atan2}(\omega L, R)$$



Magnitude : $\omega = 0$, $|H(j\omega)| = 1$

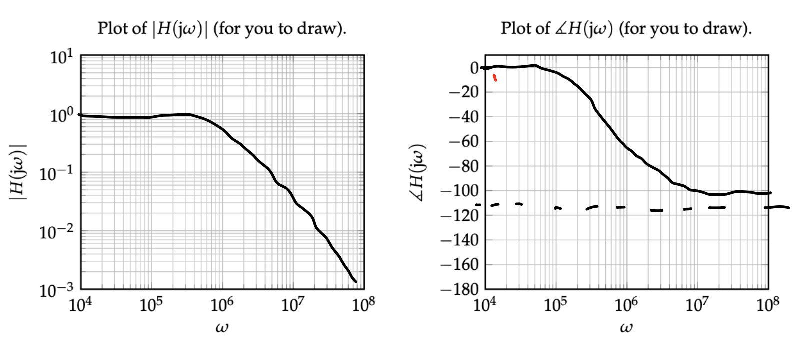
$\omega \to \infty$, $|H(j\omega)| \to 0$

Phase : $\omega = 0$, $\measuredangle H(j\omega) = -\text{atan2}(\omega L, R)$

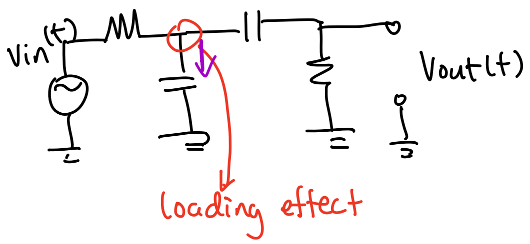
$= -0 = 0$

$\omega \to \infty$, $\measuredangle H(j\omega) \to -\frac{\pi}{2}$

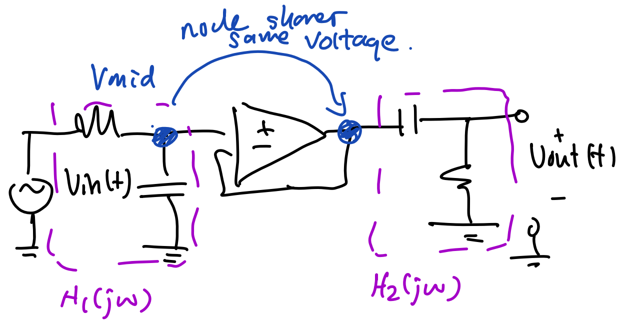
(c)



Cascading Filters



$\Rightarrow$



$\tilde{V}_{mid} = H_1(j\omega)\cdot\tilde{V}_{in}$ [at first filter]

$\tilde{V}_{out} = H_2(j\omega)\cdot\tilde{V}_{mid}$ [at second filter]

$\Big\}\ \tilde{V}_{out} = H_1(j\omega)\cdot H_2(j\omega)\cdot\tilde{V}_{in}$

cascading filters multiplies transfer functions

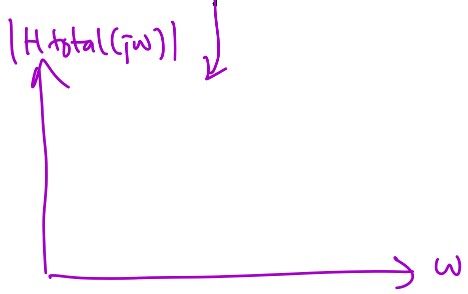
$$\tilde{V}_{out} = |H_1(j\omega)|e^{j\angle H_1}\cdot|H_2(j\omega)|e^{j\angle H_2}\cdot\tilde{V}_{in}$$

$$= |H_1(j\omega)|\cdot|H_2(j\omega)|\cdot e^{j(\angle H_1+\angle H_2)}\cdot\tilde{V}_{in}$$

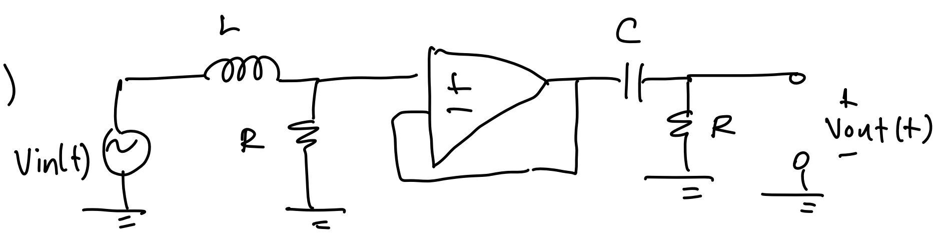
$$\boxed{V_0\cos(\omega t+\phi) \Leftrightarrow \frac{V_0}{2}e^{j\phi}}$$

$$V_{out}(t) = |H_1(j\omega)|\cdot|H_2(j\omega)|\,V_0\cos(\omega t+\phi+\angle H_1+\angle H_2)$$

What is the bode plot of $H_1(j\omega)\cdot H_2(j\omega)$ [combined filter]



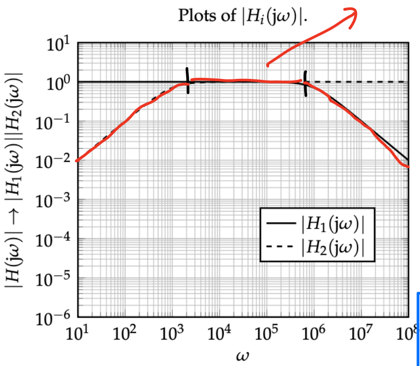
(d) (i)



Filter from (a)

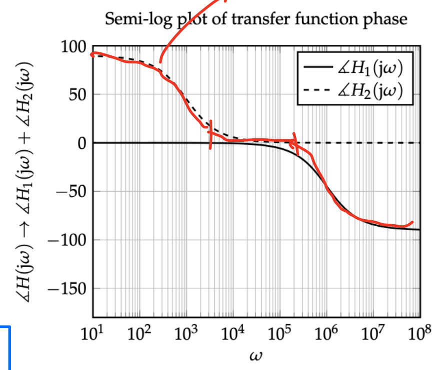
$$V_{out}(t) = |H_1(j\omega)|\cdot|H_2(j\omega)|\,V_0\cos(\omega t+\phi+\angle H_1+\angle H_2)$$

(ii)



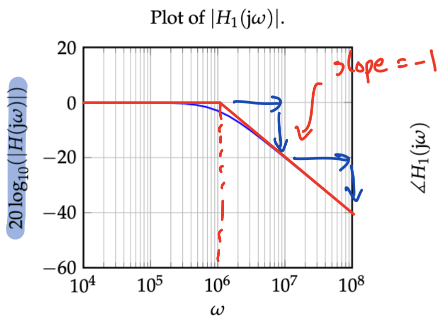
Bandpass Filter

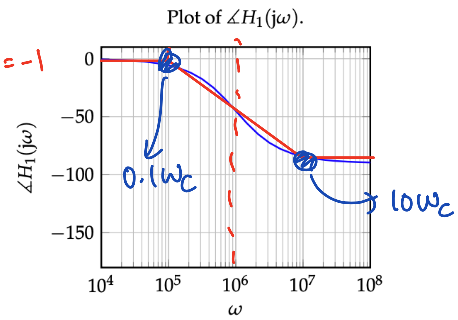
(iii)



$w_c = \frac{R}{L} = 10^6$ [filter from (a)]

2(a)



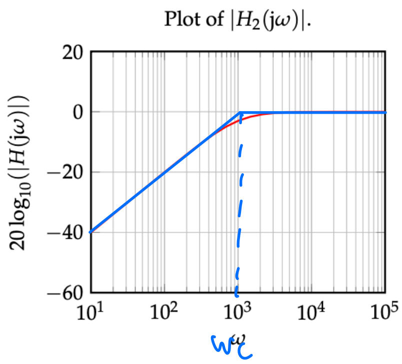


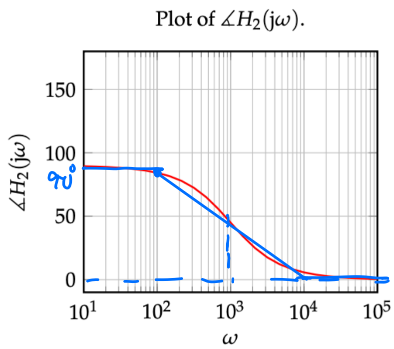
$|H(j\omega)| \to 20\log_{10}|H(j\omega)|$ (decibels dB)

Straight Line Approximation

$w_c = \frac{1}{RC} = 10^3$

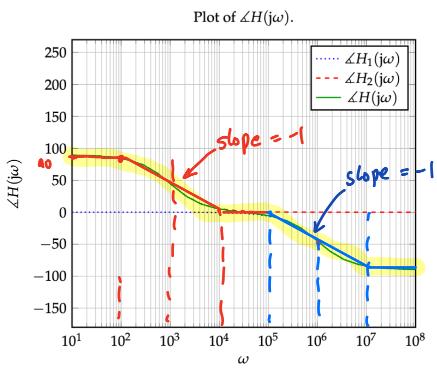
(b)

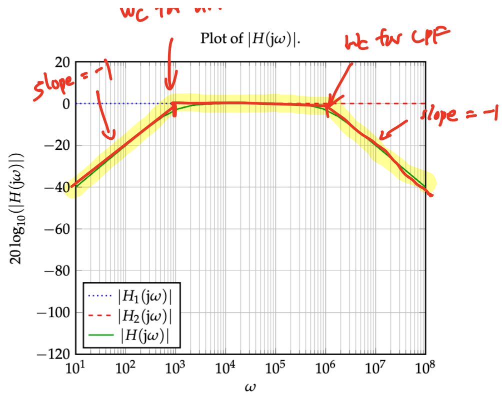




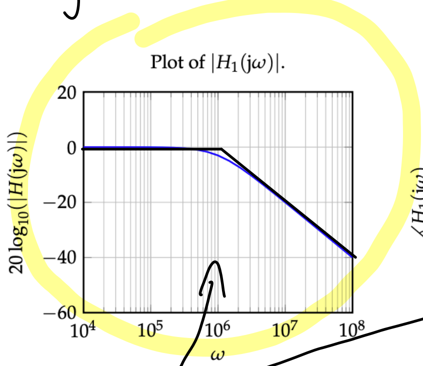
... for HPF

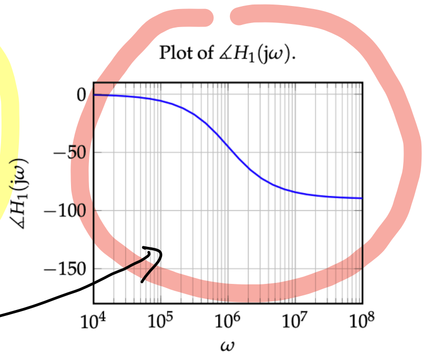
(c)

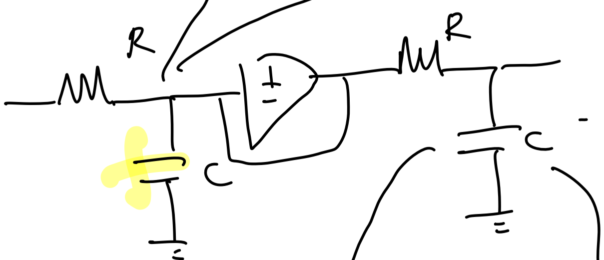


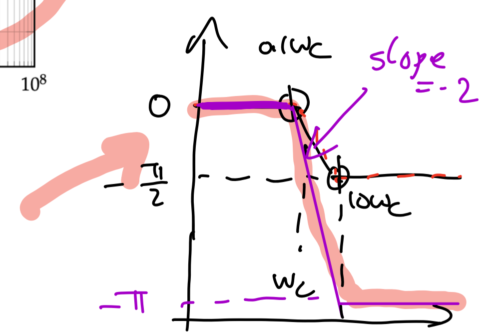


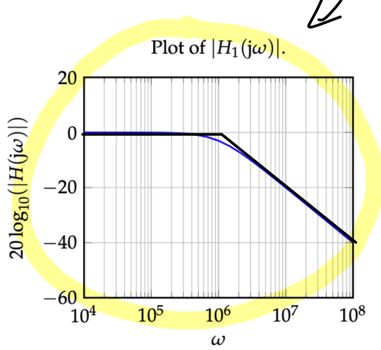
Not every filter has a slope of -1 :

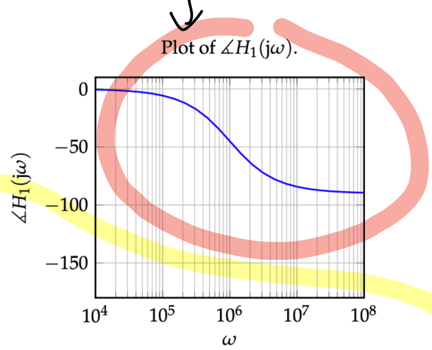




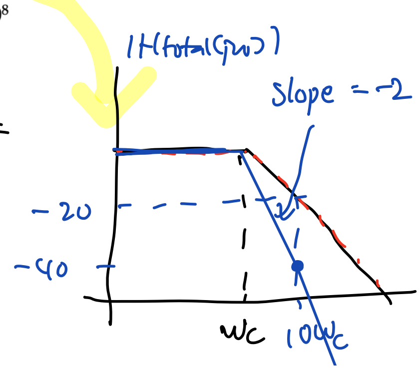








Overall transfer function $|H_{total}(j\omega)| =$

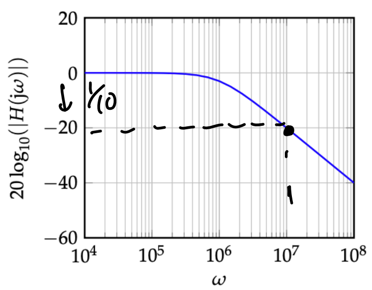


$5\cos(10^7 t)$

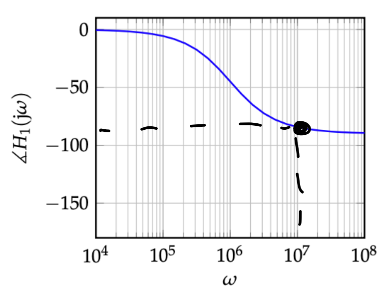
$V_{in}(t)$ → Filter 1 → $V_{out}(t) = 5 \times \frac{1}{10}\cos(10^7 t - \frac{\pi}{2})$ → Filter → $V_{out_2}(t) = \frac{5}{10} \times \frac{1}{10}\cos(10^7 t - \pi)$

$= \underline{\frac{5}{100}}\cos(10^7 t - \pi)$

Plot of $|H_1(j\omega)|$.



Plot of $\measuredangle H_1(j\omega)$.



Fa19 MT1