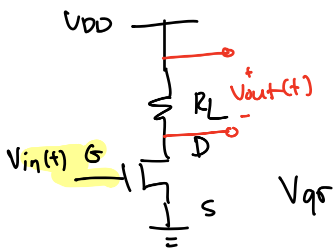
To do: Linearization Part 3



$$I_{DS}(V_{GS}) = \frac{k}{2}(V_{GS} - V_{th})^2$$

$$V_{out}(t) = V_{DD} - R_L I_{DS}(t)$$

$$V_{GS} = V_g - V_s$$
$$= V_{in}(t) - 0 = V_{in}(t)$$

(a) $V_{out}(t) = V_{DD} - R_L \cdot \frac{k}{2}(V_{GS} - V_{th})^2$

$V_{out}(t) = V_{DD} - R_L \cdot \frac{k}{2}(V_{in}(t) - V_{th})^2$

(b) $V_{in}(t)$ into $V_{in,DC} + V_{in,AC}(t)$

Recall $f(x) \approx f(x^*) + f'(x^*)(x - x^*)$, $x^* = V_{in,DC}$

Think x as $V_{in}(t)$, $f(x)$ as $V_{out}(t)$

$$V_{out}(t) \approx V_{DD} - R_L \cdot \frac{k}{2}(V_{in,DC} - V_{th})^2 + \left.\frac{dV_{out}}{dV_{in}}\right|_{x=x^*} (\underbrace{\cancel{x} - \cancel{x^*}}_{})$$

$V_{in}(t) - V_{in,DC}$

$$\left.\frac{dV_{out}}{dV_{in}}\right|_{x=x^*} = -R_L \cdot k(V_{in,DC} - V_{th})$$

$$\therefore V_{out}(t) \approx \underbrace{V_{DD} - R_L \cdot \frac{k}{2}(V_{in,DC} - V_{th})^2}_{f(x^*)} + \underbrace{(-R_L \cdot k(V_{in,DC} - V_{th}))}_{f'(x^*)} \underbrace{(V_{in}(t) - V_{in,DC})}_{(x - x^*)}$$

(c) $V_{out}(t) = V_{out,DC} + V_{out,AC}(t)$, $V_{in}(t)$ into $V_{in,DC} + V_{in,AC}(t)$ (from part (b))

$$g_m = k(V_{in,DC} - V_{th})$$

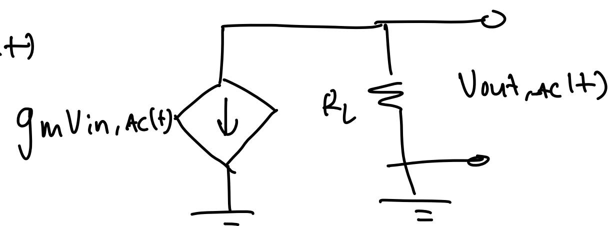
$$\hat{V}_{out}(t) = V_{DD} - R_L \cdot \frac{k}{2}(V_{in,DC} - V_{th})^2 - R_L \cdot g_m \underbrace{(V_{in}(t) - V_{in,DC})}_{V_{in,AC}(t)}$$

$$= \underbrace{V_{DD} - R_L \cdot \frac{k}{2}(V_{in,DC} - V_{th})^2}_{V_{out,DC}} \underbrace{- R_L \cdot g_m V_{in,AC}(t)}_{V_{out,AC}(t)}$$

(d) $V_{out,AC}(t) = -R_L \cdot g_m V_{in,AC}(t)$

$V_{out} \leftarrow V = IR \rightarrow R_L$

$I \rightarrow -g_m V_{in,AC}(t)$



2 (a) $\vec{x}(t) = \begin{bmatrix} x \\ \frac{dx}{dt} \\ \theta \\ \frac{d\theta}{dt} \end{bmatrix}$ $\rightarrow$ because our equations contain the second derivatives

$$\frac{d}{dt}\vec{x}(t) = \begin{bmatrix} \frac{dx}{dt} \\ \frac{d^2x}{dt} \\ \frac{d\theta}{dt} \\ \frac{d^2\theta}{dt} \end{bmatrix} \begin{matrix} \\ \leftarrow \text{Eq (3)} \\ \\ \leftarrow \text{Eq (4)} \end{matrix}$$

$$\frac{d}{dt}\vec{x}(t) = \vec{f}(\vec{x}, u)$$

$$\frac{d}{dt}\vec{x}(t) = \vec{f}(\vec{x}, u) = \begin{bmatrix} x_2 \\ \frac{1}{1+\sin^2(x_3)}(u+\sin(x_3)(x_4^2 - g\cos(x_3))) \\ x_4 \\ \frac{1}{1+\sin^2(x_3)}(-u\cos(x_3) - x_4^2\cos(x_3)\sin(x_3) + 2g\sin(x_3)) \end{bmatrix}$$

(b) $\theta = 0$ (pole upright) $\rightarrow \frac{d\theta}{dt} = 0$
$x = 0$ (cart at origin) $\rightarrow \frac{dx}{dt} = 0$

$\therefore \vec{x}^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $u^* = 0$ (plug in $\vec{x}^*$ into $\vec{f}$)

($u = F$)

(c) $J_{\vec{x}}\vec{f} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_4} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_4}{\partial x_1} & \cdots & \frac{\partial f_4}{\partial x_4} \end{bmatrix}$ 4x4 matrix

$J_u\vec{f} = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \vdots \\ \frac{\partial f_4}{\partial u} \end{bmatrix}$ 4x1 vector

(d) $$\frac{d}{dt}\vec{x}(t) = \vec{f}(\vec{x}, u) \approx \vec{f}(\vec{x}^*, u^*) + J_{\vec{x}}\vec{f}\,(\vec{x} - \vec{x}^*) + J_u f\,(u - u^*)$$

$$\approx \cancel{0} + \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_4} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_4}{\partial x_1} & \cdots & \frac{\partial f_4}{\partial x_4} \end{bmatrix} \vec{x} + \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \vdots \\ \frac{\partial f_4}{\partial u} \end{bmatrix} u$$

$$\approx \underbrace{A}_{J_{\vec{x}}f}\vec{x} + \underbrace{\vec{b}}_{J_{\vec{u}}f}u$$ [linear system]

Check stability $\rightarrow$ Check $J_{\vec{x}}f$ (A matrix)