Lecture 2

* Computing: Transistors & Logic
  * Transistor RC model
  * Solving RC circuits
  * RC transients

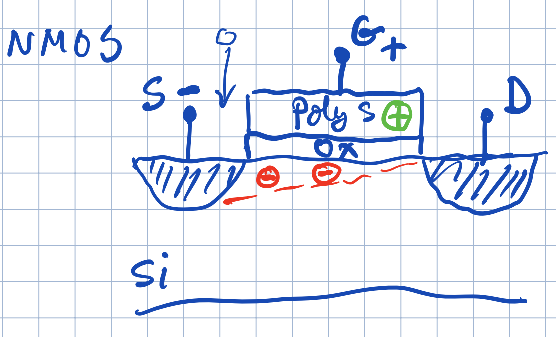
MOSFET (metal-oxide semiconductor field effect transistor)
invented in 1959-60
by Atalla & Kahng

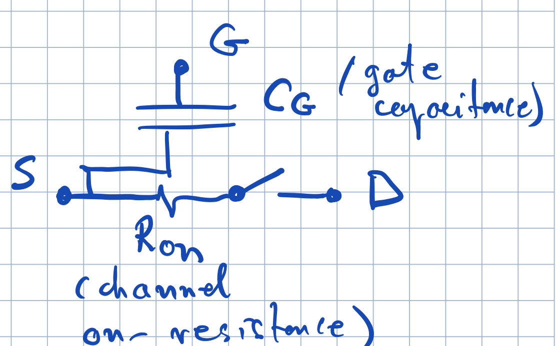
NMOS = n-channel MOSFET

PMOS = p-channel MOSFET

In the past and some present:

Planar devices

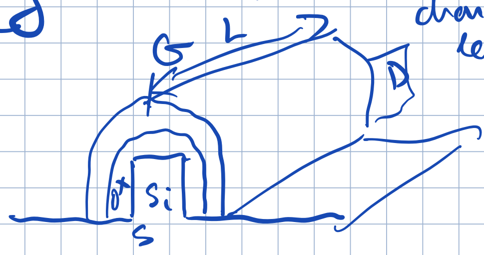




$V_{GS} \geq V_{thn} \Rightarrow$ ON

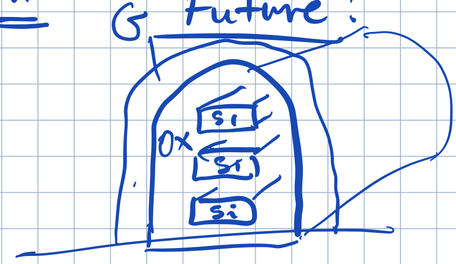
$V_{GS} < V_{thn} \Rightarrow$ OFF

Today: Fin FET



channel length 5nm

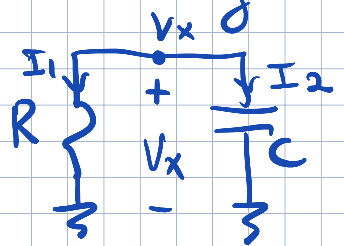
Future:



For more details take 105, 151, 130

To analyze this RC model we need to understand RC circuits.

Solving RC circuits:



Elements:

$$I_2 = C \cdot \frac{dV_x}{dt}$$

$$V_x = I_1 \cdot R$$

KCL:

$$I_1 + I_2 = 0$$

Want to find $V_x(t)$ for $t \geq 0$.

$$I_1 = \frac{V_x}{R} \qquad I_2 = C\frac{dV_x}{dt} \quad \overset{KCL}{\Longrightarrow} \quad \frac{V_x}{R} + C\frac{dV_x}{dt} = 0$$

$$\boxed{\frac{dV_x}{dt} = -\frac{V_x}{RC}}$$

first order differential equation

1 Guess: $V_x(t) = a \cdot e^{bt}$ (educated guess based on the properties of the derivative)

Initial condition:

$$t=0: \quad V_x(0) = a \cdot \cancel{e^{b\cdot 0}}^{\,1} = a$$

$$\frac{d}{dt} V_x(t) = \frac{d}{dt}\left(a \cdot e^{bt}\right) = ab \cdot e^{bt} = b \cdot V_x(t)$$

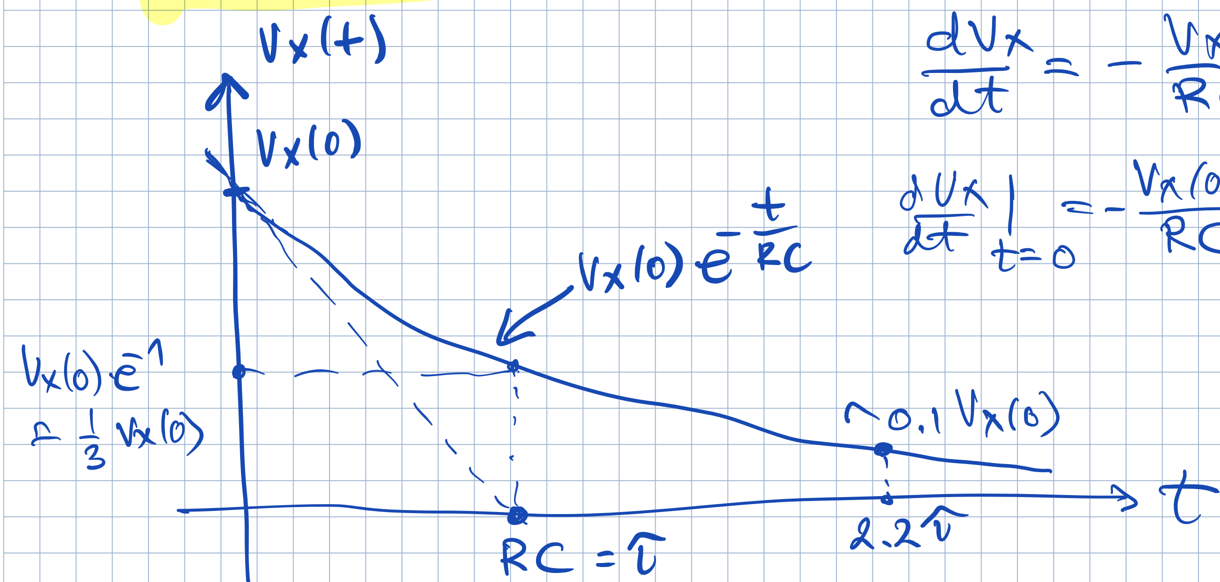
since $\frac{dV_x}{dt} = -\frac{V_x}{RC} \Rightarrow \boxed{b = -\frac{1}{RC}}$

$$\boxed{V_x(t) = V_x(0)\, e^{-\frac{t}{RC}}}$$

is a solution to

$$\frac{dV_x}{dt} = -\frac{V_x}{RC}$$

$$\left.\frac{dV_x}{dt}\right|_{t=0} = -\frac{V_x(0)}{RC}$$



τ – time constant of the RC circuit

② Check for uniqueness of the guess:

Suppose $y(t)$ which also solves D.E.

Shorter notation: $x(0) = x_0 \quad (1) \qquad V_x(t) = x(t)$

$$\frac{d}{dt} x(t) = \lambda x(t) \quad (2) \qquad \lambda = -\frac{1}{RC}$$

In ① we guessed & checked that

$$x_d(t) = x_0 \cdot e^{\lambda t} \qquad t \geq 0$$

i.e. satisfies (1) & (2)

In ② need to prove $y(t) = x_d(t)$

– i.e. the solution is unique

Either prove $\dfrac{y(t)}{x_d(t)} = 1$ or $y(t) - x_d(t) = 0$.

$$\frac{y(t)}{x_d(t)} = \frac{y(t)}{x_0 e^{\lambda t}}$$

[since $y(t)$ is a solution: from (2) $\frac{d}{dt} y(t) = \lambda y(t)$] *

$$\frac{d}{dt}\left(\frac{y(t)}{x_d(t)}\right) = \frac{d}{dt}\left(\frac{y(t)}{x_0 e^{\lambda t}}\right) = \frac{1}{x_0}\frac{d}{dt}\left(y(t)\cdot e^{-\lambda t}\right) =$$

$$= \frac{1}{x_0}\left(\frac{d}{dt}y(t)\cdot e^{-\lambda t} + y(t)(-\lambda)\cdot e^{-\lambda t}\right) =$$

$$\overset{*}{=} \frac{1}{x_0}\left(\lambda y(t) e^{-\lambda t} - \lambda y(t) e^{-\lambda t}\right)$$

$$= 0 \implies \frac{y(t)}{x_d(t)} = a \quad (\text{constant})$$

$$t \geq 0$$

From (1) $x(0) = x_0$

$y(0) = x_0$ since $y(t)$ is also a solution

$$\frac{y(0)}{x_d(0)} = \frac{x_0}{x_0 e^{\lambda 0}} = \frac{x_0}{x_0} = 1 = a$$

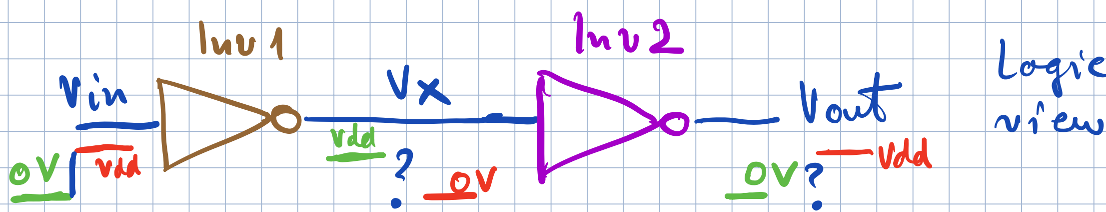
$$\Rightarrow \quad \frac{y(t)}{x_d(t)} = a = 1 \Rightarrow y(t) = x_d(t)$$

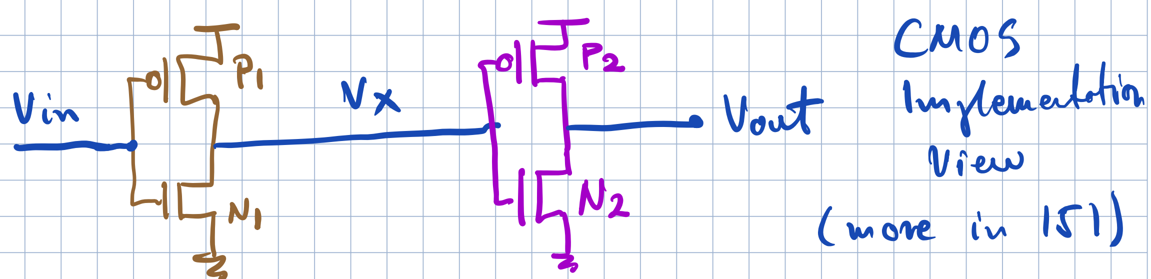
so the solution

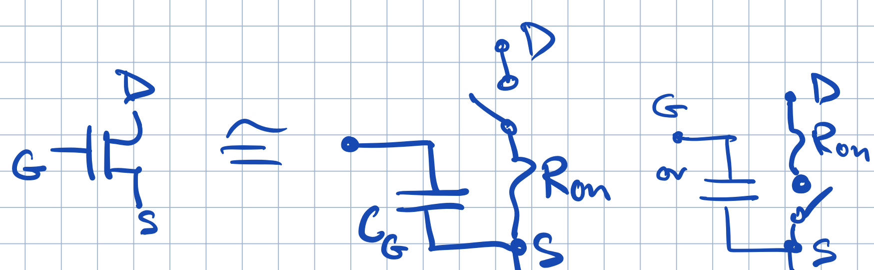
$$x_d(t) = x_0 e^{\lambda t}$$

is unique.

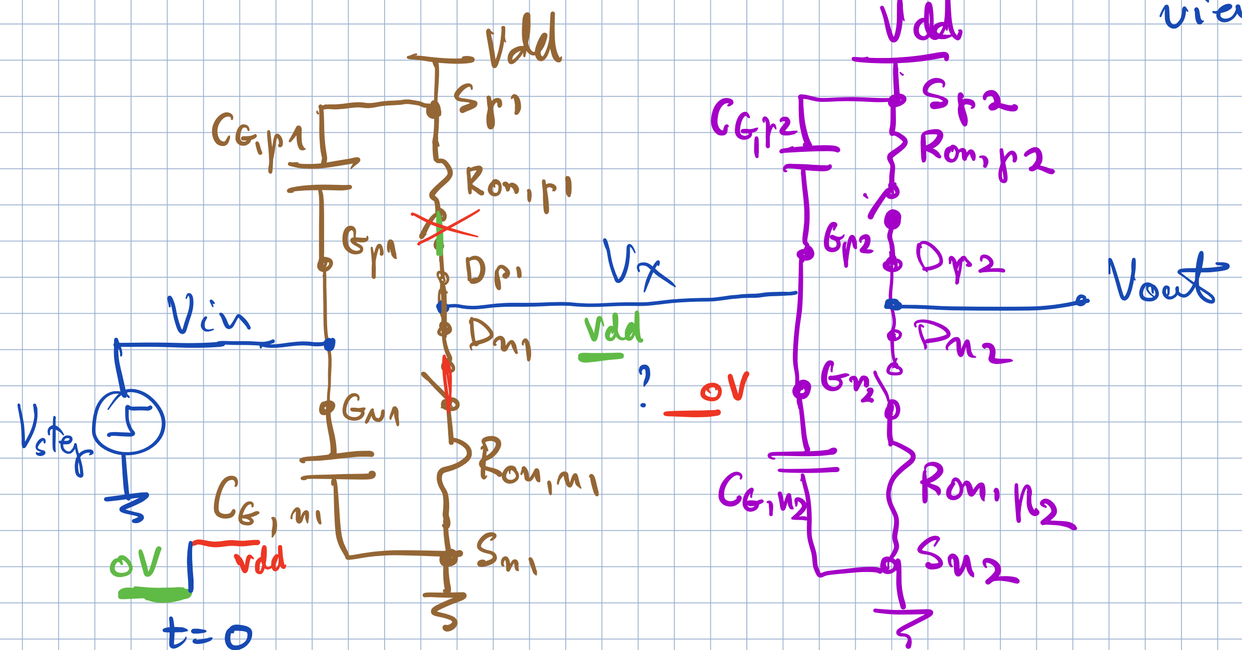
Now we can use this to solve transistor circuits with RC model.

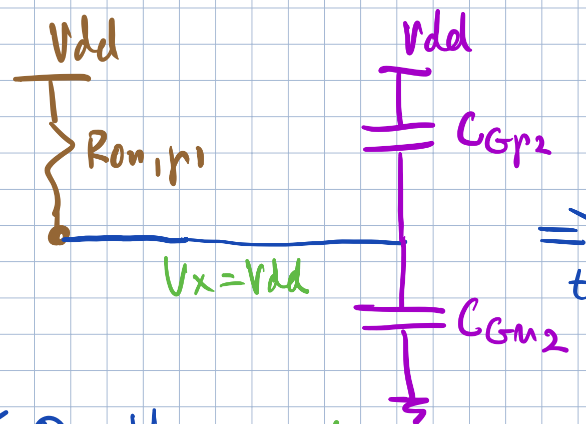




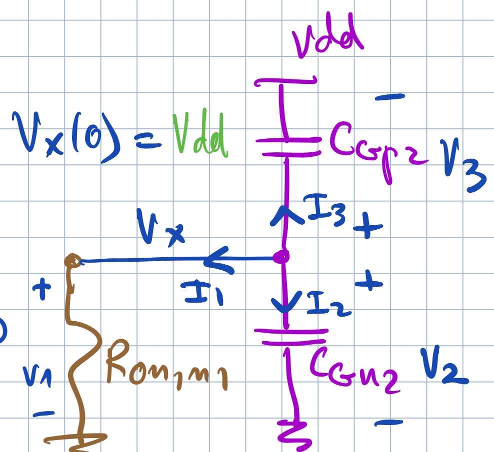


RC model view





$\Rightarrow$ $t = 0$



$t < 0$, $V_{in} = 0V$ $\qquad$ $t \geq 0$, $V_{in} = V_{dd}$

KCL: $I_1 + I_2 + I_3 = 0$

Elements: $V_1 = I_1 \cdot R_{on,n1}$

$$I_2 = C_{Gn2} \frac{dV_2}{dt}$$

$$I_3 = C_{Gp2} \frac{dV_3}{dt}$$

voltages:

$V_1 = V_x$

$V_2 = V_x$

$V_3 = V_x - V_{dd}$

Sub. into KCL:

$$\frac{V_1}{R_{on,n1}} + C_{Gn2}\frac{dV_2}{dt} + C_{Gp2}\frac{dV_3}{dt} = 0$$

$$\frac{V_x}{R_{on,n1}} + C_{Gn2}\frac{dV_x}{dt} + C_{Gp2}\frac{d}{dt}(V_x - V_{dd}) = 0$$

$$\frac{V_x}{R_{on,n1}} + C_{Gn2}\frac{dV_x}{dt} + C_{Gp2}\frac{d}{dt}V_x = 0$$

$$\frac{V_x}{R_{on,n1}} + ((C_{Gn2} + C_{Gp2})\frac{d}{dt}V_x = 0$$

$$\boxed{\frac{d}{dt}V_x(t) = -\frac{V_x}{R_{on,n1}\cdot((C_{Gn2}+C_{Gp2})}}$$

$$\tau = R_{on,n1}\,((C_{Gn2} + C_{Gp2})$$

determines the speed of transition.

Already have a solution for this:

$$t \geq 0: \quad V_x(t) = V_x(0)\, e^{-\frac{t}{\tau}}$$

$$V_x(0) = V_{dd} \Rightarrow V_x(t) = V_{dd}\cdot e^{-\frac{t}{\tau}}$$