Lecture 10

* Design example – bandpass filter

Vin components:

	freq	amplitude	
signal	600 Hz	1 mV	} desired
(alternate current) AC	60 Hz	10 mV	} interference ✗
fluorescent light	60 kHz	20 mV	

Design goal: Want to attenuate the interference (AC and fluorescent) components by 100×, but keep the signal.

$$V_{in}(t) = V_{AC}\cos(\omega_{AC}t + \phi_{AC}) + V_s\cos(\omega_s t + \phi_s) + V_F\cos(\omega_F t + \phi_F)$$

$\omega_{AC} = 2\pi \cdot 60\,\text{Hz} = 377\,\frac{\text{rad}}{s}$

$\omega_s = 2\pi \cdot 600\,\text{Hz}$

$\omega_F = 2\pi\, 60\,\text{kHz}$

strategy:

$$\tilde{V}_{in} \longrightarrow \boxed{H(j\omega)} \longrightarrow \tilde{V}_{out}$$

$\tilde{V}_{in_{AC}}$

$\tilde{V}_s$

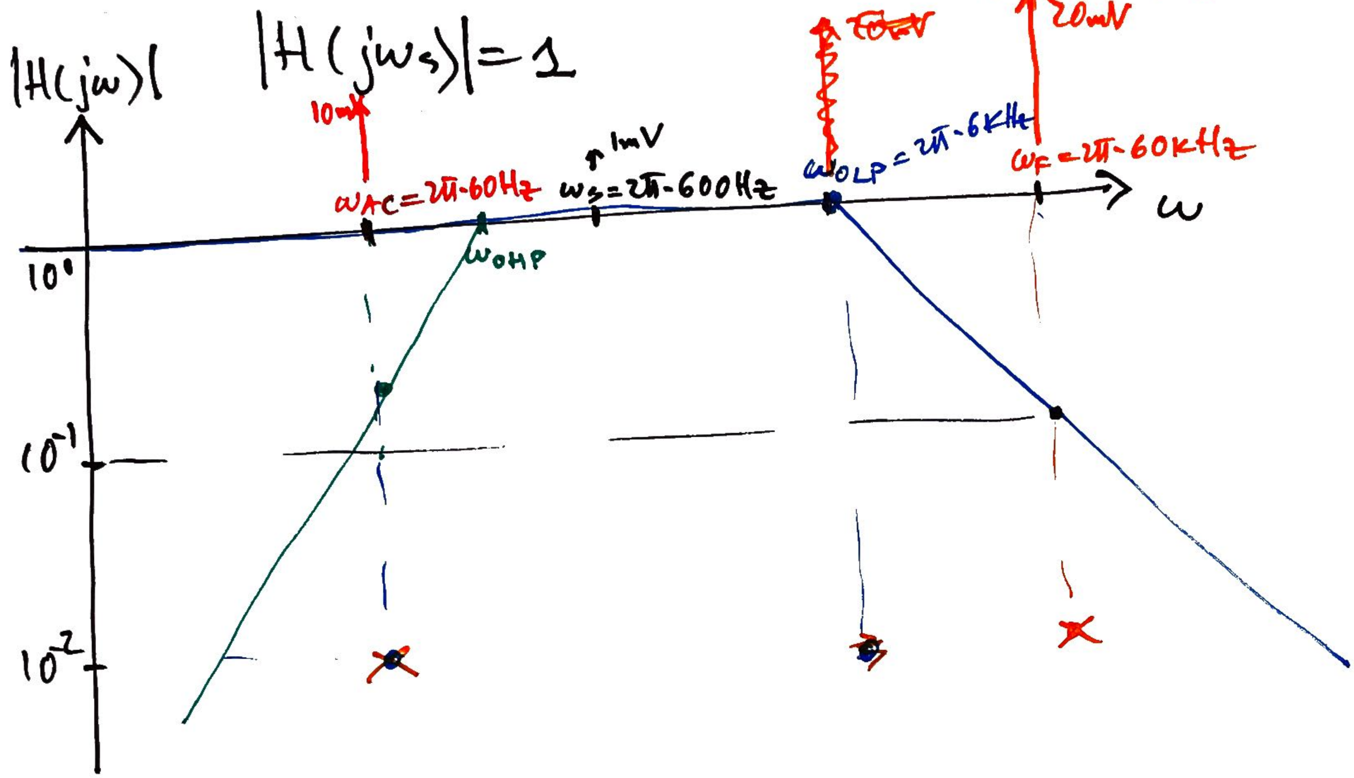
$\tilde{V}_F$

$$V_{out}(t) = |H(j\omega_{AC})| \cdot V_{AC} \cdot \cos(\omega_{AC}t + \phi_{AC} + \measuredangle H(j\omega_{AC}))$$
$$+ |H(j\omega_s)| \cdot V_s \cdot \cos(\omega_s t + \phi_s + \measuredangle H(j\omega_s))$$
$$+ |H(j\omega_F)| \cdot V_F \cdot \cos(\omega_F t + \phi_F + \measuredangle H(j\omega_F))$$

Design goal:

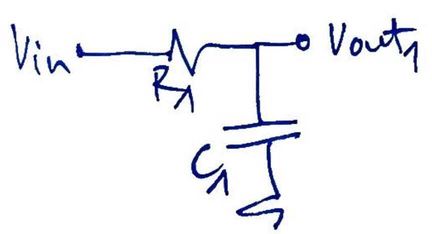
$|H(j\omega_{AC})| = \frac{1}{100}$ $\quad |H(j\omega_F)| = \frac{1}{100}$

$|H(j\omega_s)| = 1$

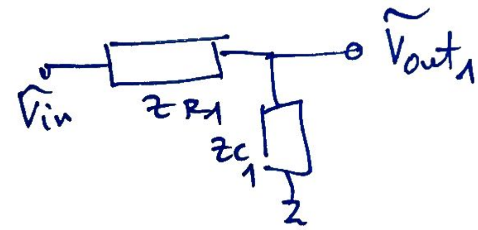


$$\tilde{V}_{in} \longrightarrow \boxed{H_{LP}(j\omega)} \xrightarrow{\tilde{V}_{out_1}} \boxed{H_{by}(j\omega)} \xrightarrow{\tilde{V}_{in_2}} \boxed{H_{HP}(j\omega)} \longrightarrow \tilde{V}_{out}$$

$H_{LP}(j\omega)$ – a "low-pass" filter

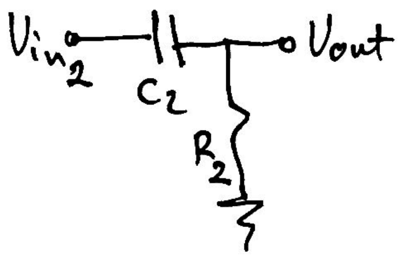


$\Rightarrow$

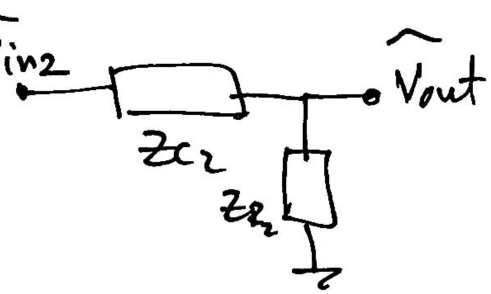


$$H_{LP}(j\omega) = \frac{\hat{V}_{out_1}}{\hat{V}_{in}} = \frac{1}{1 + j\frac{\omega}{\omega_{0LP}}} \quad , \quad \omega_{0LP} = \frac{1}{R_1 C_1}$$

$H_{HP}(j\omega)$ – a "high-pass" filter



$\Rightarrow$



$$H_{HP}(j\omega) = \frac{\tilde{V}_{out}}{\tilde{V}_{in_2}} = \frac{j\frac{\omega}{\omega_{0HP}}}{1 + j\frac{\omega}{\omega_{0HP}}} = \frac{1}{1 - j\frac{\omega_{0HP}}{\omega}}, \quad \omega_{0HP} = \frac{1}{R_2 C_2}$$

$$H(j\omega) = \frac{\tilde{V}_{out}}{\tilde{V}_{in}} = \underbrace{\frac{\tilde{V}_{out}}{\tilde{V}_{in_2}}}_{H_{HP}(j\omega)} \cdot \underbrace{\frac{\tilde{V}_{in_2}}{\tilde{V}_{out_1}}}_{H_{buf}(j\omega)} \cdot \underbrace{\frac{\tilde{V}_{out_1}}{\tilde{V}_{in}}}_{H_{LP}(j\omega)} =$$

$$= \frac{1}{1 + j\frac{\omega}{\omega_{0LP}}} \cdot \frac{1}{1 - j\frac{\omega_{0HP}}{\omega}}$$

Need to choose ω_{0LP} & ω_{0HP}.

Compromise – want to attenuate the interference w/o attenuating the signal.

$$\omega_{0HP} = \sqrt{\omega_{AC} \cdot \omega_S} \qquad \log \omega_{0HP} = \frac{1}{2}\log \omega_{AC} + \frac{1}{2}\log \omega_S$$

$$\omega_{0LP} = \sqrt{\omega_S \cdot \omega_F}$$

$$\omega_{0HP} = \frac{1}{R_2 C_2}, \quad \omega_{0LP} = \frac{1}{R_1 C_1}$$

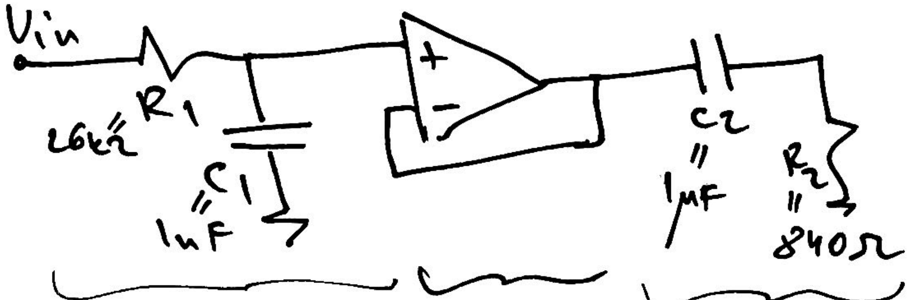
$$\omega_{0HP} = \sqrt{2\pi \cdot 60\,Hz \cdot 2\pi \cdot 600\,Hz} = 2\pi \cdot 190\,Hz$$

$$\omega_{0LP} = \sqrt{2\pi \cdot 600\,Hz \cdot 2\pi\, 60\,kHz} = 2\pi \cdot 6\,kHz$$

Pick a reasonable C:

$$C_1 = 1\,nF \Rightarrow R_1 = \frac{1}{\omega_{0LP} \cdot C_1} = 26\,k\Omega$$

$$C_2 = 1\,\mu F \Rightarrow R_2 = \frac{1}{\omega_{0HP} \cdot C_2} = 840\,\Omega$$



$H_{LP}(j\omega)$ $\quad$ $H_{1g}(j\omega)$ $\quad$ $H_{HP}(j\omega)$

Check: Evaluate the TF $H(j\omega)$ @ $\omega_{AC}, \omega_S, \omega_F$

ω	$\lvert H_{LP}(j\omega)\rvert$	$\lvert H_{HP}(j\omega)\rvert$	$\lvert H(j\omega)\rvert$	$V_{in} \cdot \lvert H(j\omega)\rvert = V_{out}$
$2\pi \cdot 60\,Hz$	$\frac{1}{\sqrt{1+\left(\frac{2\pi\, 60\,Hz}{2\pi\, 6\,kHz}\right)^2}} \approx 1$	$\frac{1}{\sqrt{1+\left(\frac{2\pi \cdot 190\,Hz}{2\pi \cdot 60\,Hz}\right)^2}} \approx 0.3$	$1 \cdot 0.3 \approx 0.3$	$10\,mV \cdot 0.3 = 3\,mV$
$2\pi \cdot 600\,Hz$	$\frac{1}{\sqrt{1+\left(\frac{2\pi \cdot 600\,Hz}{2\pi \cdot 6\,kHz}\right)^2}} \approx 1$	≈ 0.95	$1 \cdot 0.95 \approx 0.95$	$1\,mV \cdot 0.95 = 0.95\,mV$
$2\pi \cdot 60\,kHz$	$\frac{1}{\sqrt{1+\left(\frac{2\pi \cdot 60\,kHz}{2\pi\, 6\,kHz}\right)^2}} = 0.1$	≈ 1	$1 \cdot 0.1 = 0.1$	$20\,mV \cdot 0.1 = 2\,mV$

Wanted $\frac{1}{100}$ for $|H_{\cancel{HP}}(j\omega_{AC})|$ & $|H(j\omega_F)|$

but only got 0.3 & 0.1 - not quite enough.

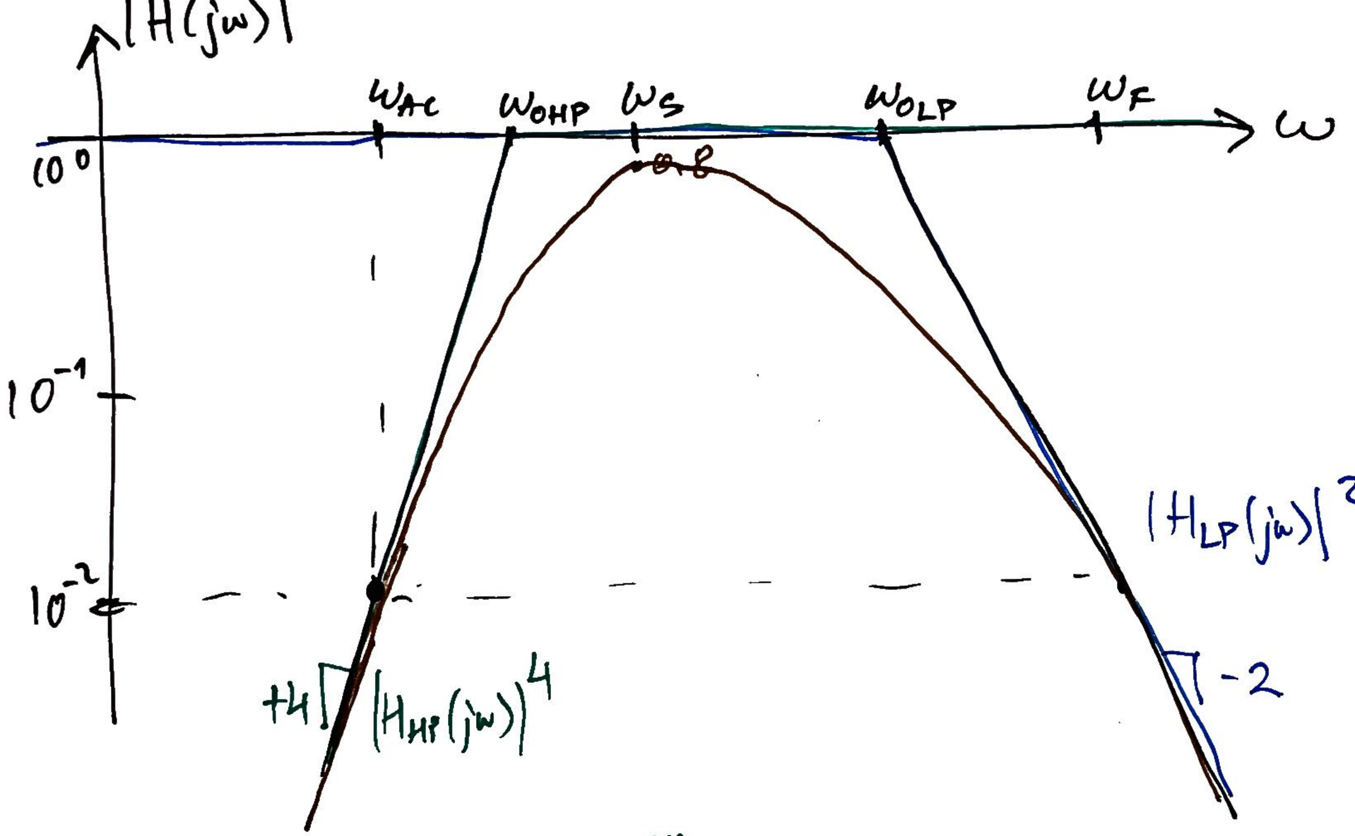
Can keep going ...

$$\underbrace{\rightarrow \boxed{H_{LP}(j\omega)} \rightarrow \boxed{H_{HP}(j\omega)} \rightarrow \boxed{H_{HP}(j\omega)} \rightarrow \boxed{H_{HP}(j\omega)} \rightarrow \boxed{H_{LP}(j\omega)} \rightarrow \boxed{H_{HP}} \rightarrow \boxed{H_{HP}} \rightarrow \cdots}$$

$$H_{tot}(j\omega) = H_{LP}^{n}(j\omega)\, H_{HP}^{m}(j\omega)$$

$$|H_{tot}(j\omega_{AC})| = \frac{1}{100} = 1^{n} \cdot 0.3^{m} \Rightarrow m = 4$$

$$|H_{tot}(j\omega_{F})| = \frac{1}{100} = 0.1^{n} \cdot 1^{m} \Rightarrow \cancel{m=2}\quad n = 2$$



$$|H_{tot}(j\omega_S)| = 1^{n} \cdot 0.95^{m} = 0.8 \quad \text{(o.k. but not great)}$$

$$\boxed{H_{tot}(j\omega) = \frac{\left(j\frac{\omega}{\omega_{OHP}}\right)^4}{\left(1 + j\frac{\omega}{\omega_{OHP}}\right)^4 \cdot \left(1 + j\frac{\omega}{\omega_{OLP}}\right)^2}}$$

<u>In general:</u>

origin zeros

$$H(j\omega) = K \frac{(j\omega)^{N_{z0}} \left(1+j\frac{\omega}{\omega_{z1}}\right)\cdots\left(1+j\frac{\omega}{\omega_{zn}}\right)}{(j\omega)^{N_{p0}} \left(1+j\frac{\omega}{\omega_{p1}}\right)\cdots\left(1+j\frac{\omega}{\omega_{pm}}\right)}$$

origin poles

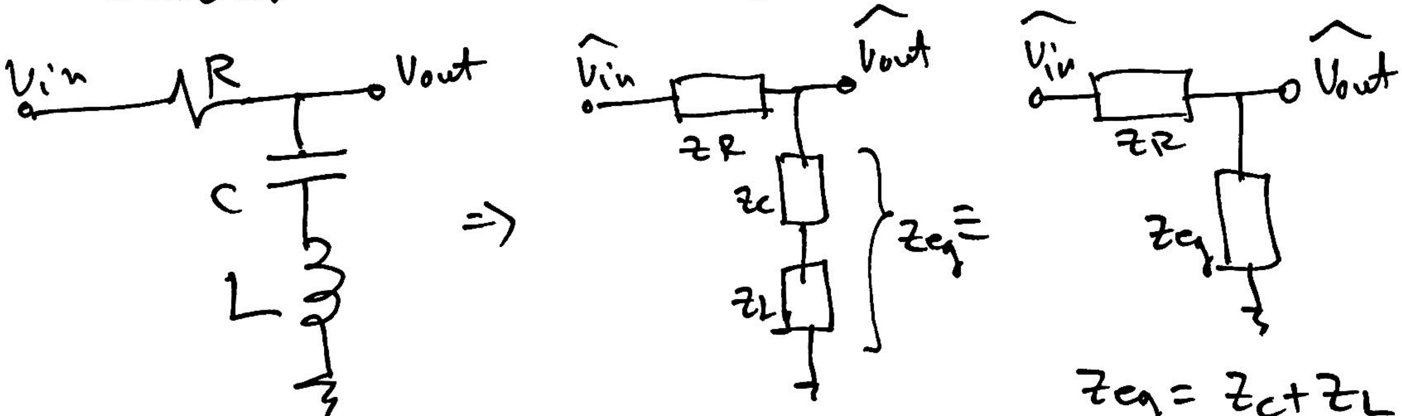
$\left.\begin{array}{l}\omega_{zn} - \text{zeroes}\\ \omega_{pn} - \text{poles}\end{array}\right\}$ control systems terminology (105, 120, ...)

What if our desired signal is at 100 Hz?

- Our previous design won't work ☹

<u>Need a different filter!</u>

Inductor to the rescue!



$Z_{eq} = Z_C + Z_L$

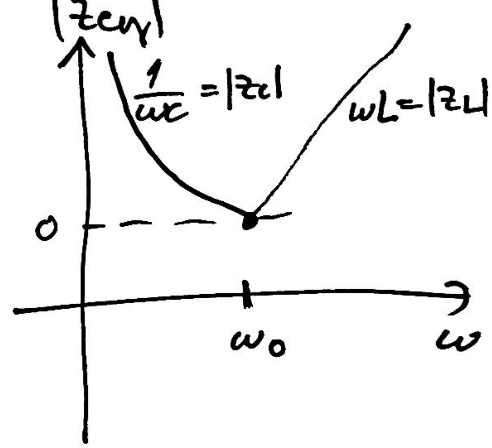
$$Z_{eq} = Z_C + Z_L = \frac{1}{j\omega C} + j\omega L = j\left(\omega L - \frac{1}{\omega C}\right)$$ <u>Fantastic!</u>

$$H(j\omega) = \frac{\widehat{V_{out}}}{\widehat{V_{in}}} = \frac{Z_{eq}}{Z_{eq} + Z_R}$$

say $Z_{eq}(j\omega_0) = 0 = j\left(\underbrace{\omega_0 L - \frac{1}{\omega_0 C}}_{0}\right)$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$|H(j\omega_0)| = \left| \frac{\overbrace{\cancel{Z_{eq}(j\omega_0)}}^{\to 0}}{\underbrace{\cancel{Z_{eq}(j\omega_0)}}_{0} + Z_R} \right| = \left| \frac{0}{\underbrace{\cancel{Z_R}}_{R}} \right| = 0$$



$$H(j\omega) = \frac{j\left(\omega L - \frac{1}{\omega C}\right)}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\omega_0 = \omega_{AC} = 2\pi \cdot 60\,Hz$$

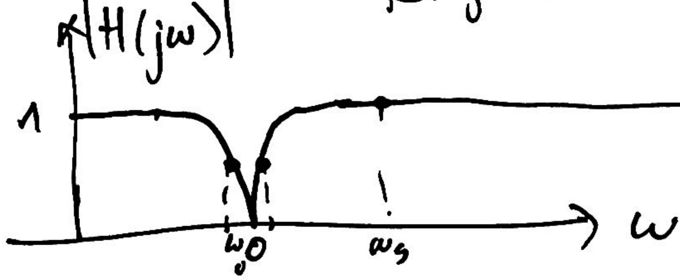
$$C = 100\,\mu F \Rightarrow L = 70\,mH$$

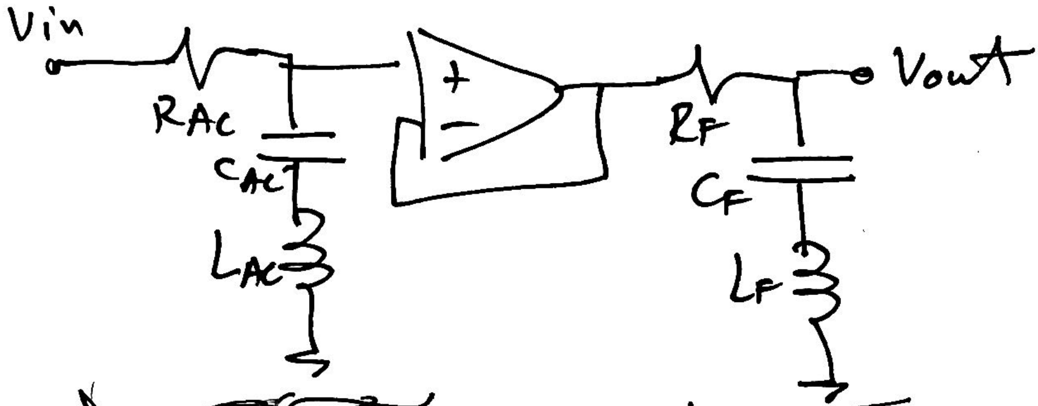
$$H(\underbrace{j\omega_0}_{2\pi\cdot 60Hz}) = 0$$

$$H(j2\pi \cdot 55Hz) = \frac{j \cdot 3.5\Omega}{R + j3.5\Omega} \quad \cancel{\approx 0.5} \text{ for } R = 3\Omega$$

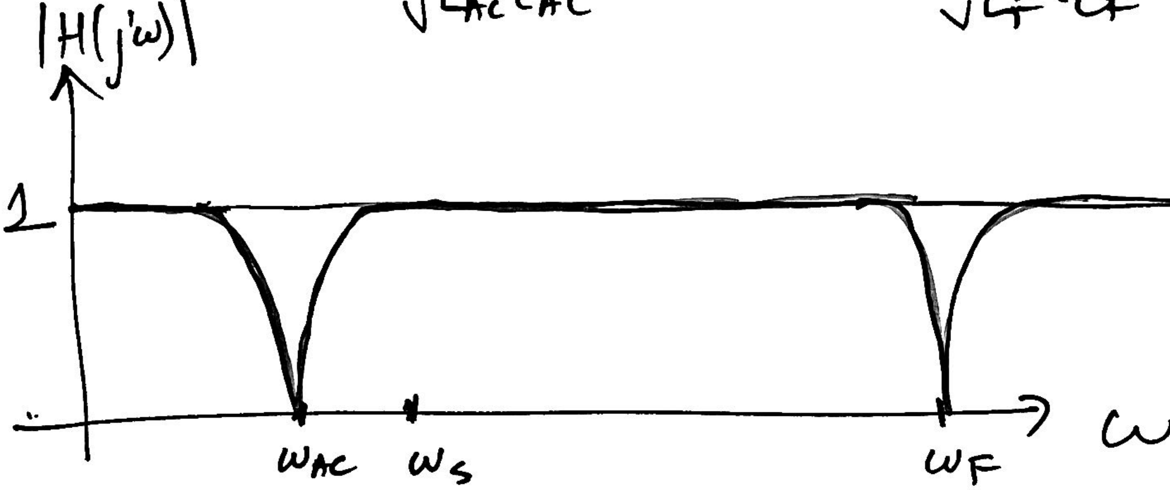
$$H(j2\pi \cdot 65Hz) = \frac{j\,4.7\Omega}{R + j\,4.7\Omega} \quad \cancel{\approx 0.5}$$

$$|H(j2\pi\,55, 65)| \approx 0.5$$





$$\omega_{AC} = \frac{1}{\sqrt{L_{AC} C_{AC}}} \qquad \omega_{F} = \frac{1}{\sqrt{L_F \cdot C_F}}$$



Band-pass: