Discussion 2B

1. Introduction to Inductors

An inductor is a circuit element analogous to a capacitor; its voltage is proportional to the derivative of the current across it. That is:

$$V_L(t) = L\frac{\mathrm{d}I_L(t)}{\mathrm{d}t} \tag{1}$$

When first studying capacitors, we analyzed a circuit where a current source was directly attached to a capacitor. In Figure 1, we form the counterpart circuit for an inductor:

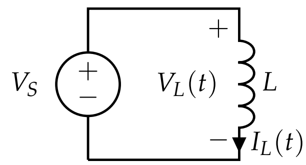


Figure 1: Inductor in series with a voltage source.

(a) **What is the current through an inductor as a function of time? If the inductance is $L = 3\,\mathrm{H}$, what is the current at $t = 6\,\mathrm{s}$?** Assume that the voltage source turns from $0\,\mathrm{V}$ to $5\,\mathrm{V}$ at time $t = 0\,\mathrm{s}$, and there's no current flowing in the circuit before the voltage source turns on, i.e $I_L(0) = 0\,\mathrm{A}$.

(b) Now, we add some resistance in series with the inductor, as in Figure 2.

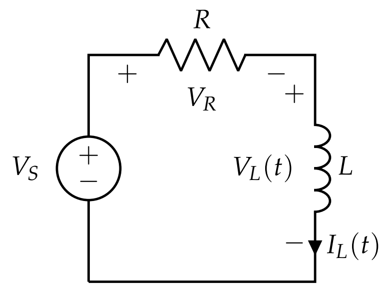
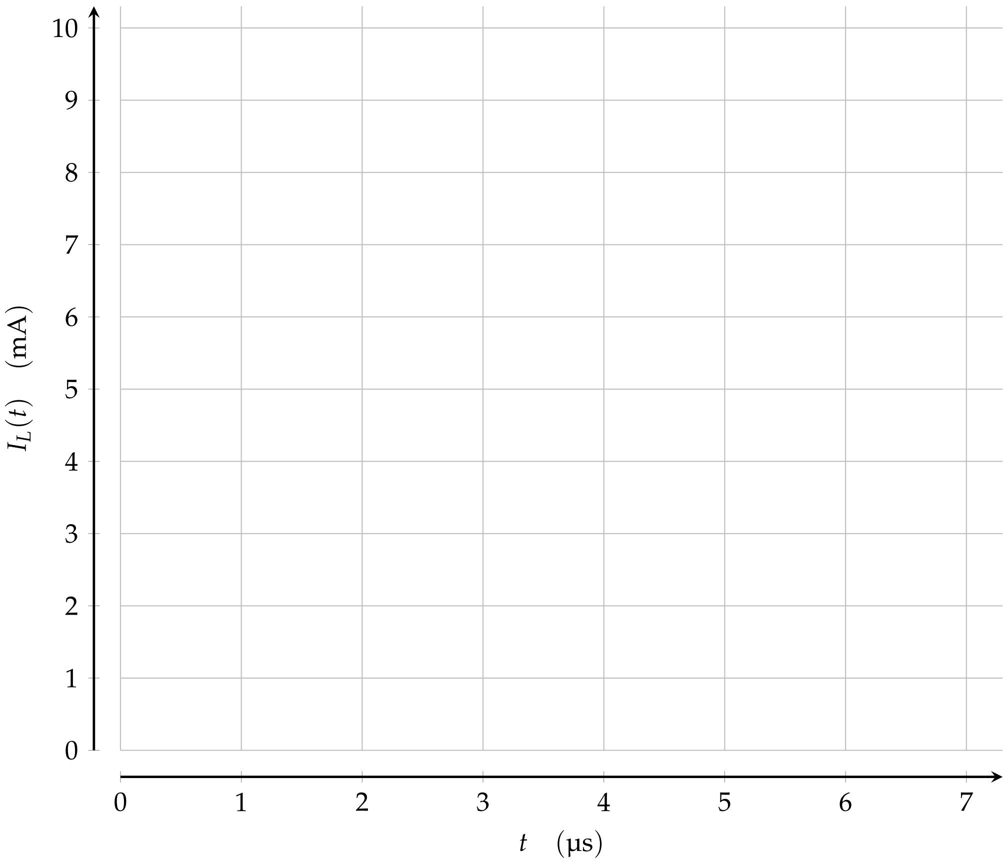


Figure 2: Inductor in series with a voltage source.

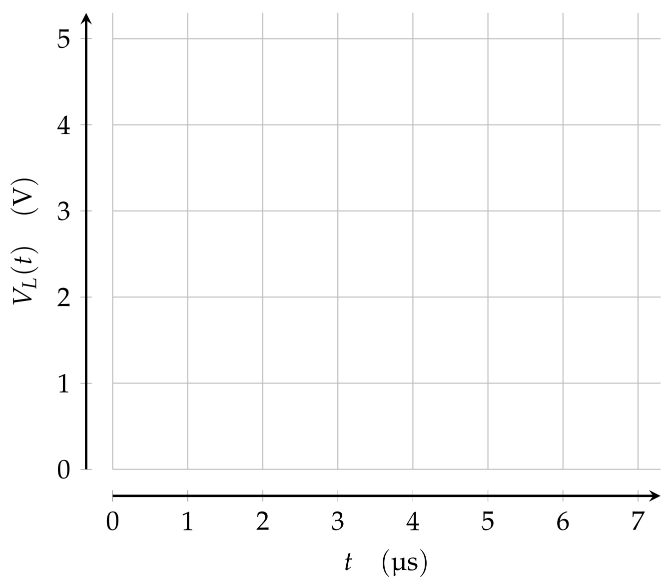
Solve for the current $I_L(t)$ and voltage $V_L(t)$ in the circuit over time, in terms of R, L, V_S, t. Note that $I_L(0) = 0\,\text{A}$. Try to solve this equation by inspection. Otherwise, you can use the following integral for the particular solution (with the proper values and functions):

$$e^{-st} \int e^{st} b(t) dt$$

(c) **Suppose $R = 500\,\Omega, L = 1\,\text{mH}, V_S = 5\,\text{V}$. Plot the current through and voltage across the inductor ($I_L(t), V_L(t)$), as these quantities evolve over time.**



$I_L(t)$ (mA)
0
1
2
3
4
5
6
7
8
9
10
t (μs)
0
1
2
3
4
5
6
7



$V_L(t)$ (V)
0
1
2
3
4
5
t (μs)
0
1
2
3
4
5
6
7

2. RL Circuit Solution Methods

Consider the following circuit:

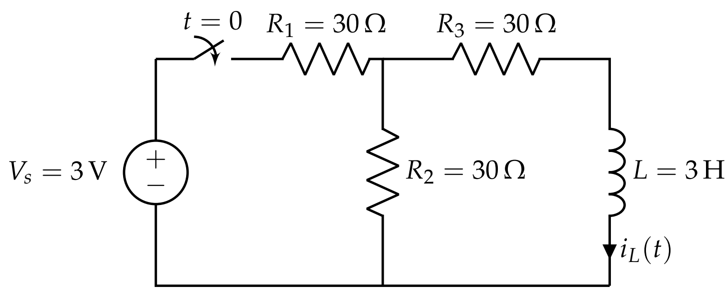


Figure 3

Before time $t = 0$, the circuit reaches a steady state. At time $t = 0$, the switch is closed. Our goal is to find the differential equation for the current through the inductor ($i_L(t)$). One method to approach this problem is to simply use Node Voltage Analysis (NVA). To start, we would define the node voltages in our circuit (including a ground node).

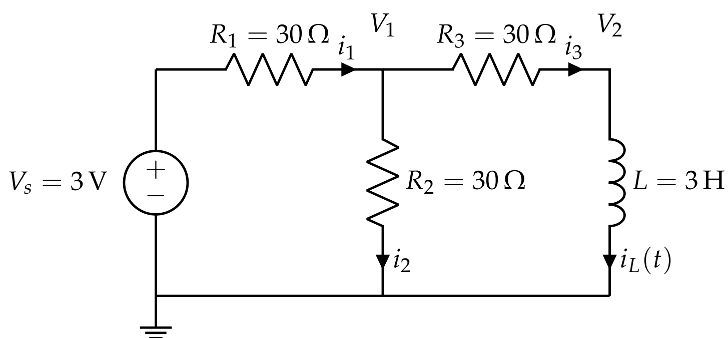


Figure 4

Then, we can set up a system of equations using KCL/KVL to find our desired differential equation. First, let's perform KCL on the node with defined voltage V_1.

$$\begin{aligned} i_1 &= i_2 + i_3 \\ \frac{V_s - V_1}{R_1} &= \frac{V_1 - 0}{R_2} + \frac{V_1 - V_2}{R_3} \\ \frac{3 - V_1}{30} &= \frac{V_1 - 0}{30} + \frac{V_1 - V_2}{30} \\ V_1 &= 1 + \frac{V_2}{3} \end{aligned}$$

Now, let's perform KCL on the node with the defined voltage V_2.

Note that $V_2 - 0 = V_2$ is the voltage across the inductor so by the inductor I-V relationship, $V_2 = L\frac{\mathrm{d}i_L}{\mathrm{d}t} = 3\frac{\mathrm{d}i_L}{\mathrm{d}t}$.

$$i_3 = i_L$$

$$\begin{aligned}
\frac{V_1 - V_2}{R_3} &= i_L \\
\frac{V_1 - V_2}{30} &= i_L \\
\frac{V_1}{30} &= \frac{V_2}{30} + i_L \\
\frac{1}{30}\left(1 + \frac{V_2}{3}\right) &= \frac{V_2}{30} + i_L \\
\frac{1}{45}V_2 + i_L &= \frac{1}{30} \\
\frac{1}{45}\left(3\frac{\mathrm{d}i_L}{\mathrm{d}t}\right) + i_L &= \frac{1}{30} \\
\frac{\mathrm{d}i_L}{\mathrm{d}t} + 15i_L &= \frac{1}{2}
\end{aligned}$$

Thus, we have found the differential equation! However, this method required solving a system of equations; is there another way?

(a) Another way to approach the problem is to use equivalence. Simplify the voltage source and resistor network into a voltage source and resistor using Thevenin equivalence. Then, reconnect the inductor and **find the differential equation for $i_L(t)$.**

For reference, here is the circuit that we want to simplify using Thevenin equivalence:

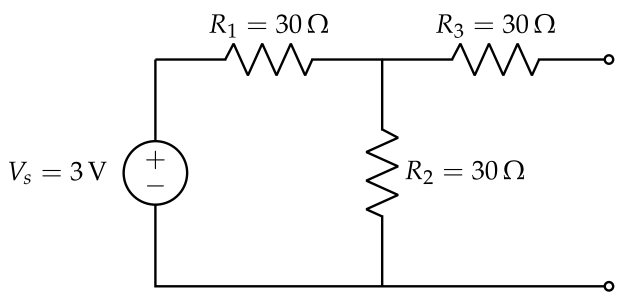


Figure 5

(HINT: Your final differential equation should be the same as the one from the problem introduction.)

(b) Now, let's start solving the differential equation. First, **find the initial condition $i_L(0)$ for our system**. Remember that the current through the inductor cannot change instantaneously (since this would correspond to infinite voltage through the inductor I-V relationship) so $i_L(0)$ will be the same as the steady state value from $t < 0$.

(HINT: If there is no voltage/current sources connected to this system, can there be any nonzero currents / voltage differences in the system during steady-state?)

(c) **(OPTIONAL)** Now that we have our differential equation and initial condition, we can now solve for the current $i_L(t)$ as a function of time. **Solve the system for $i_L(t)$**. If you can, try to solve this by inspection. Otherwise, you can use the following integral to find the particular solution (remember to use the values/functions that correspond to this specific differential equation):

$$\mathrm{e}^{-st} \int \mathrm{e}^{st} b(t) dt$$

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