Discussion 6A

1. AC Power Calculations (Hambley Example 5.7)

Suppose you are given the circuit in Figure 1, where the phasor for current $i(t)$ is calculated to be $\frac{\sqrt{2}}{10}e^{-\frac{3\pi}{4}j}$.

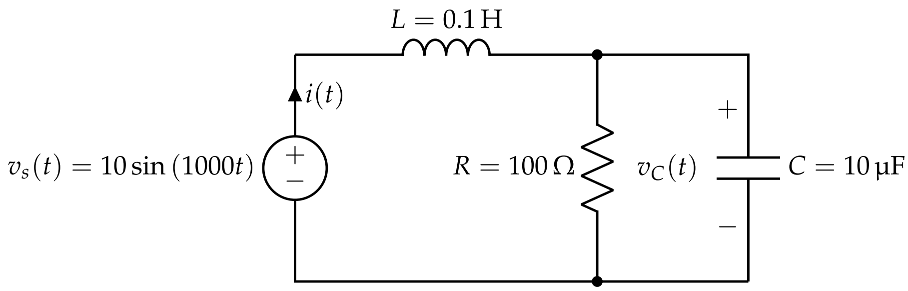


Figure 1: RLC Circuit

(a) **Compute the power and reactive power taken from the source in the circuit provided in Figure 1.**

Solution: To find the (average) power and reactive power for the source, we have to find the power angle given by the equation:

$$\phi = \phi_v - \phi_i \tag{1}$$

where ϕ_v is the phase of the voltage and ϕ_i is the phase of the current. The angle of the source voltage is $\phi_v = -90°$ (due to the convention of writing sinusoidal quantities in terms of cosine) and the angle of the current delivered by the source is $\phi_i = -135°$. Therefore, we have:

$$\phi = -90° - (-135°) = 45° \tag{2}$$

The rms source voltage and current are

$$V_{s,rms} = \frac{|\mathbf{V_s}|}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.071\,\text{V} \tag{3}$$

$$I_{rms} = \frac{|\mathbf{I}|}{\sqrt{2}} = \frac{0.1414}{\sqrt{2}} = 0.1\,\text{A} \tag{4}$$

Now, using the following equations,

$$P_{\text{avg}} = V_{s,rms} I_{rms} \cos(\phi) \tag{5}$$

$$P_{\text{react}} = V_{s,rms} I_{rms} \sin(\phi) \tag{6}$$

we can solve for the power and reactive power.

$$P_{\text{avg}} = \frac{10}{\sqrt{2}} \times 0.1 \cos(45°) = 0.5\,\text{W} \tag{7}$$

$$P_{\text{react}} = \frac{10}{\sqrt{2}} \times 0.1 \sin(45°) = 0.5\,\text{VAR} \tag{8}$$

Another Method:

Another way to compute P_{avg} and P_{react} is to find complex power and then take the real and imaginary parts. Recall, that the complex power is simply the amplitude of the full sinusoidal power function. $\frac{1}{2}\widetilde{\mathbf{V_s}}\widetilde{\mathbf{I}}^*$:

$$\mathbf{P} = \frac{1}{2}\widetilde{\mathbf{V_s}}\widetilde{\mathbf{I}}^* \tag{9}$$
$$= \frac{1}{2}\left(10e^{\frac{-\pi}{2}j}\right)\left(\frac{\sqrt{2}}{10}e^{-\frac{3\pi}{4}j}\right)^* \tag{10}$$
$$= \frac{1}{2}\left(10e^{\frac{-\pi}{2}j}\right)\left(\frac{\sqrt{2}}{10}e^{\frac{3\pi}{4}j}\right) \tag{11}$$
$$= \frac{\sqrt{2}}{2}e^{\frac{\pi}{4}j} \tag{12}$$
$$= 0.5 + j0.5 \tag{13}$$

Then, we have:

$$P_{\text{avg}} = \text{Re}\{P\} = 0.5\,\text{W} \tag{14}$$
$$P_{\text{react}} = \text{Im}\{P\} = 0.5\ \text{VAR} \tag{15}$$

(b) **Compute the power and reactive power delivered to each element in the circuit.** Assume you are given the computed currents in Figure 2.

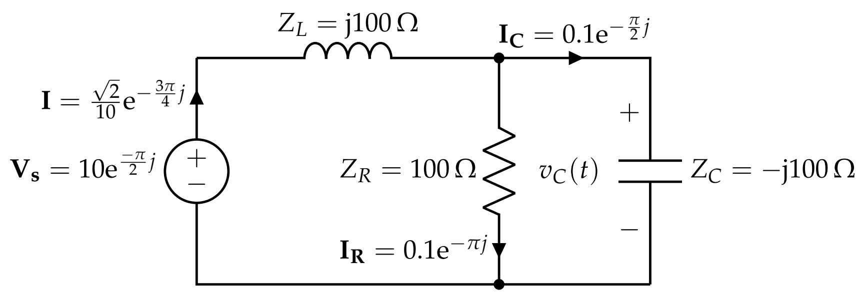


Figure 2: RLC Circuit

Solution: For the reactive power delivered to the inductor, we have:

$$P_{\text{L,react}} = I_{\text{rms}}^2 X_L \tag{16}$$
$$= (0.1)^2(100) \tag{17}$$
$$= 1.0\ \text{VAR} \tag{18}$$

Similar for the capacitor we have:

$$P_{\text{C,react}} = I_{c,\text{rms}}^2 X_C \tag{19}$$
$$= \left(\frac{0.1}{\sqrt{2}}\right)^2(-100) \tag{20}$$
$$= -0.5\ \text{VAR} \tag{21}$$

Notice that the reactance X_C of the capacitance is negative. As expected, the reactive power is negative for a capacitance. The reactive power for the resistance is zero. To verify our work, let's check that the reaction power delivered by the source is equal to the sum of the reactive powers absorbed by the inductance and capacitance:

$$P_{react} = P_{L,react} + P_{C,react} \tag{22}$$

We have that the power delivered to the resistance

$$\begin{aligned} P_{R,avg} &= I_{R,\text{rms}}^2 R && (23)\\ &= \left(\frac{|\mathbf{I_R}|}{\sqrt{2}}\right)^2 R && (24)\\ &= \left(\frac{0.1}{\sqrt{2}}\right)^2 100 && (25)\\ &= 0.5\,\text{W} && (26) \end{aligned}$$

The power absorbed by the capacitance and inductance is given by $P_L = 0$ and $P_C = 0$. Therefore, all of the power delivered by the source is absorbed by the resistance.

2. Hambley P5.83

(a) Find the Thevenin and Norton equivalent circuits for the circuit shown in Figure 3.

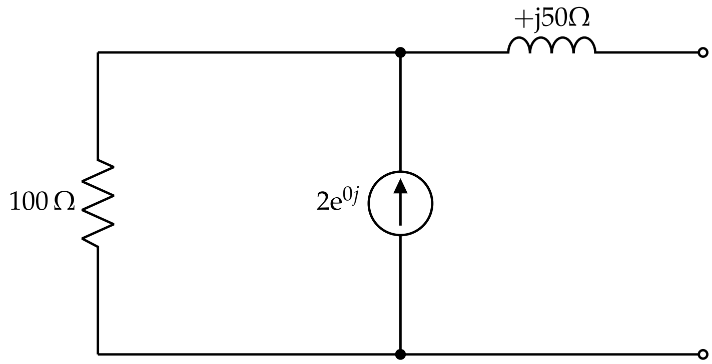
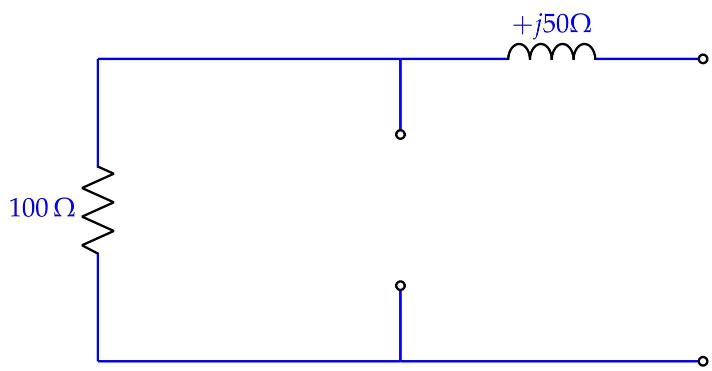
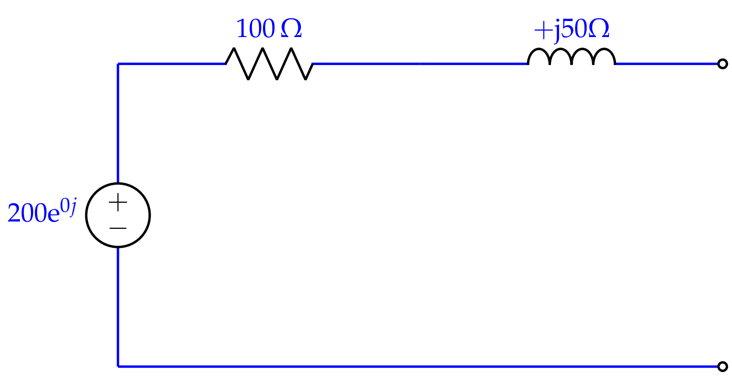


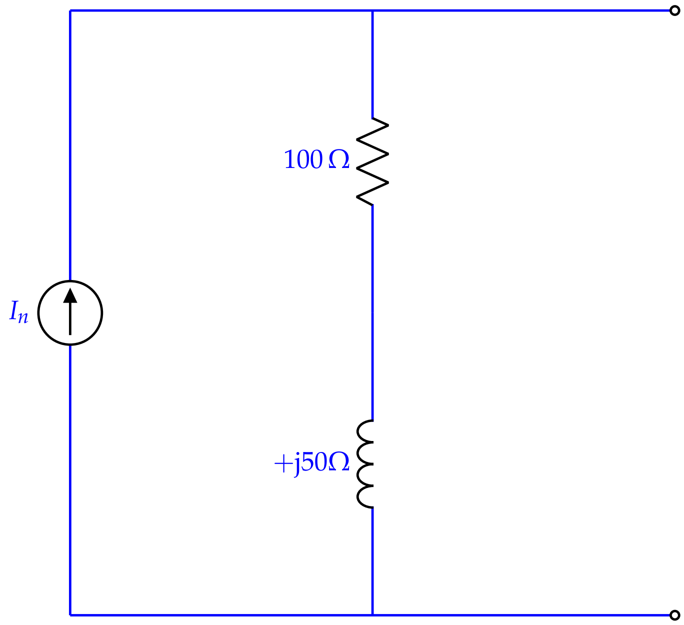
Figure 3: P5.83

Solution: Zeroing the current source, we have



Thus, the Thevenin impedance is $Z_t = 100 + 50j$. Under open circuit conditions, there is zero voltage across the inductance, since the current flows through the resistance. Hence, the Thevenin voltage is $V_t = (2e^{0j})(100) = 200e^{0j}$. From this, the Norton current is $I_n = \frac{V_t}{Z_t} = \frac{8}{5} - \frac{4}{5}j$. The Thevenin and Norton equivalent circuits are shown below respectively:





(b) Find the maximum power that this cicuit can deliver to a load if the load can have any complex impedance. *Hint: Think about which impedance value for the load would optimize the power expression.*

Solution: The load that would maximize power is $Z_{\text{load}} = (Z_t)^* = 100 - \text{j}50$. Using the Thevenin equivalent circuit, the current in this circuit would then be

$$I_{\text{load}} = \frac{V_t}{Z_t + Z_{\text{load}}} = \frac{200}{100 + \text{j}50 + 100 - \text{j}50} = 1 \tag{27}$$

so $I_{\text{load-RMS}} = \frac{1}{\sqrt{2}}$. Finally, the power would be $P_{\text{load,avg}} = R_{\text{load}}(I_{\text{load-RMS}})^2 = 100\left(\frac{1}{\sqrt{2}}\right)^2 = 50\,\text{W}$.

(c) Repeat the previous part, but this time the load is purely resistive.

Solution: The purely resistive load that would maximize power is $Z_{\text{load}} = |Z_t| = 111.8$. Again, using the Thevenin equivalent circuit, we find that

$$|I_{\text{load}}| = \left|\frac{V_t}{Z_t + Z_{\text{load}}}\right| = \left|\frac{200}{100 + \text{j}50 + 111.8}\right| = 0.9190 \tag{28}$$

so $I_{\text{load-RMS}} = \frac{0.9190}{\sqrt{2}}$. Finally, the power would be $P_{\text{load,avg}} = R_{\text{load}}(I_{\text{load-RMS}})^2 = 100\left(\frac{0.9190}{\sqrt{2}}\right)^2 = 47.21\,\text{W}$.

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