# Discussion 6B

**1. Plotting and Combining Transfer Functions**

Recall that any transfer function (which is a complex function dependent on $\omega$) can be written in polar form as

$$H(\mathrm{j}\omega) = |H(\mathrm{j}\omega)|\mathrm{e}^{\mathrm{j}\measuredangle H(\mathrm{j}\omega)} \tag{1}$$

where $|H(\mathrm{j}\omega)|$ and $\measuredangle H(\mathrm{j}\omega)$ are real functions of $\omega$ giving the magnitude and phase of the transfer function, respectively. To see how transfer functions combine, consider two transfer functions $H_1(\mathrm{j}\omega)$ and $H_2(\mathrm{j}\omega)$.

$$H_1(\mathrm{j}\omega) = |H_1(\mathrm{j}\omega)|\mathrm{e}^{\mathrm{j}\measuredangle H_1(\mathrm{j}\omega)} \tag{2}$$

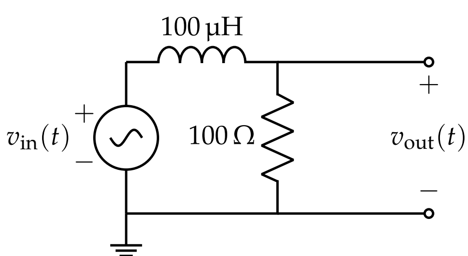
$$H_2(\mathrm{j}\omega) = |H_2(\mathrm{j}\omega)|\mathrm{e}^{\mathrm{j}\measuredangle H_2(\mathrm{j}\omega)} \tag{3}$$

$$H_1(\mathrm{j}\omega) \cdot H_2(\mathrm{j}\omega) = |H_1|\mathrm{e}^{\mathrm{j}\measuredangle H_1}|H_2|\mathrm{e}^{\mathrm{j}\measuredangle H_2} = |H_1||H_2|\mathrm{e}^{\mathrm{j}(\measuredangle H_1 + \measuredangle H_2)} \tag{4}$$

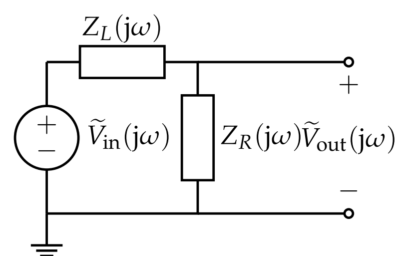
$$\frac{H_1(\mathrm{j}\omega)}{H_2(\mathrm{j}\omega)} = \frac{|H_1|\mathrm{e}^{\mathrm{j}\measuredangle H_1}}{|H_2|\mathrm{e}^{\mathrm{j}\measuredangle H_2}} = \frac{|H_1|}{|H_2|}\mathrm{e}^{\mathrm{j}(\measuredangle H_1 - \measuredangle H_2)} \tag{5}$$

As you can see, magnitudes of transfer functions multiply/divide while the phases add/subtract.

In this problem we will examine the transfer function of fig. 1a.




**(a)** An LR filter in the "time-domain".




**(b)** An LR filter in the "phasor-domain".

**Figure 1:** Circuit schematic of LR filter in both domains.

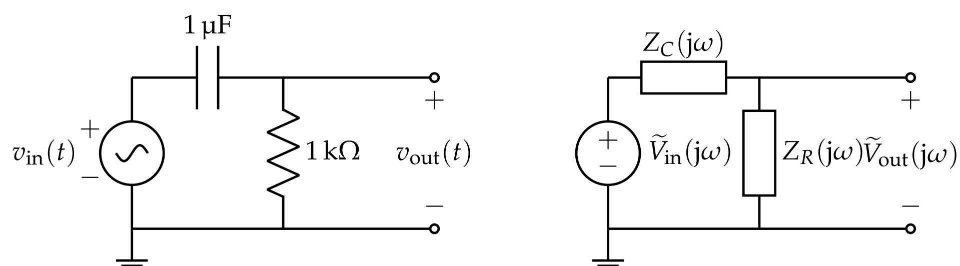
(a) First, **solve for** $H(\mathrm{j}\omega)$. Then, **write expressions for** $|H(\mathrm{j}\omega)|$ **and** $\measuredangle H(\mathrm{j}\omega)$. For now, you can keep it in terms of $R$ and $L$.

(b) **What is the cutoff frequency for this circuit?**. Note that the values of the circuit elements are given in fig. 2a.

Recall that a transfer function of the form $H(\mathrm{j}\omega) = \frac{k}{1+\frac{\mathrm{j}\omega}{\omega_c}}$ is defined to have a cutoff frequency of $\omega_c$. This is because a signal with an input frequency of $\omega_c$ exactly will have its magnitude attenuated by a factor of $\frac{1}{\sqrt{2}}$, which is a conventionally convenient number.

(c) Now suppose we want to compose the filter from fig. 2a with the filter from earlier (fig. 1a). Use $R = 1\,\mathrm{k}\Omega$ and $C = 1\,\mu\mathrm{F}$ for the RC filter. We can compose two circuits by connecting the output of the first circuit into the second circuit, through a unity gain buffer. For this problem, the transfer function of the LR filter from this worksheet fig. 1a is $H_1$, and the transfer function of the other RC filter is $H_2$. The transfer function of the composed circuit is:

$$H(\mathrm{j}\omega) = H_1(\mathrm{j}\omega) \cdot H_2(\mathrm{j}\omega) \tag{6}$$




**(a)** An RC high-pass filter in the "time-domain". **(b)** An RC high-pass filter in the "phasor-domain".

**Draw this circuit**.

*(HINT: Follow the problem description. You will need to connect the output of the first filter to the input of the second filter in some way, but doing this directly would result in loading; you will need to add one more element to prevent this.)*

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**2. Bode Plots (straight-line approximations) and filters**

Our eventual goal is to construct Bode plots of the following circuit, with $L = 100\,\mu\text{H}$, $C = 1\,\mu\text{F}$, $R_1 = 100\,\Omega$, and $R_2 = 1\,\text{k}\Omega$:

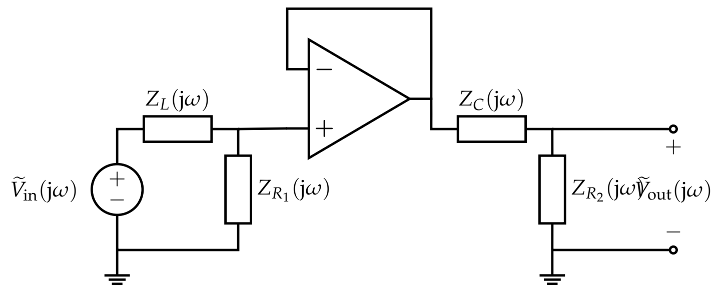
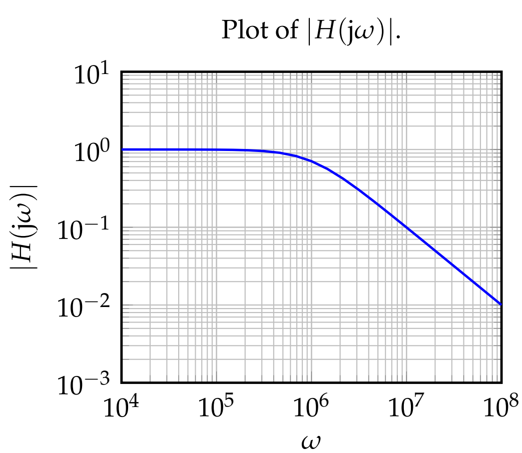


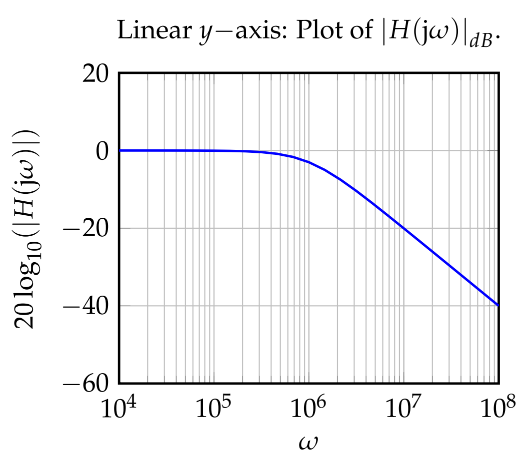

**Figure 3**

To do this we will leverage the fact that Bode plots can be composed in systematic ways.

Before we dive into the problem, let's consider a modification of the *magnitude* plot that will help us work with multiple magnitude plots at once. Namely, instead of plotting $|H(j\omega)|$ vs. $\omega$ where the $y$-axis is on a *logarithmic* scale, we plot $20\log_{10}(|H(j\omega)|)$ vs. $\omega$ instead, and now the $y$-axis is on a *linear* scale. (This is known as decibel, or dB, scale.)

Why would we want to do this? Well, when combining magnitude transfer functions, we end up multiplying them. But we really want to add two plots *graphically* for simplicity, not multiply them, so we will just plot and add the logarithms. (The constant multiple 20 is there for convention reasons, related to decibels.)




Notice that we do not need to do this for the *phase* plots, since their $y$ axes are naturally in linear scale, and combining plots can already be done by addition. Now we are ready to begin working on the problems.

(a) Consider the first half of this circuit:

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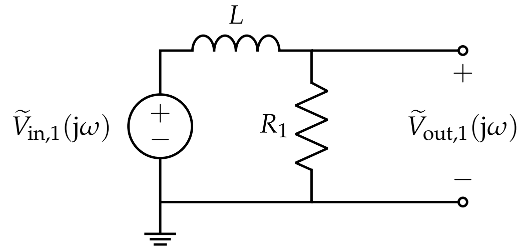



We learned in the previous problem that the transfer function is given by

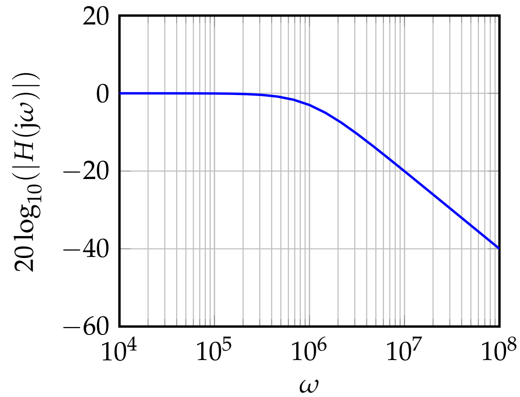
$$H_1(\mathrm{j}\omega) = \frac{\widetilde{V}_{\text{out},1}}{\widetilde{V}_{\text{in},1}} = \frac{R_1}{R_1 + \mathrm{j}\omega L} = \frac{1}{1 + \mathrm{j}\omega \frac{L}{R_1}} \tag{7}$$

and the cutoff frequency $\omega_{c,1}$ is given by

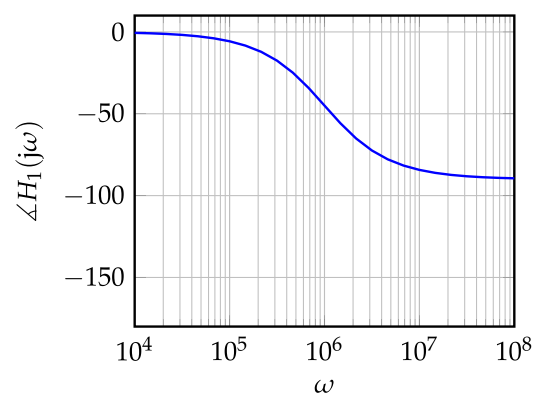
$$\omega_{c,1} = \frac{R_1}{L} = \frac{100\,\Omega}{100\,\mu\mathrm{H}} = 1 \times 10^6 \, \frac{\text{rad}}{\text{s}} \tag{8}$$

If we plot $|H_1(\mathrm{j}\omega)|_{dB}$ and $\measuredangle H_1(\mathrm{j}\omega)$ using a computer, we would get the following:

Plot of $|H_1(\mathrm{j}\omega)|_{dB}$.




Plot of $\measuredangle H_1(\mathrm{j}\omega)$.




On the above grids, **draw the Bode plots (piecewise linear approximations) for magnitude and phase**.

*(HINT: Notice that both plots seem to have natural transitions that occur around the cutoff frequency $\omega_{c,1}$. For the magnitude plot, you should have two piecewise linear segments ($\omega < \omega_{c,1}$ and $\omega > \omega_{c,1}$). For the phase plot, you should have three piecewise linear segments ($\omega < \frac{1}{10}\omega_{c,1}$, $\frac{1}{10}\omega_{c,1} < \omega < 10\omega_{c,1}$, and $\omega > 10\omega_{c,1}$).)*

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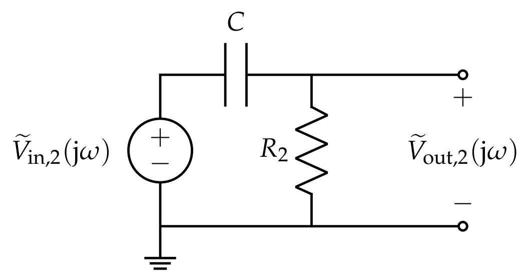
(b) Consider the second half of the circuit:



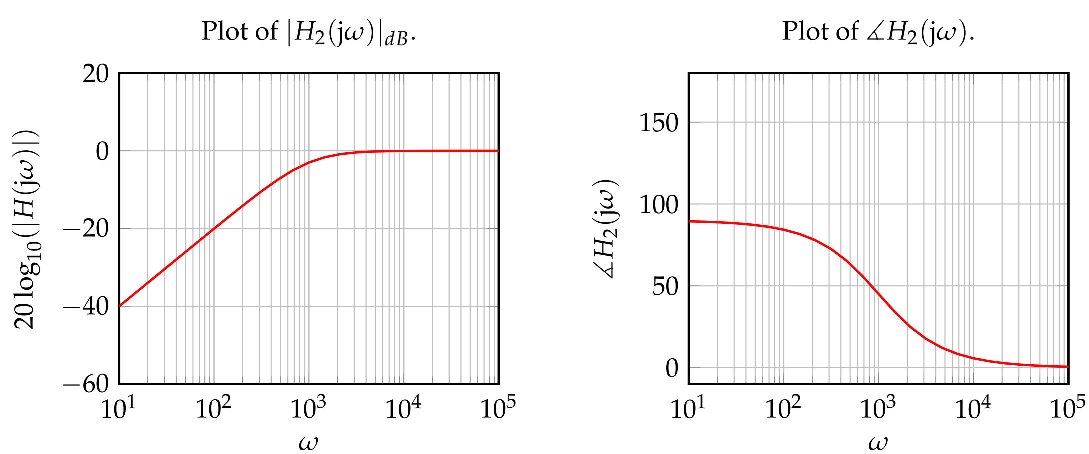

We learned in a previous discussion that the transfer function is given by

$$H_2(\mathrm{j}\omega) = \frac{\widetilde{V}_{\text{out},2}}{\widetilde{V}_{\text{in},2}} = \frac{\mathrm{j}\omega R_2 C}{1 + \mathrm{j}\omega R_2 C} \tag{9}$$

and the cutoff frequency $\omega_{c,2}$ is given by

$$\omega_{c,2} = \frac{1}{R_2 C} = \frac{1}{(1\,\mathrm{k\Omega}) \cdot (1\,\mu\mathrm{F})} = 1 \times 10^3 \, \frac{\text{rad}}{\text{s}} \tag{10}$$

If we plot $|H_2(\mathrm{j}\omega)|_{dB}$ and $\measuredangle H_2(\mathrm{j}\omega)$ using a computer, we would get the following:




On the above grids, **draw the Bode plots (piecewise linear approximations) for magnitude and phase**.

*(HINT: Same hint as the previous part.)*

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(c) Now, we will put this circuit together. Recall the diagram in fig. 3:

We saw earlier in the discussion that the transfer function is

$$H(\mathrm{j}\omega) = \frac{\widetilde{V}_{\text{out}}}{\widetilde{V}_{\text{in}}} = H_1(\mathrm{j}\omega)H_2(\mathrm{j}\omega) = \frac{\mathrm{j}\omega R_2 C}{(1+\mathrm{j}\omega\frac{L}{R_1})(1+\mathrm{j}\omega R_2 C)} \tag{11}$$

To plot $|H(\mathrm{j}\omega)|_{dB}$ and $\measuredangle H(\mathrm{j}\omega)$ we can use what we know about the plots for $H_1(\mathrm{j}\omega)$ and $H_2(\mathrm{j}\omega)$, as well as how the magnitude and phase of complex numbers change when multiplied with each other.

On the provided grids, **draw the Bode plots (piecewise linear approximations) for magnitude and phase**.

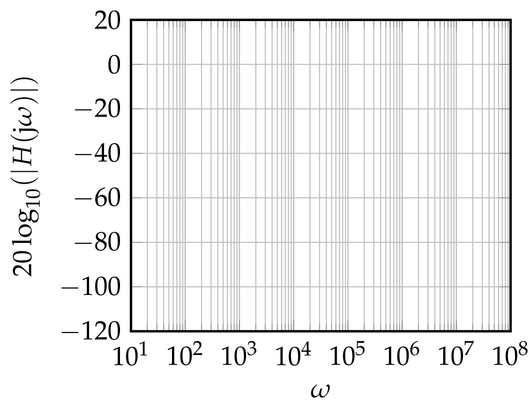
*Hint*: Recall that

$$20\log_{10}(|H(\mathrm{j}\omega)|) = 20\log_{10}(|H_1(\mathrm{j}\omega)H_2(\mathrm{j}\omega)|) = 20\log_{10}(|H_1(\mathrm{j}\omega)||H_2(\mathrm{j}\omega)|) \tag{12}$$

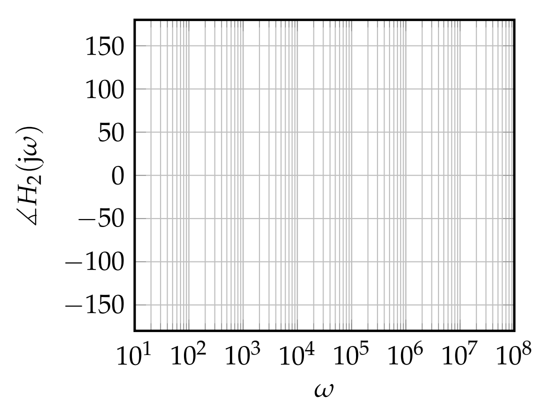
$$= 20\log_{10}(|H_1(\mathrm{j}\omega)|) + 20\log_{10}(|H_2(\omega)|) \tag{13}$$

$$\measuredangle H(\mathrm{j}\omega) = \measuredangle H_1(\mathrm{j}\omega) + \measuredangle H_2(\mathrm{j}\omega). \tag{14}$$

Plot of $|H_2(\mathrm{j}\omega)|_{dB}$.




Plot of $\measuredangle H_2(\mathrm{j}\omega)$.

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