

**This homework is due on Tuesday, January 24th, at 11:59PM.  
Self-Grades, HW Resubmissions, and HW Resubmission Self-Grades  
are due on Friday, January 27, at 11:59PM.**

**NOTE: All other homeworks follow a Friday-to-Friday cycle, in which the homework is due on a given Friday and the Self-Grades/Resubmission/Resubmission Self-Grades are due the following Friday.**

### **1. Administration**

- (a) Please fill out our introductory survey: [link to survey](#). This survey will help us understand our students' prerequisite knowledge for content creation purposes.
- (b) Please complete the Administrative Policy Quiz assignment on Gradescope. The goal is to ensure that everyone is familiar with the course policies, which you can read about [here](#). Take your percent score on the Gradescope assignment, multiply by 10 and round up to either 2, 5, 8, or 10. This is your self-grade score.
- (c) Please fill out [this](#) group formation survey if you are interested in getting matched up in a study group. We highly recommend joining a study group in order to foster a sense of community in the course and learn from others. Within a few weeks, you should get an email informing you of the group you have been matched with. Please follow up with your group members; completing this survey suggests you are interested in joining a group, after all! Please fill out the survey by this Friday so that we can get you your groups as soon as possible. The late deadline to request a group is the HW0 deadline on Tuesday, January 24. Just so you have an answer to put down for this question, write down whether you filled out the survey or not.

## 2. Least Squares

- (a) Consider the system of equations  $\vec{a}x = \vec{b}$  where  $\vec{a}, \vec{b} \in \mathbb{R}^2$  and  $x \in \mathbb{R}$ .
- When applying least squares, we want to find the  $\vec{v} \in \text{Span}(\vec{a})$  that is closest to  $\vec{b}$  in Euclidean distance.  
(HINT: It might be helpful to draw the vectors.)
    - Projecting  $\vec{b}$  onto  $\vec{a}$
    - Projecting  $\vec{a}$  onto  $\vec{b}$
    - Subtracting  $\vec{b}$  from  $\vec{a}$
    - Subtracting  $\vec{a}$  from  $\vec{b}$
    - None of the above
  - The vector  $\vec{v}$  can also be determined by minimizing the length of the error vector, represented as
    - $\vec{v} = \operatorname{argmin}_{\vec{b}} \|\vec{a} - \vec{b}\|$
    - $\vec{v} = \operatorname{argmin}_{\vec{v}} \|\vec{a} - \vec{v}\|$
    - $\vec{v} = \operatorname{argmin}_{\vec{b}} \|\vec{b} - \vec{v}\|$
    - $\vec{v} = \operatorname{argmin}_{\vec{v}} \|\vec{b} - \vec{v}\|$
- (b) For the following systems of  $A\vec{x} = \vec{b}$ , determine if they have a unique least squares solution.
- $A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \\ 0 & 0 \end{bmatrix}$ ,  $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ 
    - Yes
    - No
  - $A = \begin{bmatrix} 1 & 4 \\ 3 & 12 \\ 2 & 8 \end{bmatrix}$ ,  $\vec{b} = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$ 
    - Yes
    - No
- (c) For the following three questions, consider the system of  $A\vec{x} = \vec{b}$  with  $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$  and  $\vec{b} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$
- Can we apply the least squares formula?
    - Yes
    - No
  - What is the determinant of  $A^\top A$ ?
  - Does  $A\vec{x} = \vec{b}$  have zero, one, or more than one solution for  $\vec{x}$ ?
    - No solutions
    - One unique solution

(C) More than one solution

(d) Find the best approximation  $x = \hat{x}$  to this system of equations:

$$a_1x = b_1 \tag{1}$$

$$a_2x = b_2 \tag{2}$$

i. Write the problem into  $A\vec{x} \approx \vec{b}$  format and solve for  $\hat{x}$  using least squares. Choose the correct  $\hat{x}$ .

(A)  $\hat{x} = \frac{a_1b_1 + a_2b_2}{a_1^2 + a_2^2}$

(B)  $\hat{x} = \frac{a_1b_1 - a_2b_2}{a_1^2 + a_2^2}$

(C)  $\hat{x} = \frac{a_1b_2 + a_2b_1}{a_1^2 + a_2^2}$

(D)  $\hat{x} = \frac{a_1b_2 - a_2b_1}{a_1^2 + a_2^2}$

(E) None of the above

ii. Suppose the inner product is defined instead as a non-Euclidean  $\langle x, y \rangle = x^\top \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} y$ .

Which of the following expressions must be true with respect to the minimized least squares error vector,  $\vec{e}$ ?

(A)  $\vec{e}^\top A = \vec{0}$

(B)  $A^\top \vec{e} = \vec{0}$

(C)  $A^\top \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \vec{e} = \vec{0}$

(D)  $\left( A^\top \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} A \right)^{-1} \vec{e} = \vec{0}$

(E) None of the above

## 3. Eigenstuff

(a) You are provided the matrix  $\mathbf{A} = \begin{bmatrix} 1 & 0.4 & 0.7 \\ 0 & 0.6 & 0.2 \\ 0 & 0 & 0.1 \end{bmatrix}$ , and matrix  $\mathbf{B} = \begin{bmatrix} 1 - \alpha & 0.4 & 0.7 \\ 0 & 0.6 - \alpha & 0.2 \\ 0 & 0 & 0.1 - \alpha \end{bmatrix}$

where  $\alpha \in \mathbb{R}$ . If there exists a vector  $\vec{x} \in \mathbb{R}^3$  such that  $\mathbf{B}\vec{x} = \vec{0}$  and  $\vec{x} \neq \vec{0}$ , which of the following are true? (**Select all that apply.**)

- (A)  $\text{rank}(\mathbf{A}) = 3$
- (B)  $\vec{x}$  is in the null space of  $\mathbf{B}$
- (C)  $\vec{x}$  is in an eigenspace of  $\mathbf{B}$
- (D)  $\vec{x}$  is in an eigenspace of  $\mathbf{A}$

(b) You are given that one of the eigenvalues of  $\mathbf{A} = \begin{bmatrix} 1 & 0.4 & 0.7 \\ 0 & 0.6 & 0.2 \\ 0 & 0 & 0.1 \end{bmatrix}$  is  $\lambda = 1$ . Determine one possible eigenvector  $\vec{v}$ .

(A)  $\vec{v} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

(B)  $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

(C)  $\vec{v} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

(D)  $\vec{v} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$

(c) Now you are provided a third matrix  $\mathbf{C} = \begin{bmatrix} 0.2 & 0.8 & 0.2 \\ 0 & 0.4 & 0.2 \\ 0 & 0 & 0.8 \end{bmatrix}$  with eigenvectors  $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ ,  $\vec{v}_2 = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$ ,

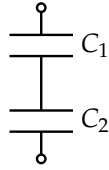
and  $\vec{v}_3 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ . Matrix  $\mathbf{C}$  is transition matrix where  $\vec{x}[t+1] = \mathbf{C}\vec{x}[t]$ . Additionally, the state vector

at timestep  $t = 1$  is  $\vec{x}[1] = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$ . **After infinite timesteps, what is the value of the state vector  $\vec{x}[t]$ ? That is, find  $\lim_{t \rightarrow \infty} \vec{x}[t]$ .**

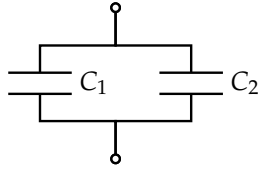
#### 4. Modeling Weird Capacitors

For parts (a) - (c) of this problem, **pick the circuit option from below that best models the given physical capacitor.**

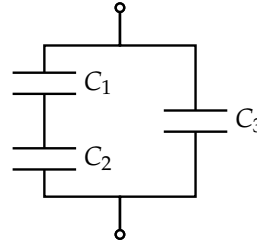
Option 1



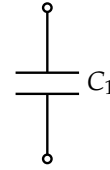
Option 2



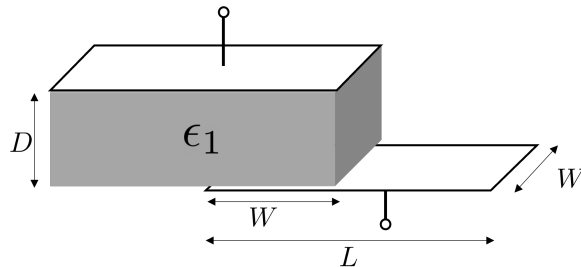
Option 3



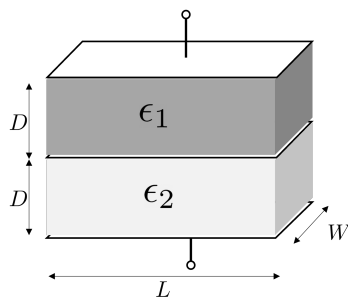
Option 4



- (a) A parallel plate capacitor with plate dimensions  $L$  and  $W$ , separated by a gap  $D$ , is filled with an insulator of permittivity  $\epsilon_1$ , with the bottom plate displaced with overlap  $W$  as shown below. You can assume  $W < L$  and  $D \ll W$ .



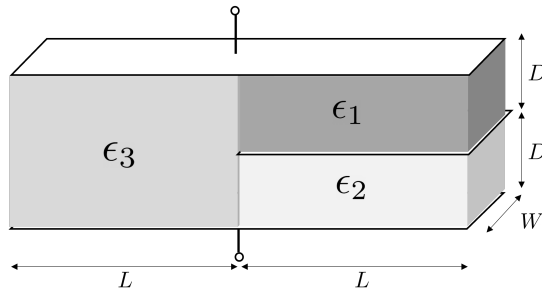
- i. What is the circuit option that best models the physical capacitor?
    - (A) Option 1
    - (B) Option 2
    - (C) Option 3
    - (D) Option 4
  - ii. What is the total capacitance,  $C$ , for this capacitor? Express your answer in terms of  $\epsilon_1$ ,  $D$ ,  $L$ , and  $W$ .
- (b) A parallel plate capacitor with plate dimensions  $L$  and  $W$ , separated by a gap  $2 \cdot D$ , is filled with two insulators of permittivities  $\epsilon_1$  and  $\epsilon_2$  as shown below. You can assume there's a plate between the two dielectrics. What is the circuit option that best models the physical capacitor?



- (A) Option 1
- (B) Option 2

- (C) Option 3
- (D) Option 4

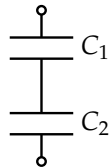
(c) A parallel plate capacitor with plate dimensions  $L$  and  $W$ , separated by a gap  $2 \cdot D$ , is filled with three different materials with permittivities  $\epsilon_1$ ,  $\epsilon_2$ , and  $\epsilon_3$  as shown in the figure below. You can assume there's a plate between the two dielectrics on the right. What is the circuit option that best models the physical capacitor?



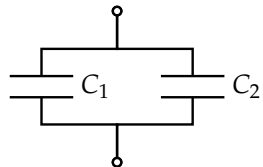
- (A) Option 1
- (B) Option 2
- (C) Option 3
- (D) Option 4

(d) For this final part, please express the equivalent capacitance,  $C_{eq}$ , between the top and bottom node for each of the following circuits from the previous parts. Feel free to include the *parallel operator* (" $\parallel$ ") in your answer.

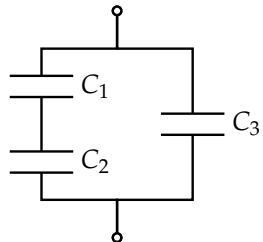
i. Option 1



ii. Option 2

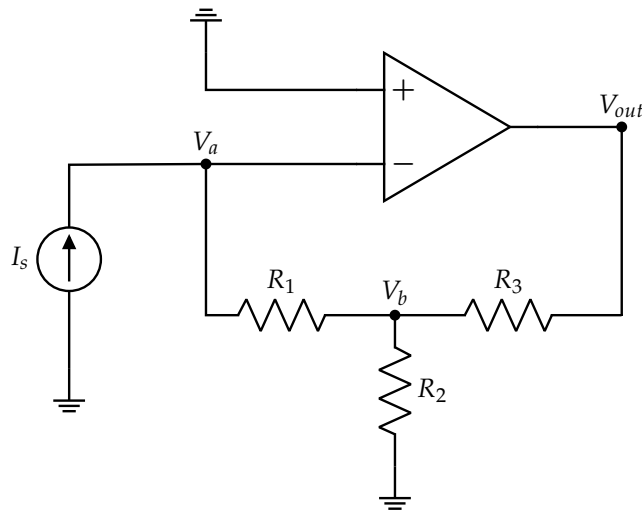


iii. Option 3



### 5. Op-Amp Analysis!

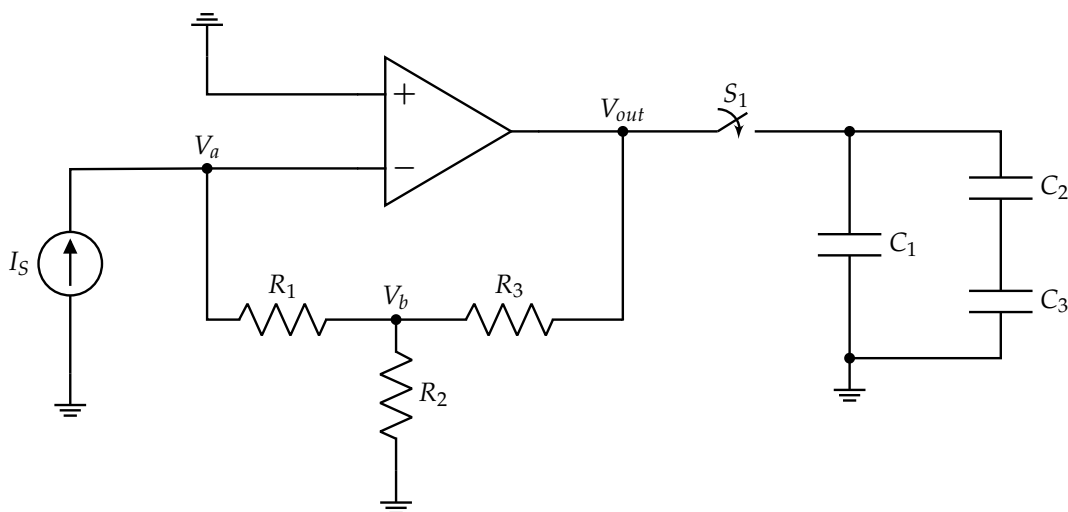
- (a) We want to find a relationship between the output voltage,  $V_{out}$ , and the input current,  $I_s$ , in the circuit below.



- i. Determine the node voltage  $V_a$  in terms of  $I_s$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .
- ii. Determine the node voltage  $V_b$  in terms of  $I_s$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .
- iii. Choose the correct expression for the output voltage  $V_{out}$  in terms of  $I_s$ ,  $V_b$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .

- (A)  $V_{out} = \left(1 - \frac{R_3}{R_2}\right) \cdot V_b - I_s \cdot R_1$   
 (B)  $V_{out} = V_b$   
 (C)  $V_{out} = \left(1 + \frac{R_3}{R_2}\right) \cdot V_b - I_s \cdot R_3$   
 (D)  $V_{out} = \frac{R_3 + R_2}{R_2} V_b$   
 (E)  $V_{out} = \left(1 - \frac{R_3}{R_2}\right) \cdot V_b - I_s \cdot (R_1 + R_3)$

- (b) Now, we will connect a set of capacitors to our previous circuit with an initially open switch  $S_1$ , as follows:



Now assume the output voltage is  $V_{out} = 5\text{ V}$ . Also, assume the capacitors  $C_1 = 4\mu\text{F}$ ,  $C_2 = 2\mu\text{F}$ , and  $C_3 = 3\mu\text{F}$  are initially discharged. In steady-state after switch  $S_1$  is closed, determine the following quantities. Please provide **numerical** values for your answers.

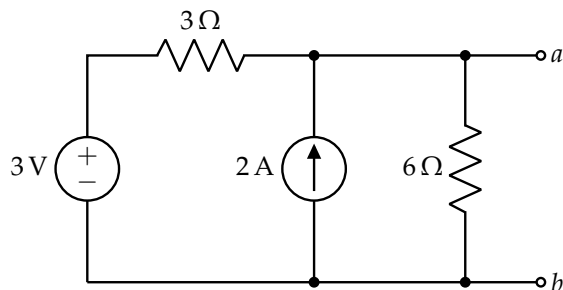
- i. What is the energy stored in **capacitor**  $C_1$ ?
- ii. What is the charge accumulated on **capacitor**  $C_3$ ?
- iii. What is the voltage across **capacitor**  $C_3$ ?



## 6. Finding Mr. Thevenin

For the following circuits, find the Thevenin and Norton equivalent resistance, voltage, and current between the nodes  $a$  and  $b$ .

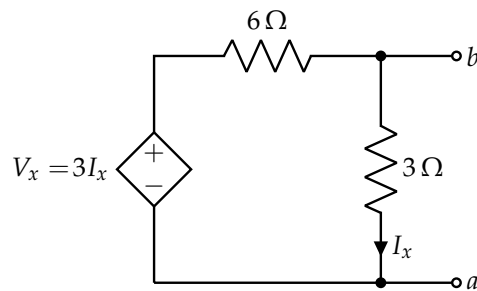
(a) Consider the circuit below:



- i. Can you turn off  $V_s$  (5V voltage source) and  $I_s$  (2A current source) to find  $R_{th}$ ?
  - (A) Yes
  - (B) No
- ii. What is  $R_{th}$ ?
  - (A)  $R_{th} = 2\ \Omega$
  - (B)  $R_{th} = 3\ \Omega$
  - (C)  $R_{th} = 4.5\ \Omega$
  - (D)  $R_{th} = 6\ \Omega$
  - (E)  $R_{th} = 9\ \Omega$
- iii. What is  $V_{th}$ ?
  - (A)  $V_{th} = 0\ \text{V}$
  - (B)  $V_{th} = 2\ \text{V}$
  - (C)  $V_{th} = 3\ \text{V}$
  - (D)  $V_{th} = 4\ \text{V}$
  - (E)  $V_{th} = 6\ \text{V}$
- iv. What is  $I_{no}$ ?
  - (A)  $I_{no} = 0\ \text{A}$
  - (B)  $I_{no} = 0.67\ \text{A}$
  - (C)  $I_{no} = 1\ \text{A}$
  - (D)  $I_{no} = 2\ \text{A}$
  - (E)  $I_{no} = 3\ \text{A}$

(b) Consider this new circuit with a current-dependent voltage source (that depends on  $I_x$ , the current through the  $3\ \Omega$  resistor):  $V_x = 3\ \Omega \cdot I_x$  [V].

(HINT: To find  $R_{th}$ , you will need to use a test voltage  $V_{test}$  (or test current) and find the relationship to its current  $I_{test}$  (or voltage).)



- i. Should you turn off  $V_x$  to find  $R_{th}$ ?
  - (A) Yes
  - (B) No
- ii. What is  $R_{th}$ ?
  - (A)  $R_{th} = 2\ \Omega$
  - (B)  $R_{th} = 3\ \Omega$
  - (C)  $R_{th} = 4.5\ \Omega$
  - (D)  $R_{th} = 6\ \Omega$
  - (E)  $R_{th} = 9\ \Omega$
- iii. What is  $V_{th}$ ?
  - (A)  $V_{th} = 0\ \text{V}$
  - (B)  $V_{th} = 2\ \text{V}$
  - (C)  $V_{th} = 3\ \text{V}$
  - (D)  $V_{th} = 4\ \text{V}$
  - (E)  $V_{th} = 6\ \text{V}$
- iv. What is  $I_{no}$ ?
  - (A)  $I_{no} = 0\ \text{A}$
  - (B)  $I_{no} = 0.67\ \text{A}$
  - (C)  $I_{no} = 1\ \text{A}$
  - (D)  $I_{no} = 2\ \text{A}$
  - (E)  $I_{no} = 3\ \text{A}$

## 7. Orthogonal Space

Let  $\vec{v}$  be a vector in  $\mathbb{R}^2$ , where  $\mathbb{R}^2$  has an inner product. We define  $W$  to be the set of all vectors orthogonal to  $\vec{v}$ , i.e.

$$W = \{\vec{w} \mid \langle \vec{v}, \vec{w} \rangle = 0\} \quad (3)$$

- (a) In the paragraph below, select the best choice for each blank to **complete the proof showing that  $W$  is a subspace**:

First, we need to show that the set contains the zero vector. We see that  $\langle \vec{v}, \vec{0} \rangle = 0$ , so this condition is fulfilled. Next, we need to show that the set (1)\_\_\_\_\_. Suppose we have  $\vec{x}, \vec{y} \in W$ , then (2)\_\_\_\_\_, so this condition is fulfilled. Finally, we need to show that the set (3)\_\_\_\_\_. Suppose we have  $\alpha \in \mathbb{R}$  and  $\vec{x} \in W$ , then (4)\_\_\_\_\_, so this condition is fulfilled. Therefore the set is a valid subspace.

- (1) (A) is closed under scalar multiplication

(B) is closed under vector addition

(C) is homogeneous

(D) is non-empty

(E) fulfills superposition

- (2) (A)  $\langle \vec{v}^\top \vec{x}, \vec{v}^\top \vec{y} \rangle = 0$

(B)  $\langle \vec{v}, \vec{x} \rangle = \langle \vec{v}, \vec{y} \rangle$

(C)  $\langle \vec{v} + \vec{x}, \vec{y} \rangle = \langle \vec{v}, \vec{x} \rangle + \langle \vec{v}, \vec{y} \rangle = 0$

(D)  $\langle \vec{v}, \vec{x} + \vec{y} \rangle = \langle \vec{v}, \vec{x} \rangle + \langle \vec{v}, \vec{y} \rangle = 0$

- (3) (A) is closed under scalar multiplication

(B) is closed under vector addition

(C) is homogeneous

(D) is non-empty

(E) fulfills superposition

- (4) (A)  $\langle \vec{v}, \alpha \vec{x} \rangle = \alpha \langle \vec{v}, \vec{x} \rangle = 0$

(B)  $\langle \alpha \vec{v}, \alpha \vec{x} \rangle = \alpha \langle \vec{v}, \vec{x} \rangle = 0$

(C)  $\langle \alpha \vec{v}^\top \vec{x}, \vec{0} \rangle = \alpha \langle \vec{v}^\top \vec{x}, \vec{0} \rangle = 0$

(D)  $\alpha \langle \vec{v}, \vec{x} \rangle = \alpha \cdot 0$

- (b) Now suppose the inner product is defined as  $\langle \vec{x}, \vec{y} \rangle = \vec{x}^\top Q \vec{y}$  for  $Q \in \mathbb{R}^{2 \times 2}$ .

- i. If  $\vec{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$  and we still define subspace  $W$  to be the set of all vectors that are orthogonal to

$\vec{v}$  from part (a), which of the following options is a basis for  $W$  if the matrix  $Q = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$ ?

(A)  $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

(B)  $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

(C)  $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(D)  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(E)  $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$

ii. What are the necessary properties for a valid inner product? **(Select all that apply.)**

(A) positive definiteness

(B) closed under scalar multiplication

(C) closed under vector addition

(D) quadratic

(E) linear

(F) non-empty

(G) symmetric

(H) contains the zero vector

iii. Which of the following choices of matrix  $Q$  results in a valid inner product  $\langle \vec{x}, \vec{y} \rangle = \vec{x}^T Q \vec{y}$ ? **(Select all that apply.)**

(A)  $\begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$

(B)  $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

(C)  $\begin{bmatrix} 15 & 0 \\ 0 & 0 \end{bmatrix}$

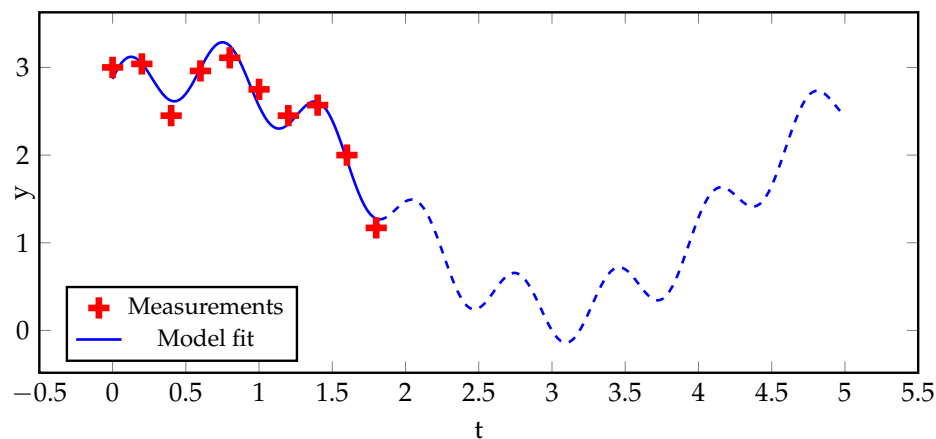
(D)  $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

### 8. Mixed signals?

Your friend set up an experiment to track the chest position while breathing in real-time by using an accelerometer placed on their chest while laying down. To their surprise, they were able to also capture some cardiac signal on top of the breathing!

They were also able to collect some (noisy) data for two heartbeat periods:

	t	y
1	0.0	3.00
2	0.2	3.04
3	0.4	2.45
4	0.6	2.96
5	0.8	3.11
6	1.0	2.75
7	1.2	2.45
8	1.4	2.57
9	1.6	2.00
10	1.8	1.17



Now, you want to find a model that fits the measurements!

(a) Your friend proposed a model for the obtained signal  $y$  as a function of time  $t$  as follows:

$$y = c_1 + c_2 \cdot \cos^2(2\pi \cdot 0.2 \cdot t) + c_3 \cdot \sin(2\pi \cdot 0.2 \cdot t) + c_4 \cdot \cos^2(2\pi \cdot 1.5 \cdot t) + c_5 \cdot \sin(2\pi \cdot 1.5 \cdot t) \quad (4)$$

As you might have noticed, we don't know all parameters in the proposed model. Here,  $c_1, c_2, c_3, c_4$  and  $c_5$  are our unknowns. Can you pose this problem as a set of linear equations to estimate our unknown parameters from the acquired data?

- (A) Yes
- (B) No

(b) You end up deciding to use a simpler model that might better fit the data:

$$y = c_1 + c_2 \cdot \cos(2\pi \cdot 0.2 \cdot t) + c_3 \cdot \sin(2\pi \cdot 0.2 \cdot t) + c_4 \cdot \cos(2\pi \cdot 1.5 \cdot t) + c_5 \cdot \sin(2\pi \cdot 1.5 \cdot t) \quad (5)$$

You setup a least squares problem  $A\vec{c} \approx \vec{y}$  to estimate our missing parameters  $\vec{c}$  that are the best fit to the acquired data. Here,  $\vec{c} \in \mathbb{R}^5$  as specified below. Let our matrix  $A \in \mathbb{R}^{10 \times 5}$  and vector  $\vec{y} \in \mathbb{R}^{10}$ , whose rows correspond to the order of the acquired data, be indexed as follows:

$$\vec{c} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} \quad A_{10 \times 5} = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,5} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,5} \\ \vdots & \vdots & \ddots & \vdots \\ a_{10,1} & a_{10,2} & \cdots & a_{10,5} \end{bmatrix} \quad \vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{10} \end{bmatrix} \quad (6)$$

i. Can you use this new model to set up linear equations to estimate our unknown parameters from the acquired data?

(A) Yes

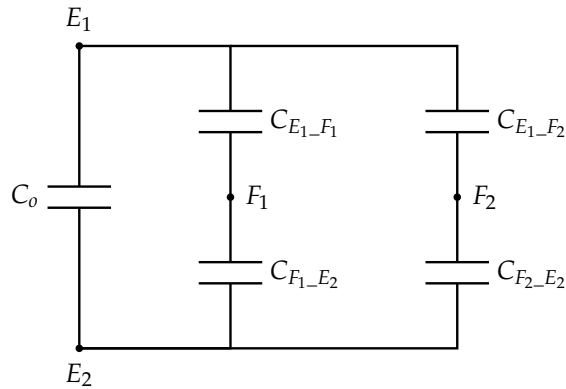
(B) No

ii. What are the numerical values for the following entries of  $y$  and  $\vec{A}$ :  $a_{1,2}, a_{1,3}, a_{3,1}, a_{6,4}, y_1, y_9$ ?  
(*HINT: We have also provided values for sine and cosine for some relevant numbers.*)

Angle	0	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
Sine	0	1	0	-1	0
Cosine	1	0	-1	0	1

9. Please don't burn your fingers

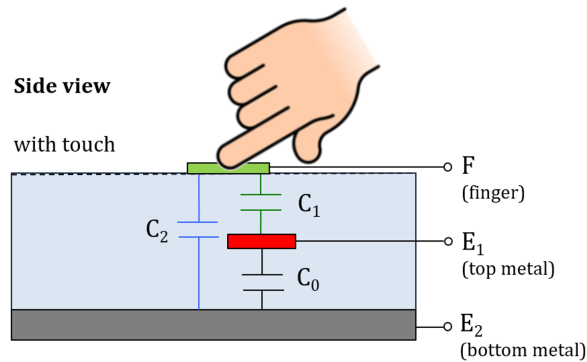
One day, hidden somewhere deep within Cory 140, you discover an ancient capacitive circuit.

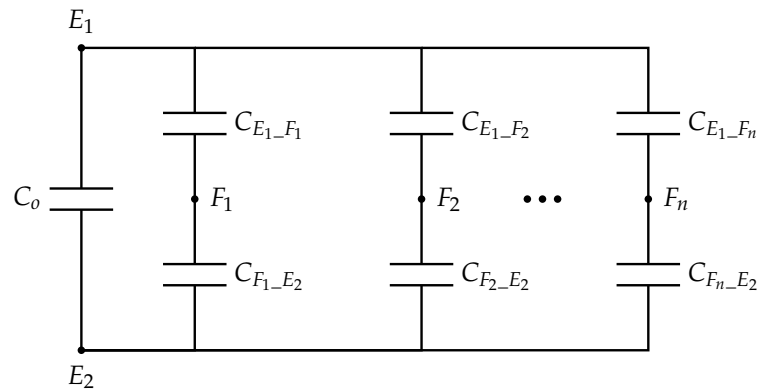


(a) Calculate the equivalent capacitance  $C_e$  between  $E_1$  and  $E_2$  given  $C_0 = C_{E_1-F_1} = C_{F_1-E_2} = C_{E_1-F_2} = C_{F_2-E_2} = 40 \text{ pF}$ .

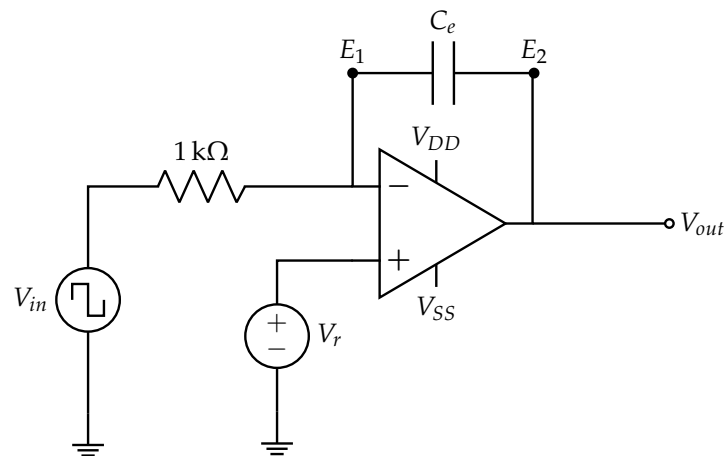
- (A) 20 pF
- (B) 40 pF
- (C) 60 pF
- (D) 80 pF
- (E) 120 pF

(b) What you found was in fact a multi-finger touchscreen that forms different capacitive circuits depending on how many fingers we place.





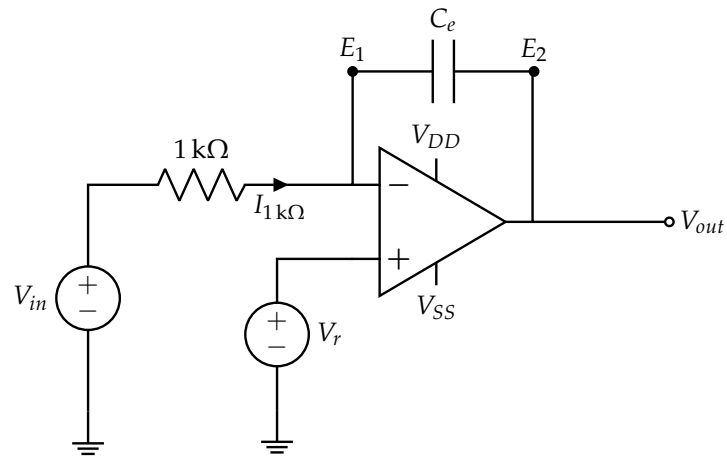
To figure out how this multi-finger touchscreen works, you decide to connect it to your op-amp setup from the Touch 3 labs. The circuit between terminals  $E_1$  and  $E_2$  is modeled as equivalent capacitance  $C_e$ , and  $V_{in}$  is a function generator with alternating square wave voltage between  $V_{in} = 0\text{ V}$  and  $V_{in} = 2V_r$ .



Assume an ideal op-amp and the circuit is in negative feedback.

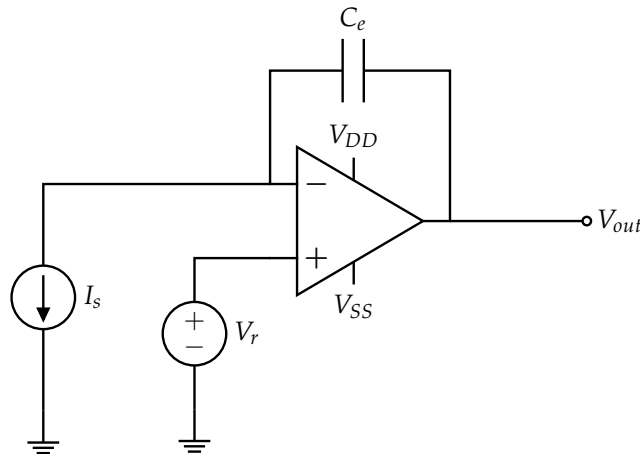
- i. After experimenting with the circuit for a bit, you notice a sudden increase in the positive peaks of  $V_{out}$ . How must the equivalent capacitance  $C_e$  have changed?
    - (A)  $C_e$  increased
    - (B)  $C_e$  decreased
  - ii. How are the equivalent capacitance  $C_e$  and the number of fingers touching related?
    - (A) More fingers increases  $C_e$
    - (B) More fingers decreases  $C_e$
    - (C)  $C_e$  does not depend on the number of fingers
- (c) Oops! Instead of a function generator, we accidentally used a constant voltage source  $V_{in}$  instead. We will find out how long it will take before the circuit breaks! Here is the circuit with the new voltage source  $V_{in}$ .





For the following problems, assume the circuit is in negative feedback.

- First, what is the current flowing in the  $1\text{ k}\Omega$  resistor ( $I_{1\text{k}\Omega}$  in the circuit)? Assume  $V_{in} = 2\text{ V}$ ,  $V_r = 1\text{ V}$ . Express your answer in mA (numerical value), and make sure your sign is correct (according to the labeled current in the circuit).
- Now assume a constant current source  $I_s$  (instead of  $V_{in}$  and the  $1\text{ k}\Omega$  resistor), as shown in the circuit below.



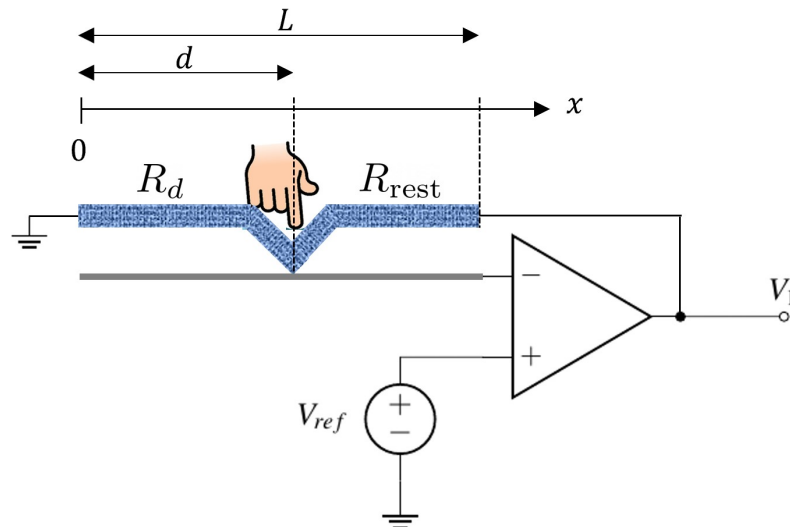
If the initial voltage across the capacitor is zero at time  $t = 0$ , what is the value of  $V_{out}$  over time? Assume the output does not saturate (i.e.,  $V_{DD} > V_{out} > V_{SS}$ ). Express your answer in terms of the variables  $I_s$ ,  $V_r$ ,  $C_e$ , and  $t$  by simplifying any integrals or derivatives (i.e. your final answer should not have any integrals or derivatives in it.)

- If the op-amp is connected to supply sources  $V_{DD} = -V_{SS}$ , **1)** how long does it take for  $V_{out}$  to saturate the op-amp? and **2)** what is the value of  $V_{out}$  in saturation? (Assume  $I_s > 0$ ,  $V_r > 0$ , and  $V_{DD} > V_r > V_{SS}$ )

- (A)  $t = C_e \frac{-V_{SS} + V_r}{I_s}$        $V_{out} = V_{SS}$
- (B)  $t = C_e \frac{V_{DD} - V_r}{I_s}$        $V_{out} = V_{DD}$
- (C)  $t = \frac{-V_{SS} + V_r}{C_e I_s}$        $V_{out} = V_{SS}$
- (D)  $t = \frac{V_{DD} - V_r}{C_e I_s}$        $V_{out} = V_{DD}$

### 10. Ask Opamps Anything

We've decided to design a 1D resistive touch-screen using an ideal opamp. The resistive touchscreen has a total length of  $L$ , a cross sectional area of  $A$  and resistivity of  $\rho$ .

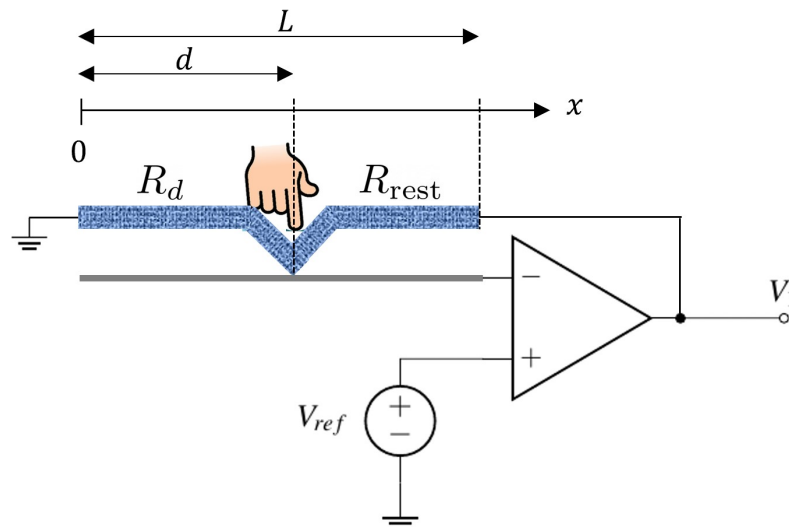


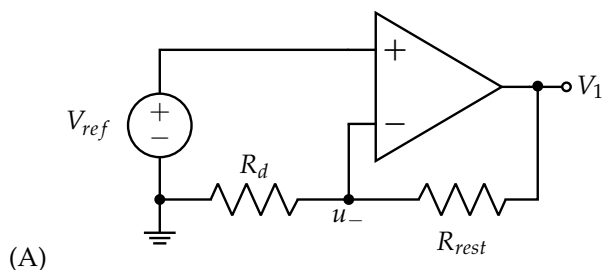
(a) First, we want to find  $V_1$ , because we will use this block in a larger design.

i. What are the values for the resistance between the touch point and ground ( $R_d$ ) and between the touch point and  $V_1$  ( $R_{rest}$ )?

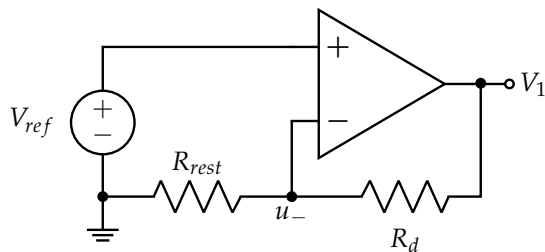
- (A)  $R_d = \rho \frac{A}{d}$        $R_{rest} = \rho \frac{A}{L-d}$   
 (B)  $R_d = \rho \frac{d}{A}$        $R_{rest} = \rho \frac{L-d}{A}$   
 (C)  $R_d = \rho \frac{L-d}{A}$        $R_{rest} = \rho \frac{d}{A}$   
 (D)  $R_d = \rho \frac{A}{L-d}$        $R_{rest} = \rho \frac{A}{d}$

ii. Identify a correct equivalent topology for this scenario:





(A)

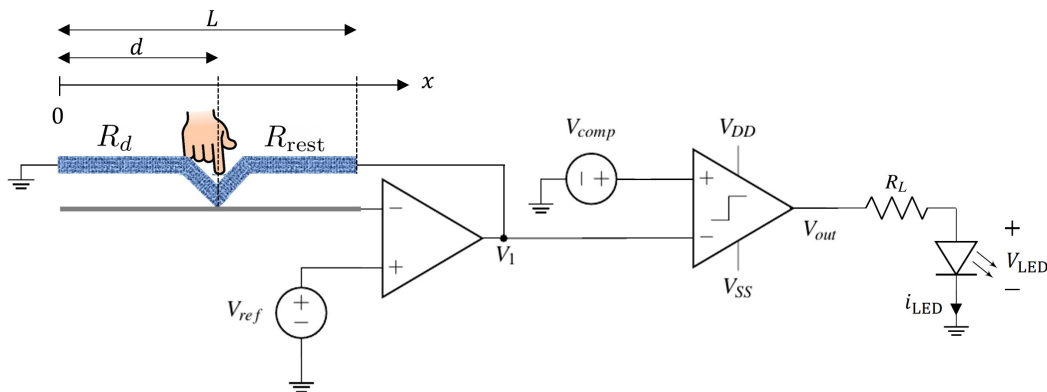


(B)

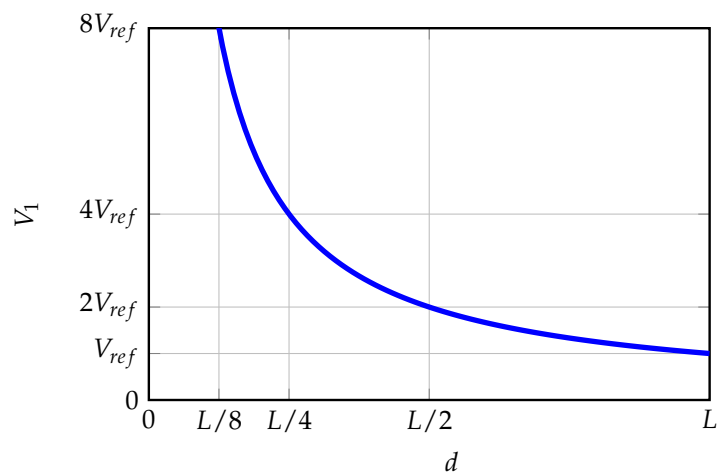
iii. What is the value of  $V_1$  if the resistive touch screen, as a function of  $R_d$  and  $R_{rest}$ ?

- (A)  $V_1 = V_{ref} \frac{R_d}{R_{rest}}$
- (B)  $V_1 = V_{ref} \frac{R_{rest}}{R_d}$
- (C)  $V_1 = V_{ref} \left(1 + \frac{R_d}{R_{rest}}\right)$
- (D)  $V_1 = V_{ref} \left(1 + \frac{R_{rest}}{R_d}\right)$

(b) Next, an LED indicator driven by a comparator is added to the output of the prior circuit.



i. You are provided the curve for the voltage  $V_1$  as a function of the touch distance  $d$ . What should the value of  $V_{comp}$  be to ensure the LED turns on when  $d > \frac{L}{2}$ ?



- (A)  $V_{comp} = +V_{ref}$   
 (B)  $V_{comp} = -V_{ref}$   
 (C)  $V_{comp} = +2V_{ref}$   
 (D)  $V_{comp} = -2V_{ref}$   
 (E)  $V_{comp} = +4V_{ref}$   
 (F)  $V_{comp} = -4V_{ref}$
- ii. When the LED shown in the diagram is turned on the voltage across it is  $V_{LED} = 1\text{ V}$ , **what is the current,  $i_{LED}$ , through it?** Consider the load resistance  $R_L = 1\text{ k}\Omega$ , and voltages supplies  $V_{DD} = 5\text{ V}$  and  $V_{SS} = 0\text{ V}$ . Your answer should be a **numerical** value.
- iii. Now, assume  $i_{LED} = 1\text{ mA}$ ,  $V_{LED} = 2\text{ V}$ ,  $R_L = 3\text{ k}\Omega$ ,  $V_{DD} = 5\text{ V}$ , and  $V_{SS} = 0\text{ V}$ . How much power  $P_{out}$  is delivered by the output of the comparator? Your answer should be a **numerical** value.