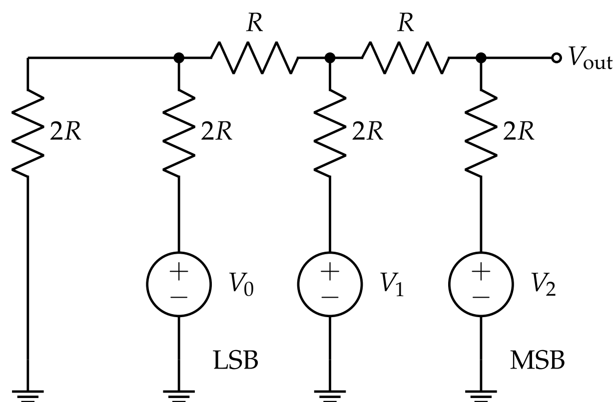
Homework 1

This homework is due on Friday, January 27, 2023, at 11:59PM. Self-grades and HW Resubmissions are due on the following Friday, Feburary 3, 2023, at 11:59PM.

1. **Digital-Analog Converter**

A digital-analog converter (DAC) is one of the key interface components between the digital and the analog world. It is a circuit for converting a digital representation of a number (binary) into a corresponding analog voltage. In this problem, we will consider a DAC made out of resistors only (resistive DAC) called the R-$2R$ ladder. This DAC will help us generate the analog voltages from the digital representation, and later will also help us digitize the analog voltages when we will be building analog to digital interfaces in Lab 3, in part based on this ladder-DAC.

Here is the circuit for a 3-bit resistive DAC.



Let $b_0, b_1, b_2 = \{0, 1\}$ (that is, either 1 or 0), and let the voltage sources $V_0 = b_0 V_{DD}$, $V_1 = b_1 V_{DD}$, $V_2 = b_2 V_{DD}$, where V_{DD} is the supply voltage.

As you may have noticed, (b_2, b_1, b_0) represents a 3-bit binary (unsigned) number where each of b_i is a binary bit. b_0 is the least significant bit (LSB) and b_2 is the most significant bit (MSB). We will now analyze how this converter functions.

(a) **Solve for V_{out} in terms of V_{DD} and the binary bits b_2, b_1, b_0.**

(b) **If $b_2, b_1, b_0 = 0, 1, 1$, what is V_{out}?** Express your answer in terms of V_{DD}.

(c) **If $b_2, b_1, b_0 = 1, 0, 1$, what is V_{out}?** Express your answer in terms of V_{DD}.

(d) **If $b_2, b_1, b_0 = 1, 1, 0$, what is V_{out}?** Express your answer in terms of V_{DD}.

(e) **If $b_2, b_1, b_0 = 1, 1, 1$, what is V_{out}?** Express your answer in terms of V_{DD}.

(f) **Explain how your results above show that the resistive DAC converts the 3-bit binary number (b_2, b_1, b_0) to the output analog voltage V_{out}.**

2. Hambley P3.16

A capacitance and the current through it are shown in Figure 1 and Figure 2 respectively. At $t = 0$, the voltage is $v_C(0) = 10\,\text{V}$. Sketch the voltage, power, and stored energy to scale versus time.

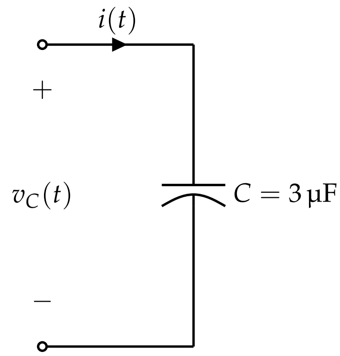


Figure 1: Circuit for P3.16

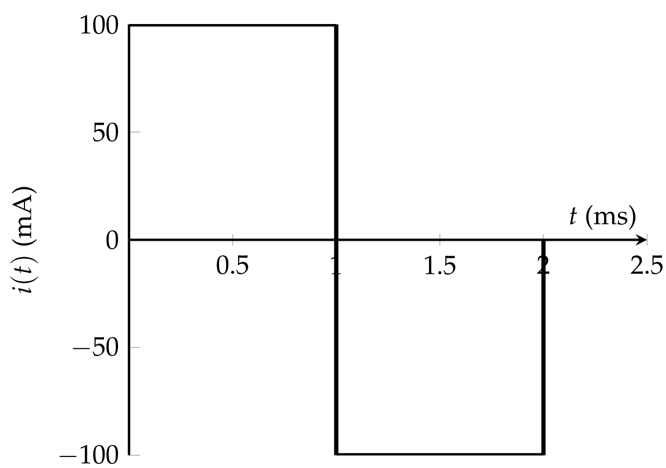


Figure 2: Current vs Time for P3.16

3. Hambley P4.3 and P4.4

(a) The initial voltage across the capacitor shown in Figure 3 is $v_c(0+) = 0$. Find an expression for the voltage across the capacitor as a function of time, and sketch it to scale versus time.

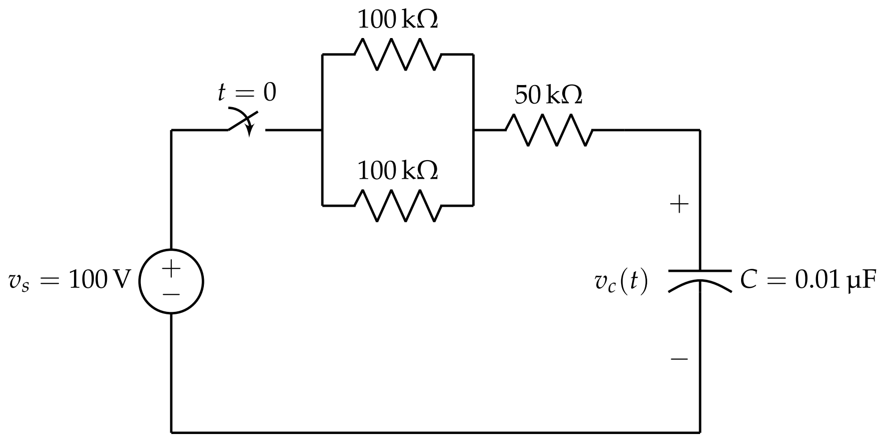
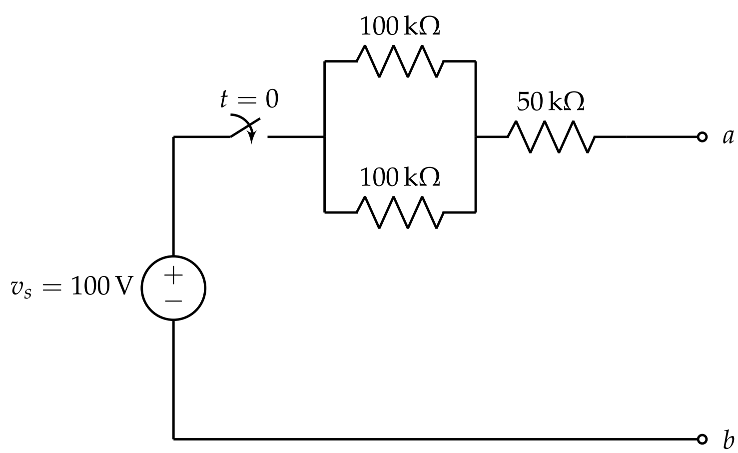


Figure 3: P4.3 Modified

(HINT: Consider simplifying the circuit using Thevenin equivalent circuits. That is, consider the following circuit, which is exactly Figure 3 without the capacitor:



Find a Thevenin equivalent circuit and use this to simplify Figure 3.)

(b) Repeat part (a) for an initial voltage $v_c(0+) = -50\,\text{V}$.

4. Capacitor Energy

Say a series R-C circuit is supplied by a constant voltage V. At $t = 0$, voltage across the capacitor was 0. We know how to find the expression for capacitor voltage as a function of time. Using this expression,

(a) Find the expression for total stored energy at $t = \infty$, $w_s = \int_0^\infty v(t)i(t)\,\mathrm{d}t$ (i.e., at steady state).

(b) We know that when a current flows through a resistor, we dissipate energy at a rate of i^2R. Using this relation, find the total dissipated energy at $t = \infty$, $w_d = \int_0^\infty i^2R\,\mathrm{d}t$ (i.e., at steady state).

(c) Find the total energy taken from the source, noting that some of it was stored and some of it was dissipated.

(d) Does the result in part (b) vary if $R = 100\,\Omega$ vs $R = 1\,\mathrm{k}\Omega$? What about $R = 0\,\Omega$? *OPTIONAL: Can you explain this result?*

5. Hambley P4.7

The capacitor shown in Figure 4 is charged to a voltage of 50 V prior to $t = 0$.

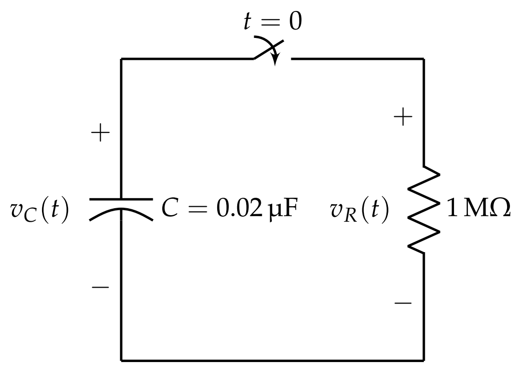


Figure 4: P4.7

(a) Find expressions for the voltage across the capacitor $v_C(t)$ and the voltage across the resistor $v_R(t)$.

(b) Find an expression for the power delivered to the resistor.

(c) Integrate the power from $t = 0$ to $t = \infty$ to find the energy delivered.

(d) Show that the energy delivered to the resistor is equal to the energy stored in the capacitor prior to $t = 0$.

6. Hambley P4.46

Consider the circuit shown in Figure 5. The voltage source is known as a **ramp function**, which is defined by

$$v(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases} \tag{1}$$

Assume that $v_C(0) = 0$. Derive an expression for $v_C(t)$ for $t \geq 0$. Sketch $v_C(t)$ to scale versus time.

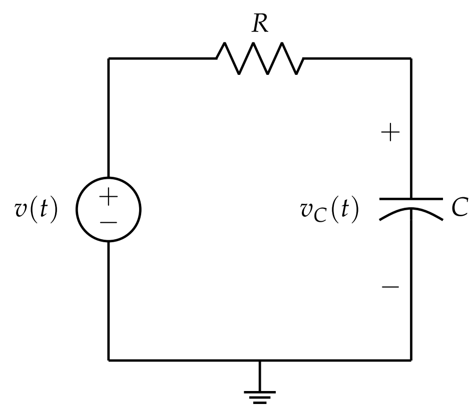


Figure 5: P4.46

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