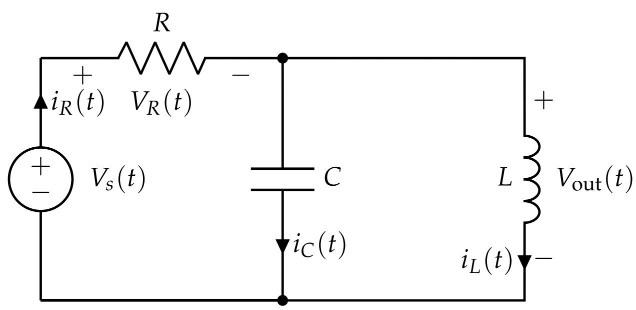
Homework 5

This homework is due on Friday, February 24, 2023, at 11:59PM. Self-grades and HW Resubmissions are due on the following Friday, March 3, 2023, at 11:59PM.

1. **Phasor-Domain Circuit Analysis**

The analysis techniques you learned previously in 16A for resistive circuits are equally applicable for analyzing circuits driven by sinusoidal inputs in the phasor domain. In this problem, we will walk you through the steps with a concrete example.

Consider the following circuit where the input voltage is sinusoidal. The end goal of our analysis is to find an equation for $V_{\text{out}}(t)$.



The components in this circuit are given by:

$$V_s(t) = 10\sqrt{2}\cos\left(100t - \frac{\pi}{4}\right) \tag{1}$$
$$R = 5\,\Omega \tag{2}$$
$$L = 50\,\text{mH} \tag{3}$$
$$C = 2\,\text{mF} \tag{4}$$

(a) **Give the amplitude V_0, input frequency ω, and phase ϕ of the input voltage V_s.**

(b) Transform the circuit into the phasor domain. **What are the impedances of the resistor, capacitor, and inductor? What is the phasor $\widetilde{V}_S$ of the input voltage $V_s(t)$?**

(c) Use the circuit equations to **solve for** $\widetilde{V}_{\text{out}}$, the phasor representing the output voltage.

2. Hambley P6.55

Consider the circuit shown in Figure 1. The input signal is given by

$$v_{\text{in}}(t) = 5 + 5\cos(2000\pi t) \tag{5}$$

Find an expression for the output $v_{\text{out}}(t)$ in steady-state conditions.

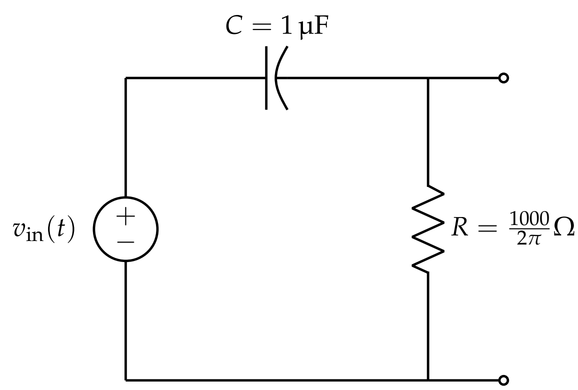


Figure 1: P6.55

(HINT: Use superposition. That is, find $v_{\text{out},1}(t)$ which is the output voltage if the input is $v_{\text{in},1}(t) = 5$, and then find $v_{\text{out},2}(t)$ which is the output voltage if the input is $v_{\text{in},2}(t) = 5\cos(2000\pi t)$. What is $v_{\text{out}}(t)$ in terms of $v_{\text{out},1}(t)$ and $v_{\text{out},2}(t)$?)

3. Hambley P6.74

Derive an expression for the resonant frequency of the circuit shown in Figure 2. We define the resonant frequency to be the frequency at which the impedance is purely real (no imaginary component).

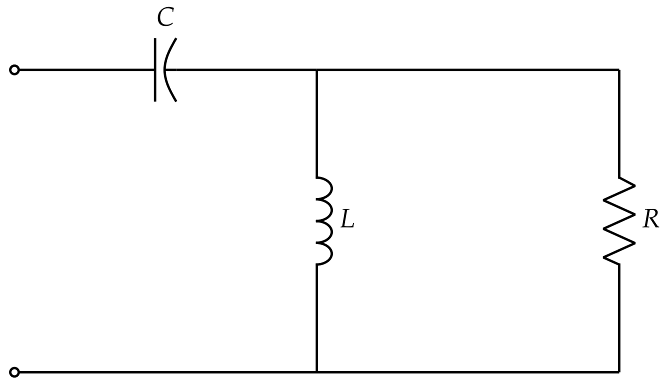


Figure 2: P6.74

4. Phasors and Eigenvalues

Suppose that we have the two-dimensional system of differential equations expressed in matrix/vector form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{x}(t) = A\vec{x}(t) + \vec{b}u(t) \tag{6}$$

where for this problem, the matrix A and the vector $\vec{b}$ are both real.

(a) Give a necessary condition on the eigenvalues λ_k of A such that any impact of an initial condition will eventually completely die out. (i.e. the system will reach steady-state.) You don't have to prove this. *(HINT: Read Lecture 9 Slide 6.)*

(b) Now assume that $u(t)$ has a phasor representation $\widetilde{U}$. In other words, $u(t) = \widetilde{U}\mathrm{e}^{+\mathrm{j}\omega t} + \overline{\widetilde{U}}\mathrm{e}^{-\mathrm{j}\omega t}$. Assume that the vector solution $\vec{x}(t)$ to the system of differential equations (6) can also be written in phasor form as

$$\vec{x}(t) = \vec{\widetilde{X}}\mathrm{e}^{+\mathrm{j}\omega t} + \overline{\vec{\widetilde{X}}}\mathrm{e}^{-\mathrm{j}\omega t}. \tag{7}$$

Derive an expression for $\vec{\widetilde{X}}$ involving $A, \vec{b}$, $\mathrm{j}\omega$, $\widetilde{U}$, and the identity matrix I. (Here, we assume that $\mathrm{j}\omega$, and $-\mathrm{j}\omega$ are not eigenvalues of A, which indicates that $\det(\mathrm{j}\omega I - A)$ and $\det(-\mathrm{j}\omega I - A)$ are non-zero.)

(HINT: Plug eq. (7) into eq. (6) and simplify, using the rules of differentiation and grouping terms by which exponential $\mathrm{e}^{\pm\mathrm{j}\omega t}$ they multiply.

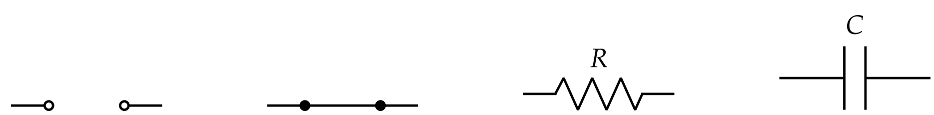
)

5. Circuit Design

In this problem, you will find a circuit where several components have been left *blank* for you to fill in.

Assume that the op-amp is *ideal*. A special note on op amps in frequency domain analysis: The op-amps you learned about in 16A can be used in exactly the same way for setting up differential equations and even Phasor analysis in 16B. Treat them as ideal op-amps and invoke the Golden Rules.

You have at your disposal *only one of each* of the following components (not including R_1):



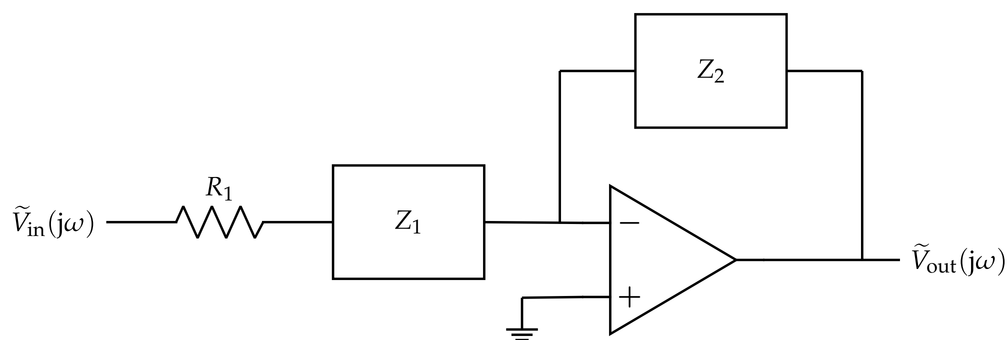
(a) an open circuit

(b) a short circuit

(c) a resistor (you choose from the values $R = 1\,\mathrm{k\Omega}, 10\,\mathrm{k\Omega}, 20\,\mathrm{k\Omega}$)

(d) a capacitor (you choose from the values $C = 0.5\,\mu\mathrm{F}, 1\,\mu\mathrm{F}, 2\,\mu\mathrm{F}$)

Consider the circuit below. The labeled voltages $\widetilde{V}_{\text{in}}(\mathrm{j}\omega)$ and $\widetilde{V}_{\text{out}}(\mathrm{j}\omega)$ are the phasor representations of $v_{\text{in}}(t)$ and $v_{\text{out}}(t)$ respectively, where $v_{\text{in}}(t)$ has the form $v_{\text{in}}(t) = v_0 \cos(\omega t + \phi)$. The transfer function $H(\mathrm{j}\omega)$ is defined as $H(\mathrm{j}\omega) = \frac{\widetilde{V}_{\text{out}}(\mathrm{j}\omega)}{\widetilde{V}_{\text{in}}(\mathrm{j}\omega)}$.



(a) Let $Z_1(\mathrm{j}\omega)$ and $Z_2(\mathrm{j}\omega)$ are the impedances of the boxes shown in the circuit diagram. **Write the expression of the transfer function $H(\mathrm{j}\omega)$.**

(b) Let R_1 be $1\,\mathrm{k\Omega}$. We have to find Z_1 and Z_2, such that the circuit's transfer function $H(\mathrm{j}\omega)$ has the following properties:

- $|H(\mathrm{j}0)| = 0$.
- $|H(\mathrm{j}\infty)| = 10$.
- $|H(\mathrm{j}10^3)| = \sqrt{50}$.

Using the fact that the circuit is a high pass filter, infer the components (we will find values later) of Z_1 and Z_2. Write the transfer function $H(\mathrm{j}\omega)$ using these components.

(HINT: Try method of elimination: figure out what Z_2 cannot be. Once you find what Z_2 is, what does Z_1 have to be for the circuit to be a filter?)

(c) **Now use the facts that $|H(\mathrm{j}\infty)| = 10$ and $R_1 = 1\,\mathrm{k\Omega}$ to find the component value of Z_2.**

(d) **Finally use the fact that $|H(\mathrm{j}10^3)| = \sqrt{50}$ and the values of R_1 and Z_2 to find the component value of Z_1.**

Contributors:

- Geoffrey Négiar.
- Regina Eckert.
- Antroy Roy Chowdhury.
- Nathan Lambert.
- Jaijeet Roychowdhury.
- Anant Sahai.
- Yen-Sheng Ho.
- Sidney Buchbinder.
- Ayan Biswas.
- Druv Pai.

