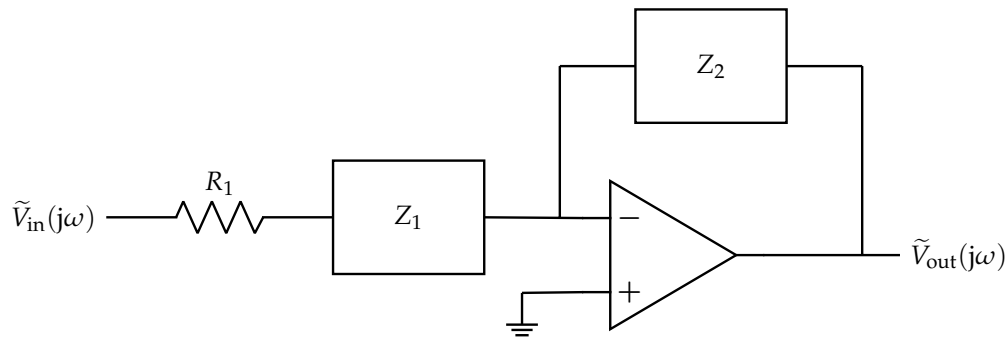


## Homework 6

This homework is due on Friday, March 3, 2023, at 11:59PM. Self-grades and HW Resubmissions are due on the following Friday, March 10, 2022, at 11:59PM.

## 1. Circuit Design Part 2

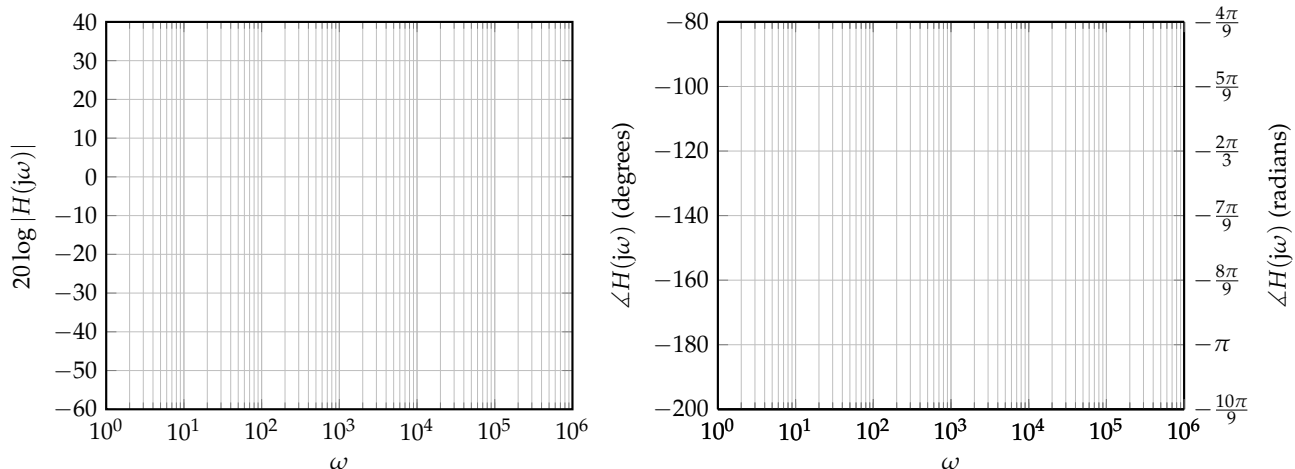
In the previous homework, you analyzed the following circuit in phasor domain:



You (hopefully) determined that  $Z_2 = R = 10 \text{ k}\Omega$  and  $Z_1 = \frac{1}{j\omega C}$  with  $C = 1 \mu\text{F}$ . This gave you the following transfer function:

$$H(j\omega) = -\frac{R}{R_1} \cdot \frac{1}{1 - \frac{j}{\omega CR_1}} \quad (1)$$

- (a) Draw the magnitude and phase Bode plots (straight-line approximations to the transfer function) of this transfer function. Blank plots are provided here for you to use.



**2. Hambley P5.15**

Determine the rms value of  $v(t) = A \cos(2\pi t) + B \sin(2\pi t)$ .

**3. Hambley P5.65**

Consider a load that has an impedance given by  $Z = 100 - j50\Omega$ . The current flowing through the load is  $I = 15\sqrt{2}e^{j\frac{\pi}{6}}$  A. Determine the power and reactive power delivered to the load.

**4. Hambley P6.53**

A transfer function is given by

$$H(j\omega) = \frac{100}{1 + j\frac{\omega}{1000}} \quad (2)$$

Sketch the asymptotic magnitude and phase Bode plots to scale. What is the value of the half-power frequency?

**5. Hambley P6.55**

Sketch the asymptotic magnitude and phase Bode plots to scale for the transfer function

$$H(j\omega) = 10 \frac{1 - j\frac{\omega}{100}}{1 + j\frac{\omega}{100}} \quad (3)$$

## 6. Bandpass Filter: Lowpass and Highpass Cascade

Consider an input signal that is composed of the superposition of:

- $A_p := 20$  mV level pure tone at frequency  $f_p := 60$  Hz and phase  $\phi_p$  corresponding to power line noise.
- $A_v := 1$  mV level pure tone at frequency  $f_v := 600$  Hz and phase  $\phi_v$  corresponding to a voice signal.
- $A_f := 10$  mV level pure tone at frequency  $f_f := 60$  kHz and phase  $\phi_f$  corresponding to fluorescent light control electronics noise.

We would like to keep the 600 Hz tone, which could correspond to a voice signal.

*NOTE:* The phases  $\phi$  are symbolic – we do not provide numerical values – but the amplitudes  $A$  are not symbolic.

- (a) Write the  $V_{\text{in}}(t)$  that describes the above input in time domain, in the following format:

$$V_{\text{in}}(t) = A_p \cos(2\pi f_p t + \phi_p) + A_v \cos(2\pi f_v t + \phi_v) + A_f \cos(2\pi f_f t + \phi_f) \quad (4)$$

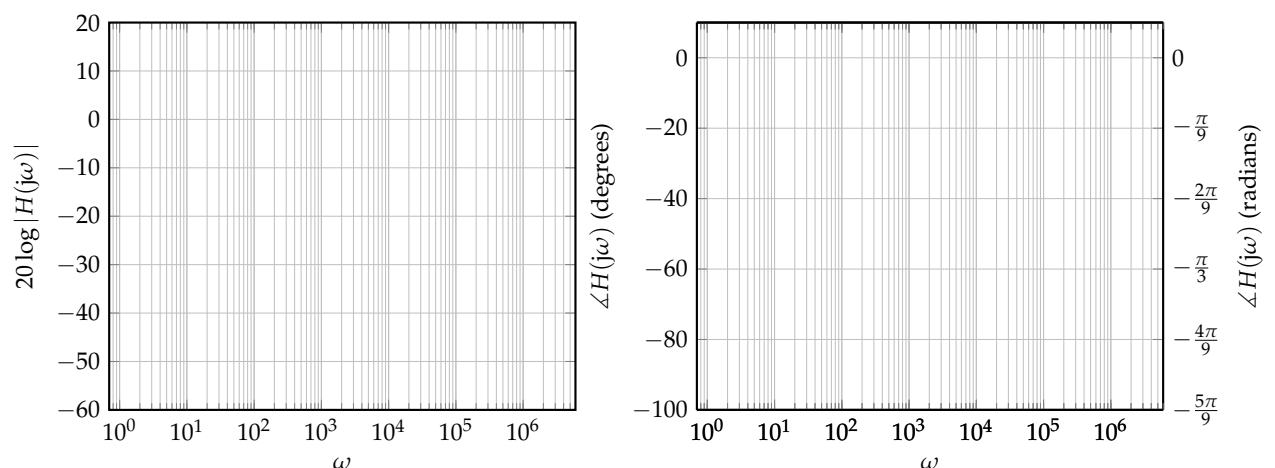
- (b) What are the angular frequencies (i.e.,  $\omega_p, \omega_v, \omega_f$ ) involved and the phasors associated with each tone? Remember that the frequencies of the tones are provided in Hz. To convert these frequencies to angular frequencies, we use  $\omega = 2\pi f$ .

*NOTE:* This scenario is common in applications; usually, the data collected is in "regular" frequencies, but the analysis requires angular frequencies.

- (c) To achieve your goal of keeping the voice tone but rejecting the noise from the power-lines and fluorescent lights, at what frequency do you want to have the cutoff frequency for the lowpass filters?

(*HINT:* To arrive at a unique solution consider computing the geometric mean (the analogous quantity to the arithmetic mean on a log scale) of the two frequencies of interest.)

- (d) Draw the Bode plot (straight-line approximations to the transfer function) for the magnitude (using  $20 \log |H(j\omega)|$ ) and phase of the lowpass filter.

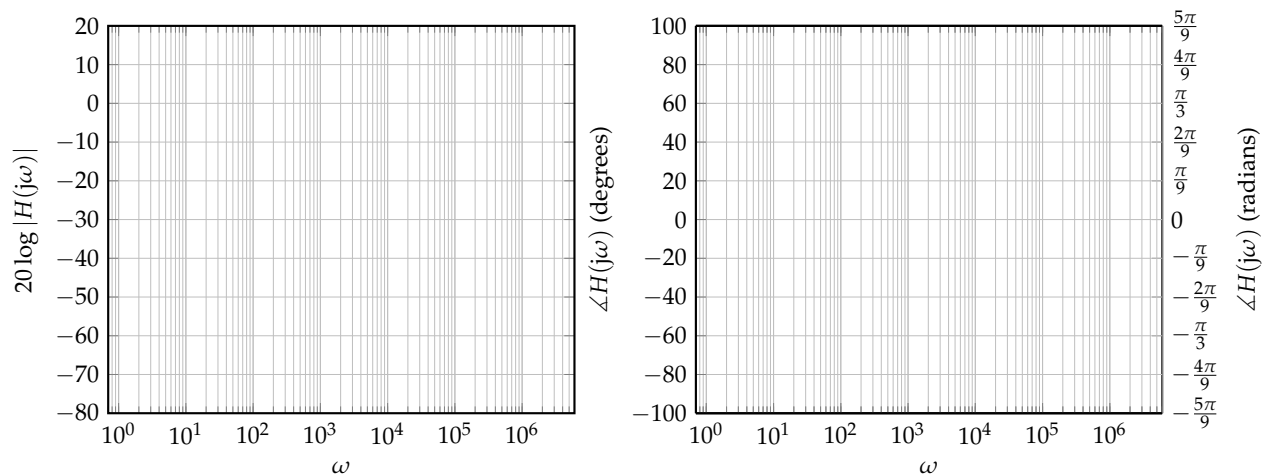


- (e) To achieve your goal of keeping the voice tone but rejecting the noise from the power-lines and fluorescent lights, **at what frequency do you want to have the cutoff frequency for the highpass filters?**

(HINT: To arrive at a unique solution consider computing the geometric mean (the analogous quantity to the arithmetic mean on a log scale) of the two frequencies of interest.)

- (f) **Draw the Bode plot (straight-line approximations to the transfer function) for the magnitude (using  $20 \log |H(j\omega)|$ ) and phase of the highpass filter.**
- (g) For the following questions, assume your cut-off frequencies for lowpass and highpass are 6 kHz and 189 Hz respectively. Suppose that you only had 1  $\mu\text{F}$  capacitors to use. **What resistance values would you choose for your highpass and lowpass filters so that they have the desired cutoff frequencies?**
- (h) The overall bandpass filter that is created by cascading the lowpass and highpass with ideal buffers in between. **Draw the Bode plot (straight-line approximations to the transfer function) for the magnitude and phase of the overall bandpass transfer function.**

(HINT: You should think about how the Bode plot of a cascade of two filters can be derived based on the Bode plots of the lower-level filters.)



- (i) Suppose that the bandpass filter does not have enough suppression at 60 Hz and 60 kHz. You decide to use a cascade of three bandpass filters (with unity-gain buffers in between) (as shown in Figures 1 and 2). **What are the phasors for each of the frequency tones after all three bandpass filters?**

(HINT: Remember how you determined the transfer function of the bandpass filter from the transfer functions of the lowpass and highpass filters.)

Feel free to use a computer to help you evaluate both the magnitudes and the phases here.

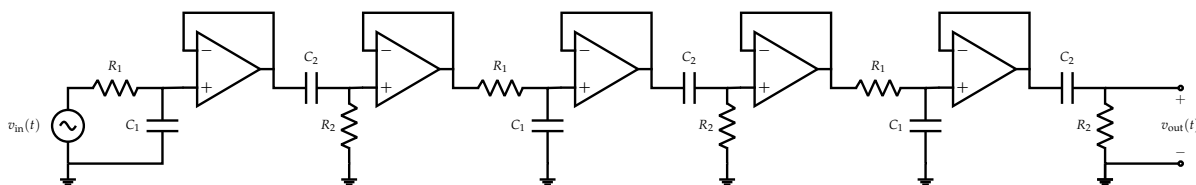


Figure 1: "Time-domain" circuit: Cascade of the three bandpass filters, using buffers to avoid loading.

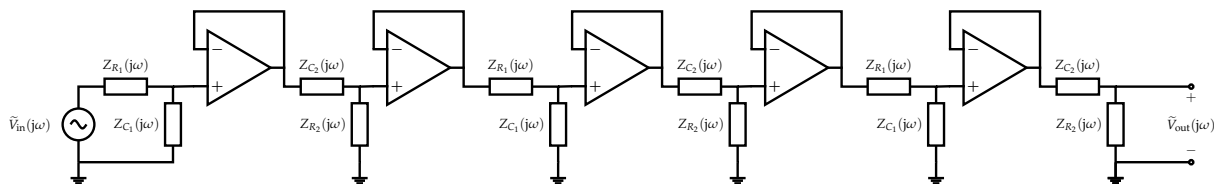
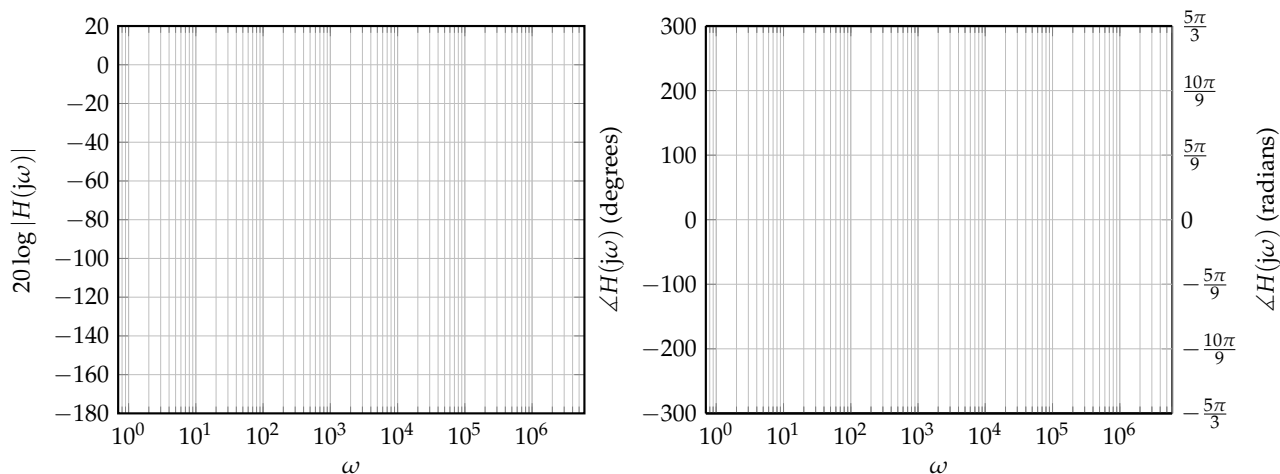


Figure 2: "Phasor-domain" circuit: Cascade of the three bandpass filters, using buffers to avoid loading.

- (j) Draw the Bode plots (straight-line approximations to the transfer function) for the magnitude and phase of the 3<sup>rd</sup> order bandpass filter. To highlight the difference between the 3<sup>rd</sup> and 1<sup>st</sup> order filters, please draw both Bode plots on a single figure.



- (k) Write the final time domain voltage waveform that would be present after the filter.

**Contributors:**

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