Lab 5: Sensing Part 2

Part 0: Introduction

   In this lab, you will further design filters to determine the frequency spectrum of the mic board. To do this, you will select your desired cutoff frequencies and calculate the appropriate resistor and capacitor values to build filters with said cutoff frequencies, somewhat to making your own color organ!

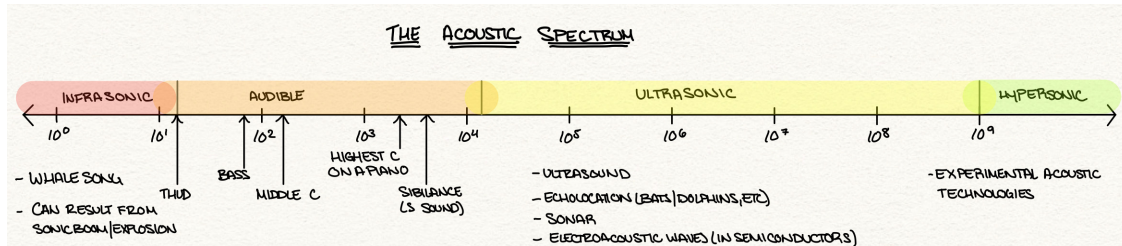   The audible range is actually a somewhat small spectrum of frequencies, as demonstrated below:
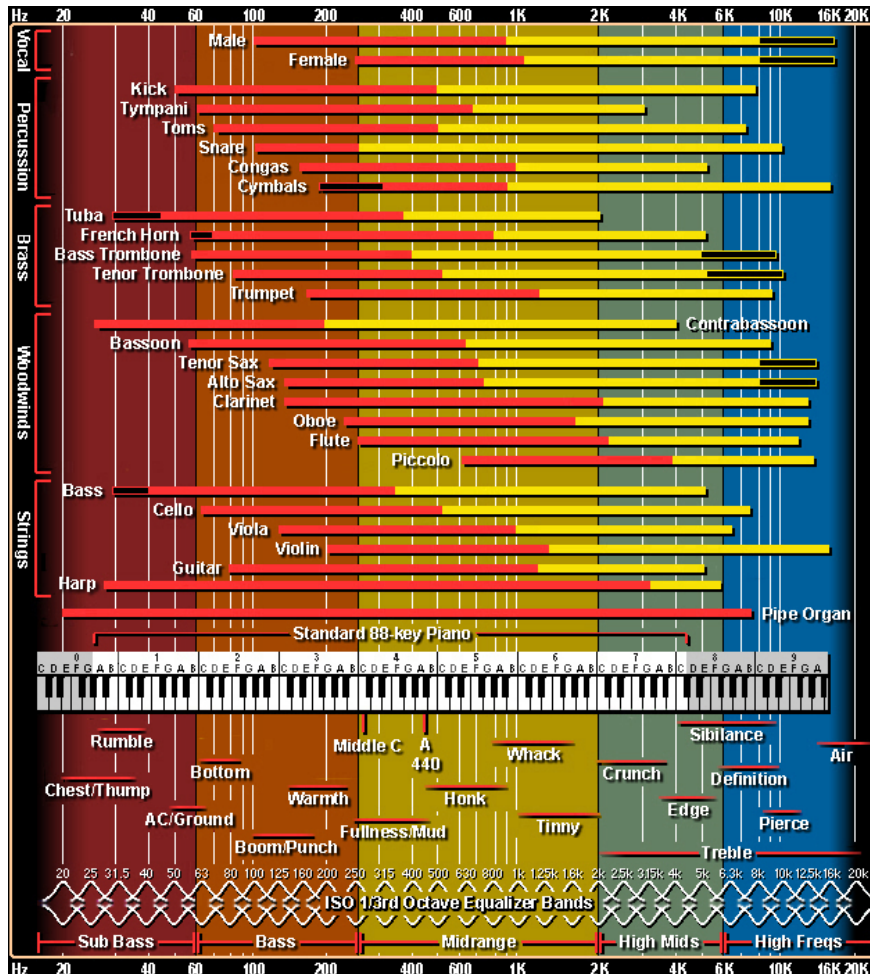

Figure 1: Sketch of the acoustic spectrum.

Figure 2: Expansion of the audible range of the acoustic spectrum.

> **Sanity check question:** *What challenge does the relatively small size of the audible spectrum create? Remember, the word "cutoff" in the phrase "cutoff frequency" is somewhat of a misnomer; the cutoff frequency indicates the point at which the signal power is attenuated by half or the voltage gain is attenuated by $\frac{1}{\sqrt{2}}$, not the point at which the signal is fully eliminated. **Hint**: Think about separation in frequency domain.*

Note: Acoustic waves are *not* electromagnetic waves: sound waves are mechanical and therefore need a medium through which to propagate, whereas EM waves do not need a medium[1]: they can propagate through the vacuum of space.

You will be targeting part of the midrange section (about 1000-2500 Hz) depicted in Figure 2 above, which we define as follows:

Bass	0-500 Hz
Midrange	1000-5,000 Hz
Treble	6,000-20,000 Hz

Ultimately, these frequency ranges are **guidelines**: the goal of this lab is to independently light up your LEDs (with little to no overlap — two LEDs should not light up at the same pure frequency, and "dead zones" should be minimal/imperceptible). Also, you may notice that these ranges do not align exactly with those displayed in figure 2. After completing Part 1 of the lab, you will see why that is.
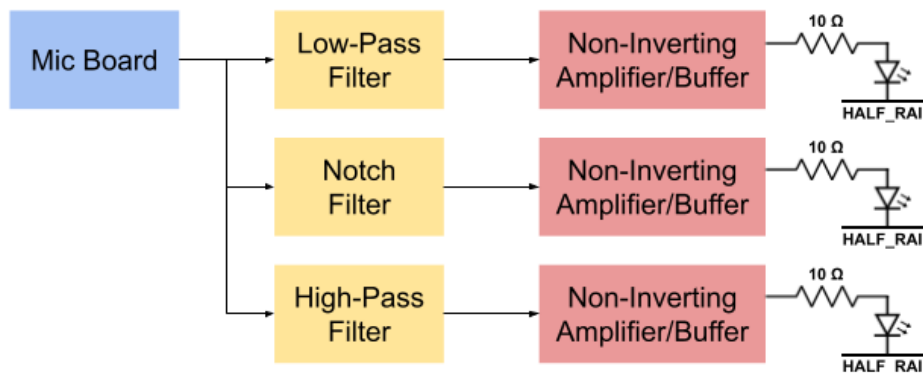

Figure 3: High-level overview of completed micboard color organ system.

This is a significantly larger circuit than the circuits you have built in previous labs, so **be sure to plan ahead when constructing your circuit, and keep your circuit clean!** You don't want to building the band-pass filter only to find you don't have enough space!

Part 1: Frequency Response of the Speaker-Microphone System

Generalizing the first-order filter

The system you are building today is not limited to just the color organ circuit: you must also consider the ability of your speaker to reliably reproduce the desired frequency at a volume large enough to excite the microphone, and the ability of your microphone to respond to the desired frequency. You must also consider that in some ranges, the signal will be highly attenuated, if picked up at all, and compensate for that when you design your color organ (*How can I add gain to some frequencies and not others? Which frequencies should I choose? How can I create sharp cutoffs to minimize both overlap and dead zones?*). To gain the necessary information to design a working color organ, we first identify the speaker-microphone system's frequency response. We will do this empirically: you will sweep over a range of audio frequencies and record the amplitude of the received wave at that frequency.

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[1]This bothered early scientists, so they came up with the concept of the aether (subsequently decommissioned in 1897).

Part 2A: High-Pass Filter

As part of our color organ filter, we will also build a first-order high-pass filter to isolate the frequencies above the desired cutoff frequency.

In the lab ipynb, there is a schematic of how to make a simple RC high-pass filter. Note that in the schematic, we ask you to connect the other end of the resistor to OS2 instead of gnd. This is different from the low-pass filter, where we asked you to connect the other end of the capacitor to gnd, not OS2.

> *Sanity check question: Why do we have you connect the second component of the filter to OS2 instead of gnd for the high-pass filter but not the low-pass filter? Hint: The input to our filters, which is the output of the micboard, has both a sinusoidal/fluctuating component with a frequency and a DC offset component. What does a low-pass filter do to each component? What about a high-pass filter?*

Please refer to Appendix A of this note to familiarize yourself with the derivation of high-pass RC filters.

Part 2B: Caught in the Notch

In the final part of this lab, we will be building a notch filter to pass all frequencies besides a frequency of your choice. The notch filter can be created by placing a resistor, capacitor, and inductor in series.
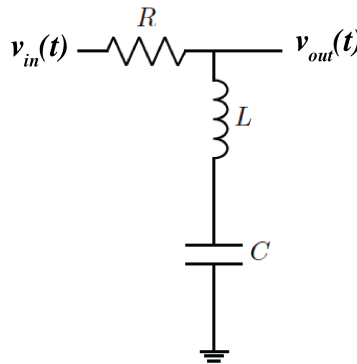
Your circuit should look something like this:

Figure 4: Notch filter.

Its transfer function can be written as:

$$H(j\omega) = \frac{\widetilde{V}_{out}}{\widetilde{V}_{in}} = \frac{Z_L + Z_C}{Z_L + Z_C + Z_R} = \frac{j\omega L + \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{j(\omega L - \frac{1}{\omega C})}{R + j(\omega L - \frac{1}{\omega C})}$$

It's obvious that $|H(j\omega)|$ will be equal to 0 if $\omega L = \frac{1}{\omega C}$, which means that, if we know the exact frequency of the interference signal, we can use this type of RLC filter (notch filter), to create a notch at that frequency and then completely filter the interference signal out, which is impossible to be achieved by an RC filter. The frequency of the notch location, or the resonant frequency of the LC tank, is $\omega_c = \frac{1}{\sqrt{LC}}$. From a circuit perspective, the resonant frequency for this circuit makes both the inductor and capacitor act as shorts, therefore all the voltage drop is in the resistor.

A notch filter has one resonant frequency ω_c and a characteristic called the Q factor, where for an RLC circuit in series,

$$Q = \frac{\omega_c L}{R}$$

At a high level, the Q factor represents the quality of a notch filter. The higher the Q factor, the smaller the bandwidth around the resonant frequency, which means that there will be sharper attenuation at the frequency of choice. The Q

factor can also be used to determine whether a system is underdamped, overdamped, or critically damped. A higher Q factor corresponds to underdamping, whereas a small Q factor corresponds to overdamping. You can think of the resistor in the circuit as what causes the damping. Energy is dissipated in our resistor, whereas inductors and capacitors both store energy in the form of magnetic and electric fields respectively. Therefore the larger the resistance, the higher the damping of the system (how does this relate to frequency attenuation in our magnitude response?). Please refer to Appendix B for more information about notch filters.

> ***Sanity check question:*** *Does the resistor value in the notch filter matter? I.e. how does it impact the response of the notch filter?*

Buffers

You can think of a buffer as providing an impedance transformation between two *cascaded* circuits. When you observe an undesired loading effect between two circuits, placing a buffer between them changes the load impedance of the first circuit to a very high value and the source impedance of the second to a very low value in accordance with (approximately) ideal op-amp characteristics. As the op-amp does not allow any current to flow into its input terminals, this prevents the second filter from drawing current from the first filter and affecting the frequency response. Instead, the second filter draws its current from the output of the op-amp, which is a replica of the first filter's output due to this being a buffer circuit. This allows you to build very modular circuits easily, without having to do lots of ugly algebra.

By placing a buffer in between our two filters that make up the band-pass filter, *cascading them does not change the transfer functions of the individual circuits* and the overall transfer function of the cascade is simply the product of the transfer functions of the individual circuits. This is why buffers are so useful in filter design. However, op-amps (and therefore these buffer circuits) do have their limits; they are not quite as perfect as the golden rules describe them to be.

Amplifier Loading

Recall the impedance characteristics of the ideal op-amp: its input impedance is infinite (no current flows into its inputs) and its output impedance is 0. This allows it to act like an ideal voltmeter at the input and supply infinite current at its output. But what happens if we don't have those characteristics?

Let's assume we have a noninverting amplifier with a gain of 100, with a finite input impedance Z_I and non-zero output impedance Z_o, as in the schematic below. The middle box represents the amplifier.
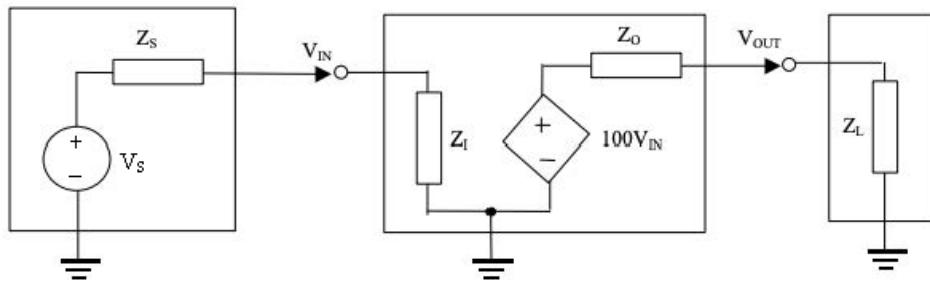
We now see that V_{IN} depends on the source output impedance Z_S and the amplifier input impedance Z_I (because of the voltage divider formed by the two), and V_{out} depends on the amplifier output impedance Z_O and the load impedance Z_L. Recalling the voltage divider equation,

$$V_{IN} = V_S \frac{Z_I}{Z_I + Z_S}$$

$$V_{out} = (100 V_{IN}) \frac{Z_L}{Z_L + Z_O}$$

But $100 V_{IN}$ is our desired V_{out}! To keep that approximately correct and avoid "loading" the output and reducing the voltage noticeably from what we expect, we need Z_L to be considerably larger than Z_O to keep $\frac{Z_L}{Z_L + Z_O}$ as close to 1 as possible.

This is why you set the "output load" on the signal generator to "High-Z": by doing so, you are telling it to expect a high-impedance load. The function generator has a 50-ohm output impedance, while the oscilloscope probes are high-impedance, so when the function generator is set to "High-Z," you can probe it with the oscilloscope and see the output voltage you expect (the one you explicitly set). If you set the function generator to "50 Ohm", it expects a 50 Ohm load. Since this is equal to its output impedance, V_{out} would be halved, so the function generator compensates for this by doubling its output voltage in 50 Ohm mode.
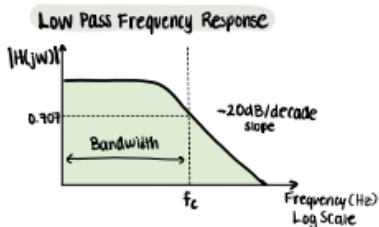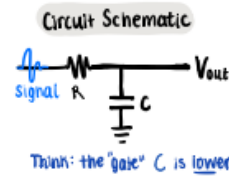
Note that in addition to the output loading effect, the amplifier itself can also load its input (e.g. the output of a preceding circuit) if it has a finite input impedance, reducing the V_{IN} from the desired value of V_S. We thus also need Z_I to be much larger than Z_S as well so that the input is not affected (keep $\frac{Z_I}{Z_I+Z_S}$ as close to 1 as possible).

For our circuits, including the ones in this lab, you may assume that the load impedance is always sufficiently large enough and the op-amps' output impedance is small enough so that the loading effect is negligible. You may also assume that the input impedance of our op-amps is sufficiently large enough to prevent the op-amp from loading its own input. Thus, we can safely connect the op-amp without worrying about it loading the output of the previous circuit, and the outputs of our op-amp circuits, like the buffers we're using in this lab, will remain unaffected by whatever we connect it to, allowing us to safely use a buffer to build our band-pass filter.

Appendix A: Derivation of first and second order RC filters

Building Filters

Lowpass Filter

Circuit Schematic

signal R → Vout, with C to ground

Think: the "gate" C is lower.

Low Pass Frequency Response

$|H(j\omega)|$, 0.707, Bandwidth, $-20dB/decade$ slope, f_c, Frequency (Hz) Log Scale

$$\hat{V}_{out} = \hat{V}_{in} \cdot \frac{Z_C}{Z_R + Z_C} = \hat{V}_{in} \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \hat{V}_{in} \frac{1}{j\omega RC + 1}$$

$$\frac{V_{out}}{V_{in}} = H(j\omega) \text{ and cutoff frequency is at half power, where } \frac{|\hat{V}_{out}|}{|\hat{V}_{in}|} = \frac{1}{\sqrt{2}} = 0.707.$$

$$|H(j\omega)| = \frac{1}{\sqrt{2}} = \frac{\sqrt{1}}{\sqrt{(\omega RC)^2 + (1)^2}} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$
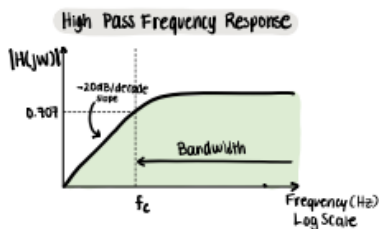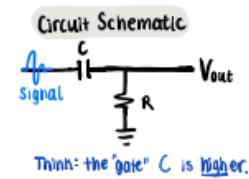
$$2 = 1 + (\omega RC)^2$$

$$1 = \omega RC$$

$$\boxed{\omega = \frac{1}{RC}} \quad \text{angular cutoff frequency}$$

$$\boxed{f_c = \frac{1}{2\pi RC}} \quad \text{cutoff frequency}$$

Conceptually: as $\omega \to \infty$, $|H(j\omega)| \to 0$
as $\omega \to 0$, $|H(j\omega)| \to 1$

Everything that is less than f_c gets through. Note that our cutoff isn't clean & perfect because the attenuation is gradual.

High Pass Filter

Circuit Schematic

signal C → Vout, with R to ground

Think: the "gate" C is higher.

High Pass Frequency Response

$|H(j\omega)|$, 0.707, $-20dB/decade$ slope, Bandwidth, f_c, Frequency (Hz) Log Scale

$$\hat{V}_{out} = \hat{V}_{in} \cdot \frac{Z_R}{Z_R + Z_C} = \hat{V}_{in} \frac{R}{\frac{1}{j\omega C} + R}$$

$$|H(j\omega)| = \frac{|\hat{V}_{out}|}{|\hat{V}_{in}|} = \frac{1}{\sqrt{2}} = \frac{\sqrt{R^2}}{\sqrt{(\frac{1}{\omega C})^2 + R^2}}$$

$$\frac{1}{2} = \frac{R^2}{(\frac{1}{\omega C})^2 + R^2}$$

$$\left(\frac{1}{\omega C}\right)^2 + R^2 = 2R^2$$

$$\left(\frac{1}{\omega C}\right)^2 = R^2$$

$$\boxed{\omega = \frac{1}{RC}} \quad \text{angular cutoff frequency}$$

$$\boxed{f_c = \frac{1}{2\pi RC}} \quad \text{cutoff frequency}$$

Conceptually: as $\omega \to \infty$, $|H(j\omega)| \to 1$
as $\omega \to 0$, $|H(j\omega)| \to 0$
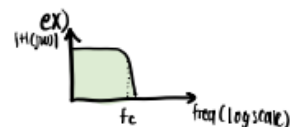
Everything higher that f_c gets through.

Thought: What happens to DC Voltage in a high pass filter?
↳ It gets destroyed, $\omega = 0$!
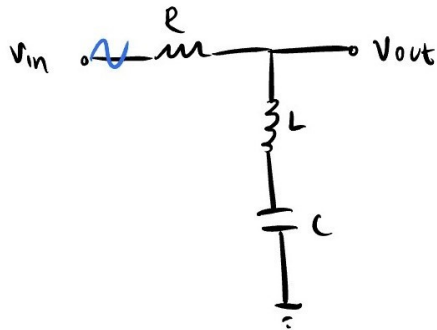
Thought: How can we make attenuation faster?
↳ multiple filters cascaded. Our transfer functions multiply, making the drop-off faster.
↳ make sure to place a unity gain buffer in between to prevent loading
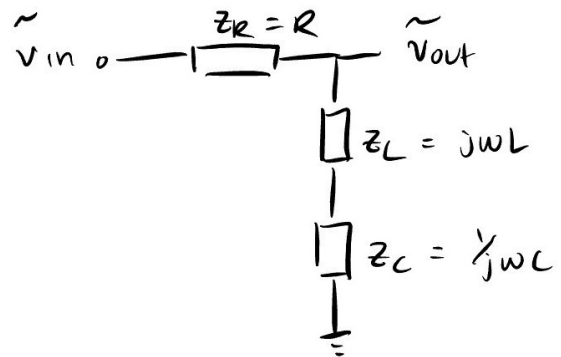
ex) $|H(j\omega)|$ vs freq (log scale), f_c

Appendix B: Derivation of second order RLC Notch
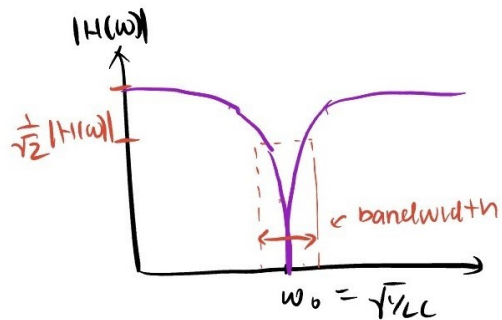
Notch filters

Time Domain

Phasor Domain

Phasor Domain Analysis:

$$\frac{\tilde{v}_{out}}{\tilde{v}_{in}} = \frac{j\omega L + \frac{1}{j\omega L}}{R + j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{j^2 \omega^2 LC + 1}{Rj\omega C + j^2 \omega^2 LC} = \frac{-\omega^2 LC + 1}{j\omega RC - \omega^2 LC}$$

$$\frac{\tilde{v}_{out}}{\tilde{v}_{in}} = 0 \quad \longrightarrow \quad -\omega^2 LC + 1 = 0$$
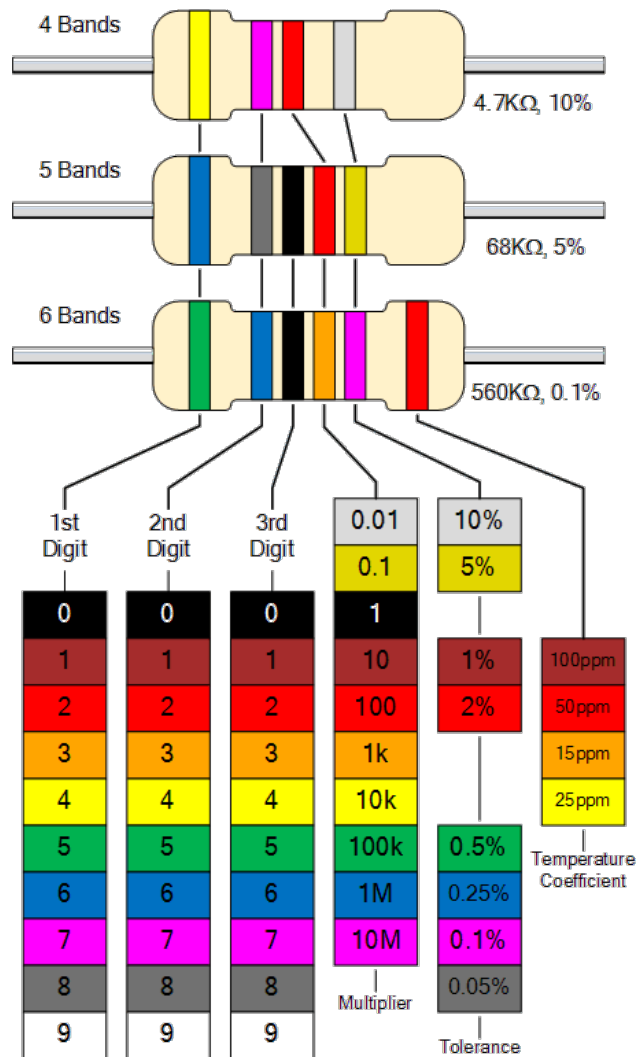
$$1 = \omega^2 LC$$

$$\frac{1}{LC} = \omega^2$$
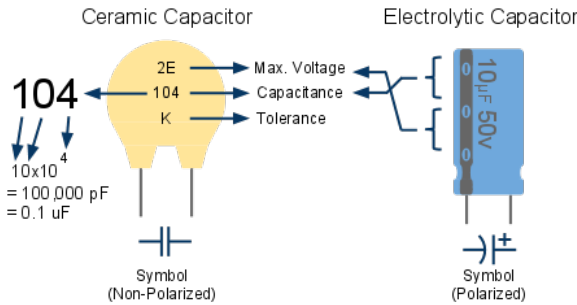
$$\boxed{\sqrt{\frac{1}{LC}} = \omega_0}$$

$$Q \text{ factor} = \frac{\omega_0}{\text{bandwidth}} = \frac{\omega_0 L}{R} \quad \text{or} \quad \frac{1}{\omega_0 RC}$$

Appendix C: Resistor Color Code

Appendix D: Capacitor Codes

Capacitors

Ceramic Capacitor

Electrolytic Capacitor

104

2E	→ Max. Voltage
104	→ Capacitance
K	→ Tolerance

10x10⁴
= 100,000 pF
= 0.1 uF

Symbol
(Non-Polarized)

10µF 50V

Symbol
(Polarized)

Max. Operating Voltage

Code	Max. Voltage
1H	50V
2A	100V
2T	150V
2D	200V
2E	250V
2G	400V
2J	630V

Capacitance Conversion Values

Microfarads (µF)		Nanofarads (nF)		Picofarads (pF)
0.000001 µF	↔	0.001 nF	↔	1 pF
0.00001 µF	↔	0.01 nF	↔	10 pF
0.0001 µF	↔	0.1 nF	↔	100 pF
0.001 µF	↔	1 nF	↔	1,000 pF
0.01 µF	↔	10 nf	↔	10,000 pF
0.1 µF	↔	100 nF	↔	100,000 pF
1 µF	↔	1,000 nF	↔	1,000,000 pF
10 µF	↔	10,000 nF	↔	10,000,000 pF
100 µF	↔	100,000 nF	↔	100,000,000 pF

Tolerance

Code	Percentage
B	± 0.1 pF
C	±0.25 pF
D	±0.5 pF
F	±1%
G	±2%
H	±3%
J	±5%
K	±10%
M	±20%
Z	+80%, -20%

References

Horowitz, P. and Hill, W. (2015). *The Art of Electronics*. 3rd ed. Cambridge: Cambridge University Press, ch 1.
Sedra, A. and Smith, K. (2015). *Microelectronic Circuits*. 7th ed. New York: Oxford University Press, ch 17.
https://www.electronics-tutorials.ws/wp-content/uploads/2018/05/resistor-res5.gif
https://www.bragitoff.com/wp-content/uploads/2015/09/CapacitorsCheatSheet.png