Lecture 2 Key Concepts

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KC: Key Concepts. **BK:** Background material.

1 Lecture 2, Module 1

1.1 Slide 25

BK: We build capacitors in an IC process using various structures such as multi-finger comb capacitors as opposed to parallel plate capacitors. We get better capacitor density using such structures as opposed to parallel plate capacitors because IC process technology is optimized to allow very close spacing between metals and very thin lines whereas we don't want to stack metals too close because it causes *parasitic* capacitance. So we usually prefer a lateral capacitor to a vertical one[1].

1.2 Slide 26-27

KC: When we place a dielectric between the capacitor plates, the capacitance goes up. This occurs because the electric fields inside the capacitors distort the electrons and produce dipoles. These dipoles cancel out inside the dielectric but at the boundary where they meet the conductors, they actually cancel the charge partially, allowing more charge to be placed onto the plates.

1.3 Slide 28

KC: Capacitor current is easily derived from the definition of capacitance, $q = CV$, so

$$I = \frac{dq}{dt} = C\frac{dV}{dt}$$

KC: the voltage-current relation of a capacitor can produce positive or negative power $p(t)$ since dV/dt can be positive or negative. If it's positive, we're charging the capacitor, and the capacitor is storing energy. When dV/dt is negative, the capacitor is delivering energy to the rest of the circuit, like a battery.

KC: Capacitors cannot dissipate energy, they only store it. You can later get the energy back.

[1] Some technologies have a specialized thin oxide for building high density parallel plate capacitors, but this is the exception rather than the rule.

1.4 Slide 30

KC: We find that for a system of conductors, we can define a matrix of capacitors which represents the amount of charge induced on each conductor. We don't need this formation in this class, simply be aware that when multiple conductors are in close proximity, we can model the system as having many mutual capacitances.

1.5 Slide 32

KC: Placing capacitors in parallel results in an equivalently larger capacitor of value $C_1 + C_2 + \cdots$. This is intuitive if we that adding capacitors in parallel is like putting water tanks in parallel, it increases the capacity to store water (charge).

1.6 Slide 33

KC: Series capacitors are a bit tricky because there's a floating node in between and no net charge can flow onto the floating node. We show the equivalent capacitance is dominated by the smallest in series, and the formula is the same is resistors in parallel.

1.7 Slide 34

KC: The physical picture of series capacitors shows us very clearly that the middle plates do not really do anything and the capacitor is equivalent to a new capacitor with bigger spacing between the plates.

1.8 Slide 35

KC: There are capacitors everywhere, like it or not! When we put traces on a PCB, or when we route signals on an IC, the metal wires overlap or go over a ground plane, resulting in capacitance to ground and coupling capacitance (mutual capacitors) between the different conductors.

2 Lecture 2, Module 2

2.1 Slide 4

KC: If we run a constant current into a capacitor, the voltage ramps up linearly. The capacitor has memory. It will ramp up from it's initial value to some final value.

We introduce the water tank analogy here and imagine filling a water tank at a constant rate, resulting in a linearly increasing water level in the tank. Note that we must pump the water from the bottom because it takes energy to raise the water level, similar to charging a capacitor that has an initial voltage. It takes energy to add more charge onto the plates.

2.2 Slide 6

Here we setup the differential equation using KVL/KCL for a simple RC circuit. We get a differential equaiton.

KC: When there are no sources present, RC circuits results in homogeneous differential equations. The solution is determined by the initial conditions (the initial charge/voltage on the capacitor).

2.3 Slide 7

KC: The best way to solve a differential equation is to guess the solution and to verify it's correct.

KC: It's pretty clear that an exponential decay will satisfy the differential equation. We try it and it works.

KC: For any homogeneous differential equation, substitute an exponential e^{st} and solve for s from the resulting algebraic equation. The complete solution is then given by applying initial conditions.

2.4 Slide 8

KC: Capacitors have memory and remember the past ! There's no instantaneous input-output equation in a capacitor like we have for a resistor.

2.5 Slide 9

KC: We view our RC circuit is a system with inputs and outputs.

2.6 Slide 10

KC: The forced response is the solution to the system (differential equation) for a given input. The input can be DC, a sine wave, or any arbitrary waveform.

2.7 Slide 11

KC: Superposition applies to linear differential equations.

2.8 Slide 12

KC: We prove that the solution of the homogeneous differential equation is unique.

KC: The general solution to the differential equation is the forced response plus the homogenous solution. The unknown constants of the solution can be determined by initial conditions.

2.9 Slide 13-14

KC: We use a nice trick to solve the general first-order differential equation with an input. This is the integrating factor. Basically we take advantage of the well known product rule:

$$(uv)' = u'v + v'u$$

If we can put our system into this form, we can solve it immediately by integrating both sides. In our case, we solve it with a exponential function integrating factor.

2.10 Slide 16

KC: We solve the special but important case of an RC circuit with DC inputs. While the integrating factor is used here, it's fairly obvious what the solution should look like. Later we'll use the concept of DC steady-state to write the solution by inspection.