EECS 16B

Designing Information Devices and Systems II

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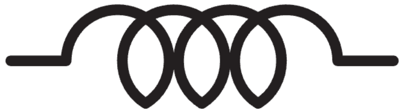
Module 3: Inductors and Inductance

EECS 16B

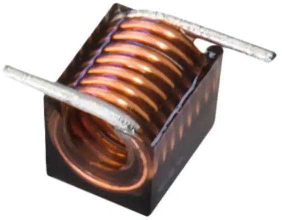
Outline

- Inductance
- Inductors
- Differential Equations
- Mutual Inductors
- Transformers

Inductors

Symbol: 

- Inductors store energy in the magnetic field
- Current carrying coils wound around a magnetic core material (popular materials are various types of iron oxides – often called ferrites) or “air core” inductors for higher frequencies



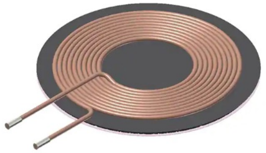


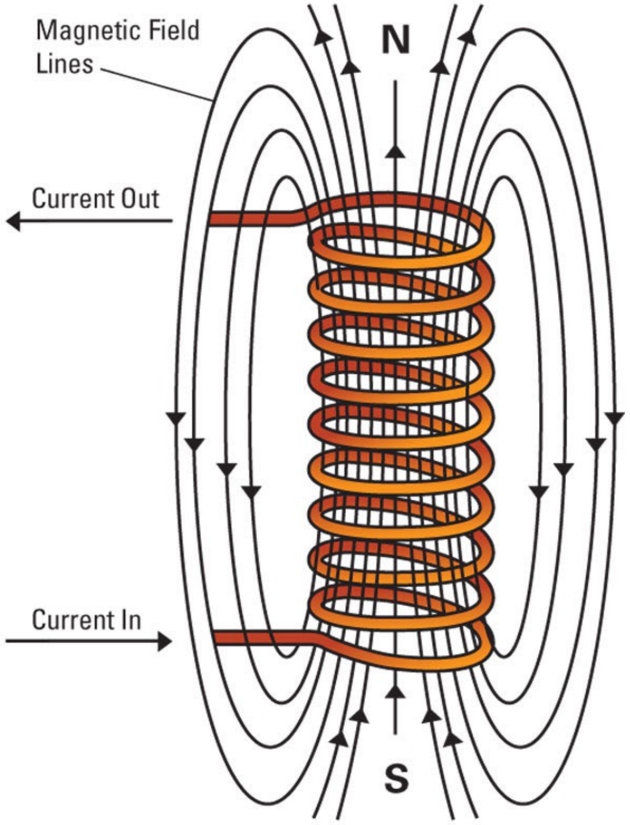
Image source: Digikey

Magnetic Flux

- Governed by Faraday's law of electromagnetic induction which states that a time-varying magnetic flux linking a coil induces a voltage across the coil which is proportional to the rate of change in the current. This proportionality constant is the inductance

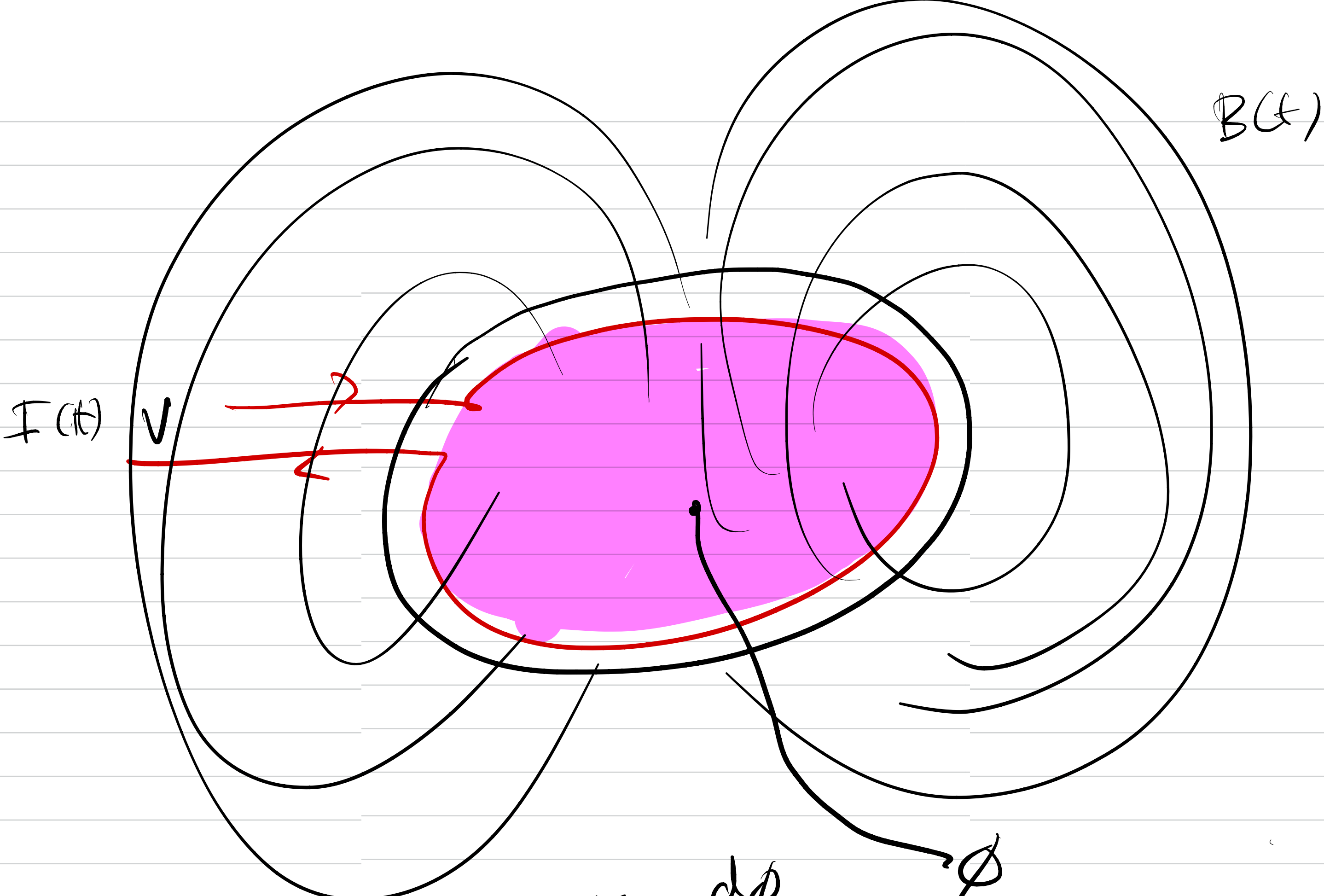
Reciprocity says that a time-varying current flowing through a coil will create a time-varying magnetic flux

$$v(t) = L\frac{di}{dt}$$



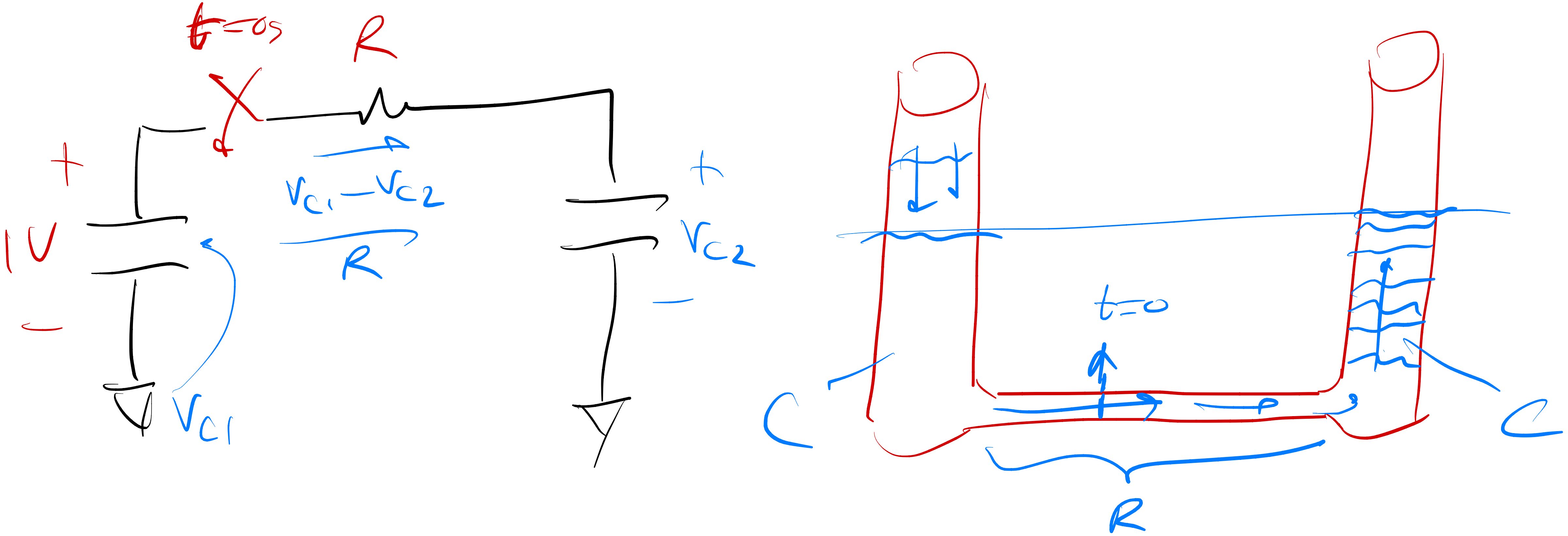
Charge q → $\frac{dq}{dt} = i$ $q = C v$

magnetic flux ϕ → $\frac{d\phi}{dt} = v$ $\phi = L i$

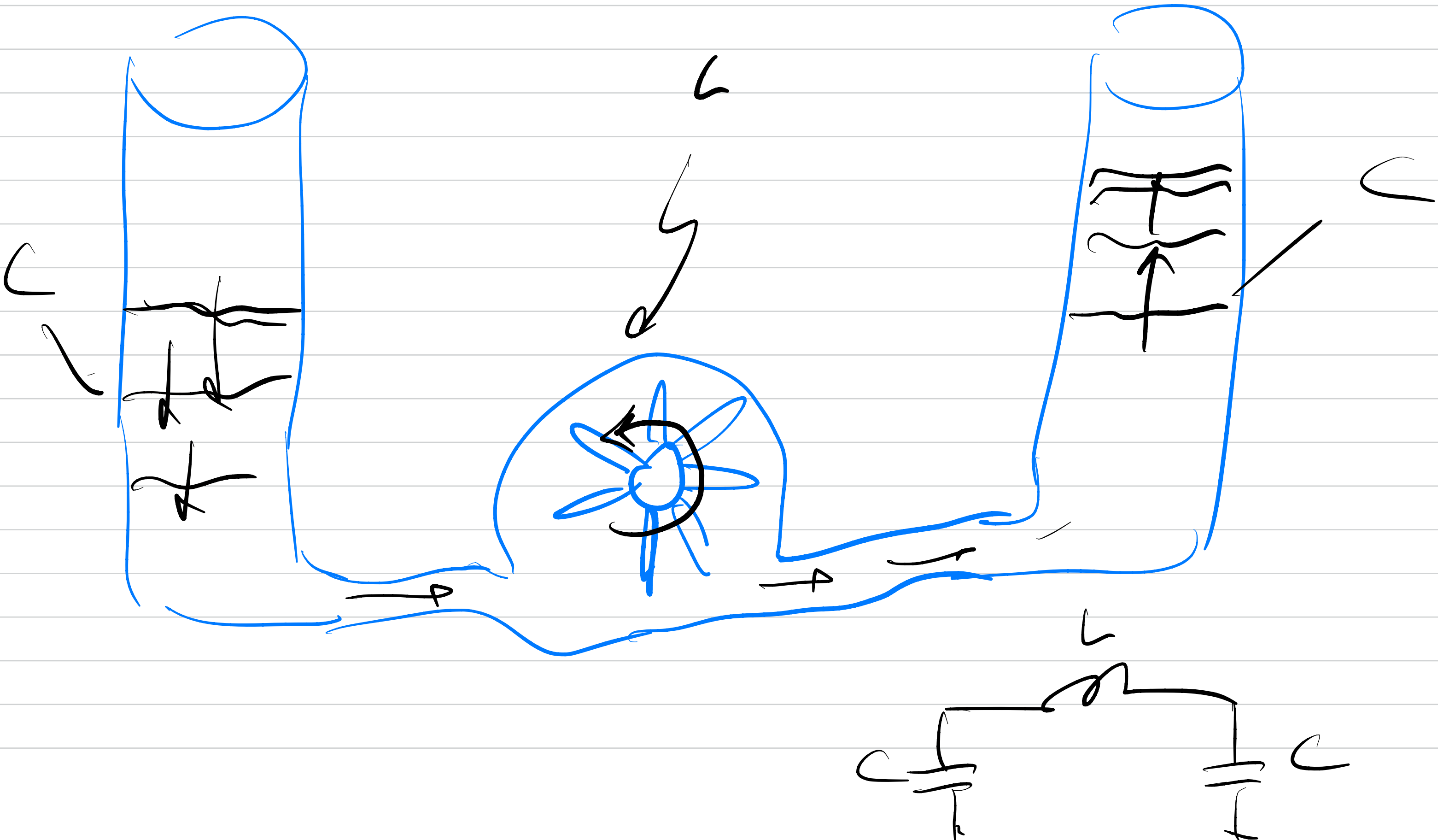


$$V = -\frac{d\phi}{dt}$$

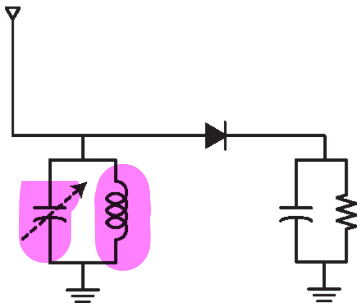
Waterwheel Analogy

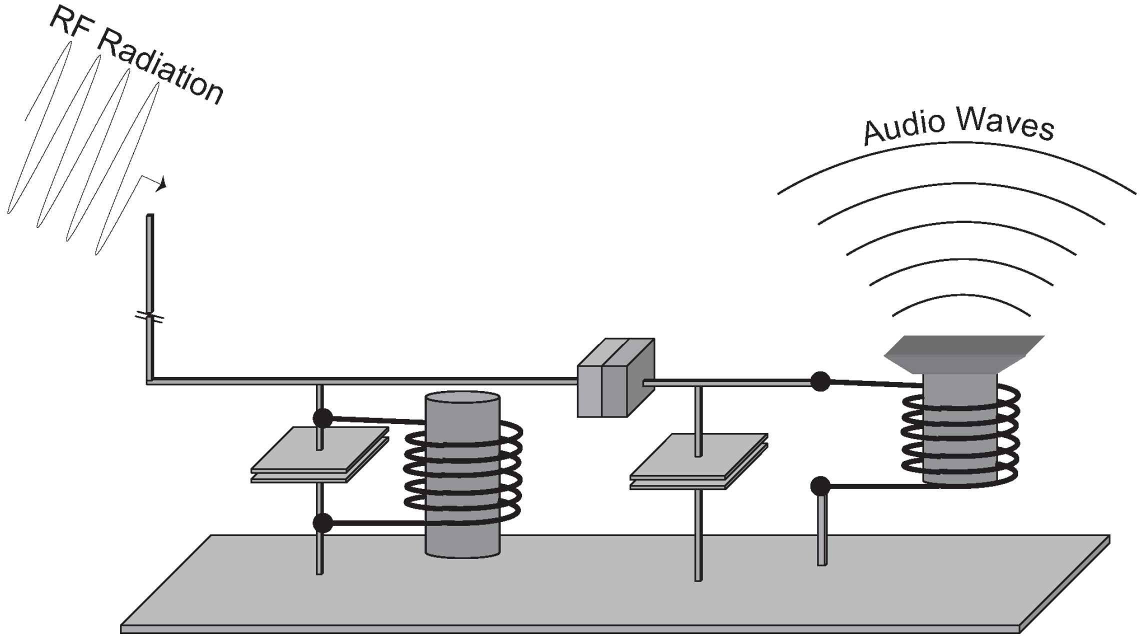


- Angular momentum of waterwheel keeps current flowing !

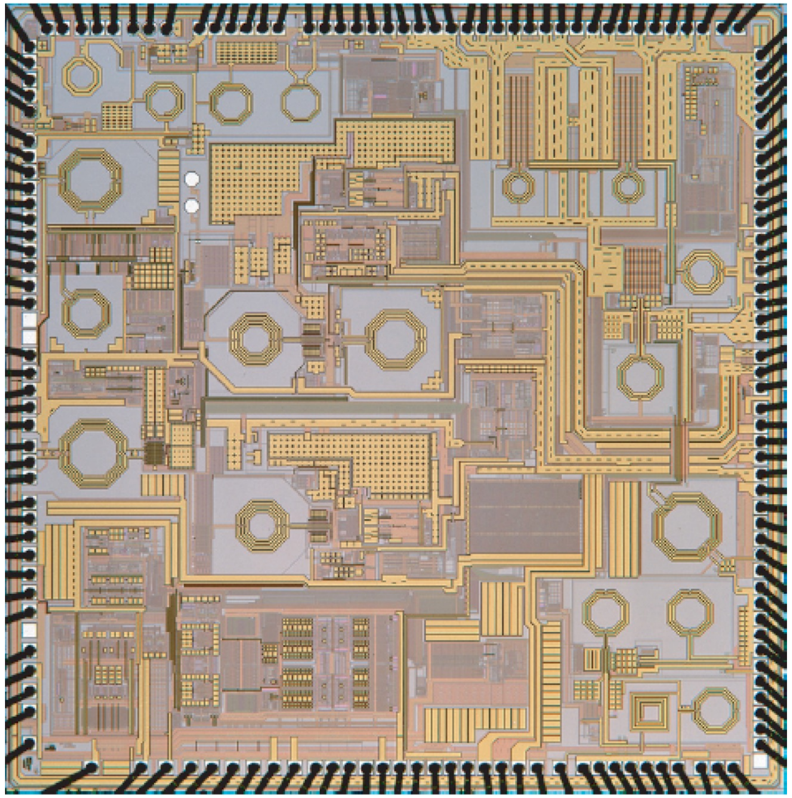

C
L
C
L
C
C

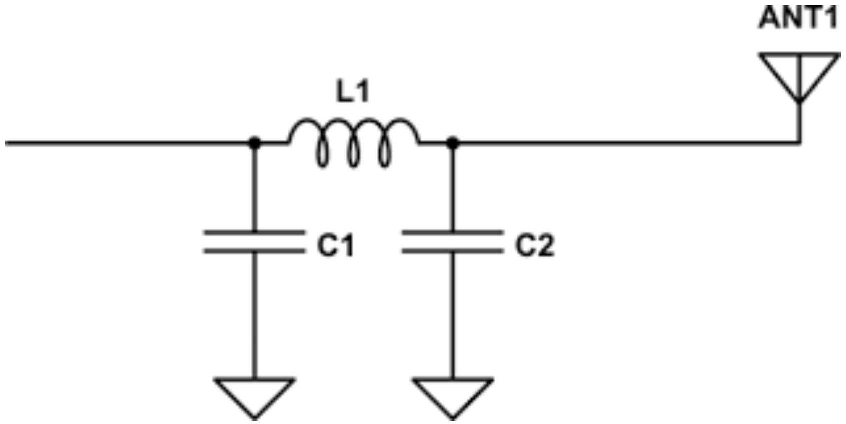
Classic AM Radio





Inductors



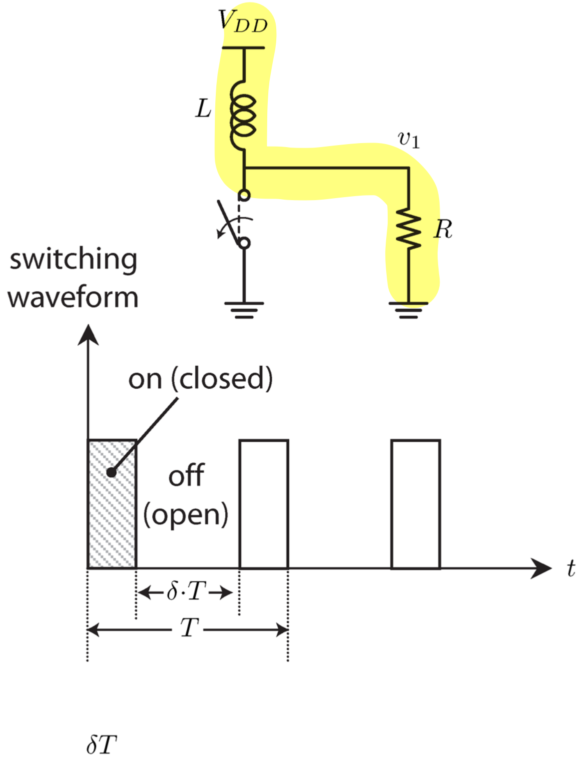


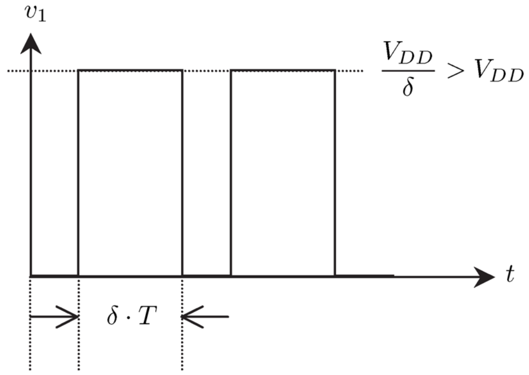
Communication needs sending and receiving of

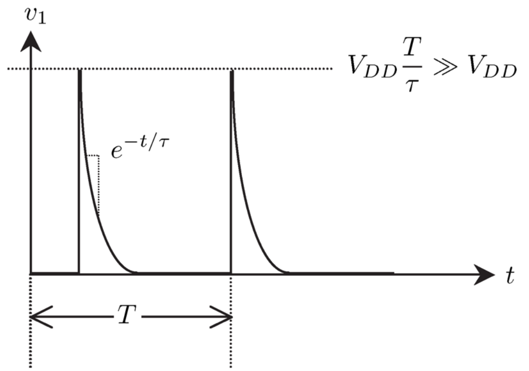
C L

Electro-Magnetic Wave

DC-DC Convertor







- Inside of virtually every electronic device including microprocessors
- By varying the duty cycle of the switching waveform, we can "boost" the DC voltage. We can also step down the voltage with a "buck" converter.

Current in an Inductor

$\phi = L\, i$

$v = \frac{d\phi}{dt}$

$v = \frac{d}{dt}(L i)$

$v = L \frac{di}{dt}$

$$v(t) = L\frac{di}{dt}$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v\ dt + i(t_0)$$

$q = C v$

$i = \frac{dq}{dt}$

Remember the capacitors

$$i = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i\ dt + v(t_0)$$

DUAL

$i \leftrightarrow v$

Stored Energy in an Inductor

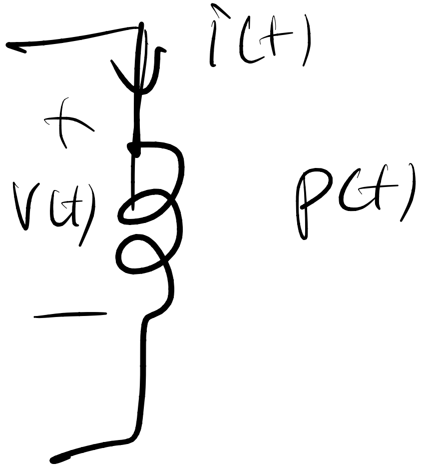
$$v(t) = L\frac{di}{dt}$$

$$E = \frac{1}{2}Cv^2$$

$$p(t) = v(t)i(t)$$

$$p(t) = i(t)\left(L\frac{di(t)}{dt}\right) = L\frac{i(t)di(t)}{dt}$$

$$p(t)dt = Li(t)di(t)$$



$$p(t) = L\, i(t)\frac{di(t)}{dt}$$

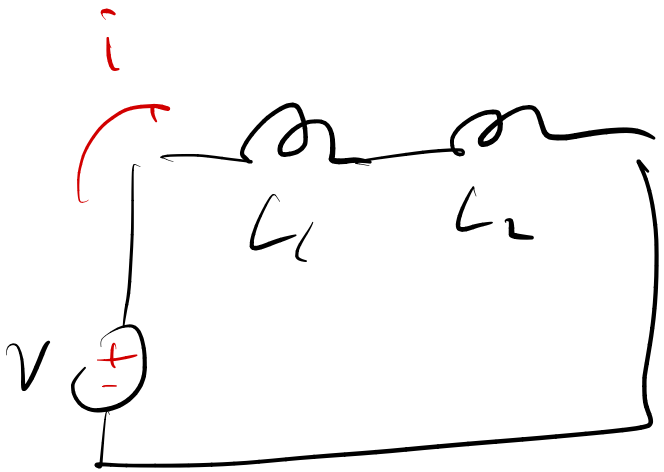
$$E = \int p(t)\,dt = \int L\, i(t)\,di = L\int i\,di$$

$$= L\left.\frac{i^2}{2}\right|_{i_0}^{i} = \frac{Li^2}{2}$$

$$i_0 = 0$$

Inductances in Series and Parallel

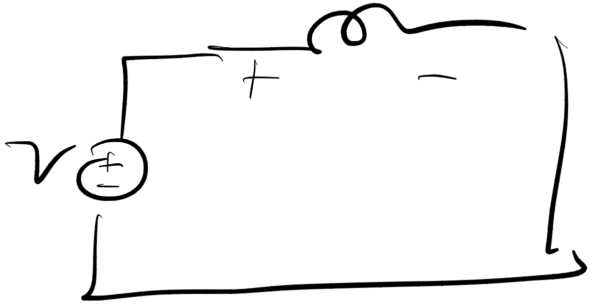
Series: Common Current



$V_{L1} = L_1 \frac{di}{dt}$

$V_{L2} = L_2 \frac{di}{dt}$

L_{eq}

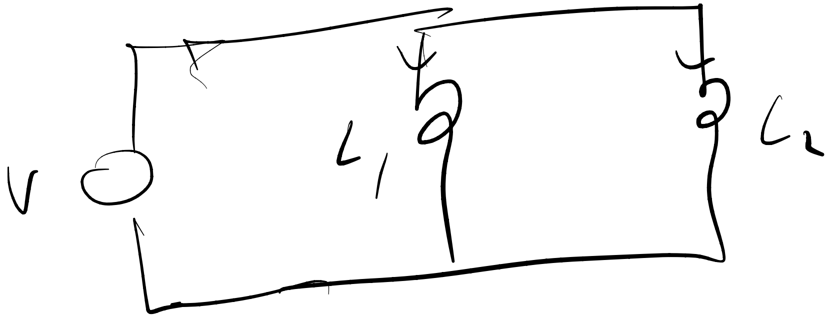


$V = V_{L1} + V_{L2}$

$= L_1 \frac{di}{dt} + L_2 \frac{di}{dt}$

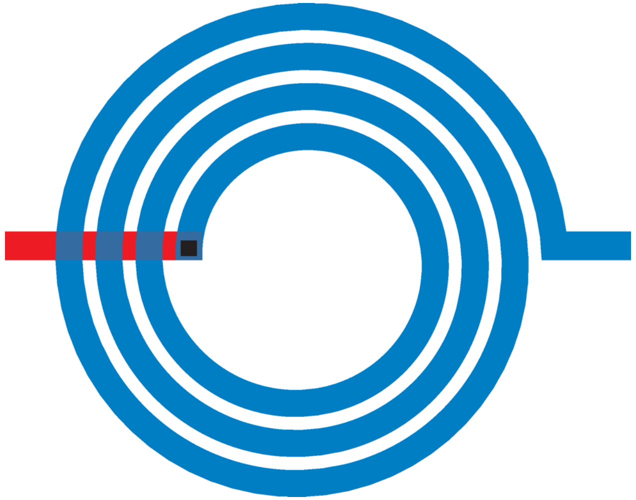
$= (L_1 + L_2) \frac{di}{dt}$

Parallel: Common Voltage



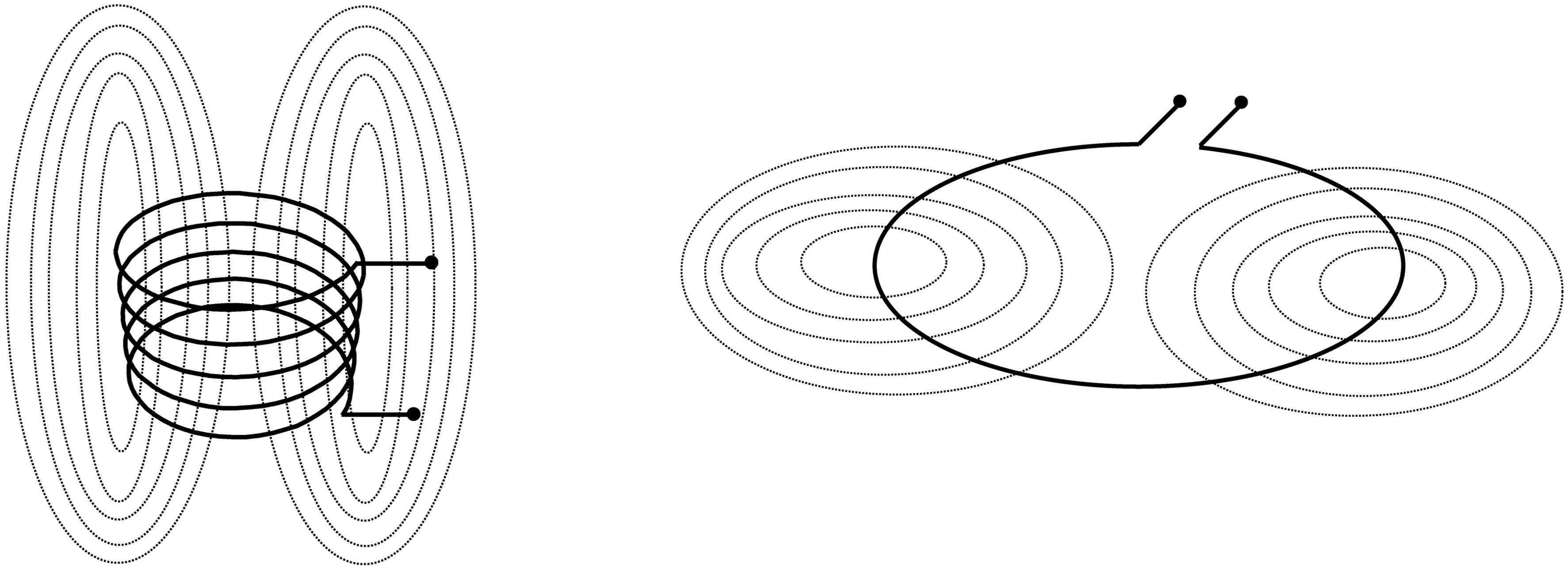
$$L_{eq} = \frac{L_1 \parallel L_2}{L_1 + L_2}$$

Integrated Circuit Inductors



- Can’t build “3D” solenoid types so typically build spiral inductors. These are “tiny” (radius ~ thickness of hair)

Inductance of Circuits



- Even if we try to avoid building an inductor, any closed loop circuit has intrinsic inductance !

Summary

Capacitors:

$$i = C\frac{dv}{dt}$$

$$w = \frac{1}{2}Cv^2$$

- v cannot charge instantaneously
- i **can** charge instantaneously (do not short circuit a charged capacitor)
- N capacitors in series $\frac{1}{C_{eq}} = \sum_{i=1}^{N} \frac{1}{C_i}$
- N capacitors in parallel $C_{eq} = \sum_{i=1}^{N} C_i$

Inductors:

$$v = L\frac{di}{dt}$$

$$w = \frac{1}{2}Li^2$$

- i cannot charge instantaneously
- v **can** charge instantaneously (do not open an inductor with current)
- N inductors in series $L_{eq} = \sum_{i=1}^{N} L_i$
- N inductors in parallel $\frac{1}{L_{eq}} = \sum_{i=1}^{N} \frac{1}{L_i}$

General Solution of the Differential Equation

For a first order, linear differential equation of the form:

$$\frac{dy}{dt} + ay(t) = b(t)$$ where we assume a to be a constant

Homogeneous Solution

$$\frac{dy}{dt} + ay(t) = 0$$
$$\Rightarrow \frac{dy}{y} = -a$$
$$\Rightarrow \ln(y) = -at + C$$
$$\Rightarrow y(t) = Ke^{-at}$$

Particular Solution (Integrating Factor Method):

$$\frac{dy}{dt} + ay(t) = b(t)$$

We want to find a multiplier function *f(t)* such that

$$f(t)\frac{dy}{dt} + af(t)y(t) = b(t)f(t)$$

can be written as

$$\frac{d}{dt}[y(t)f(t)] = b(t)f(t) \quad \text{(A)}$$

For equation (A) to hold

$$\frac{df(t)}{dt} = af(t)$$
$$\Rightarrow f(t) = e^{at}$$

Then from (A)

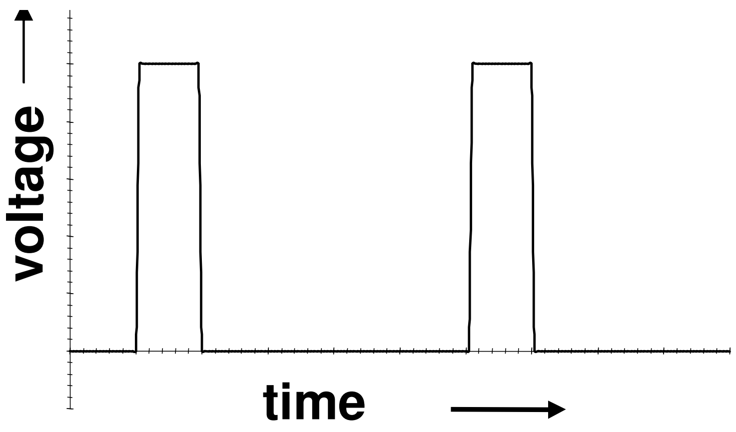
$$y(t) = \frac{1}{f(t)}\int b(t)f(t)dt$$
$$\Rightarrow y_p(t) = e^{-at}\int e^{at}b(t)dt$$

$$\boxed{y(t) = Ke^{-at} + e^{-at}\int e^{at}b(t)dt}$$

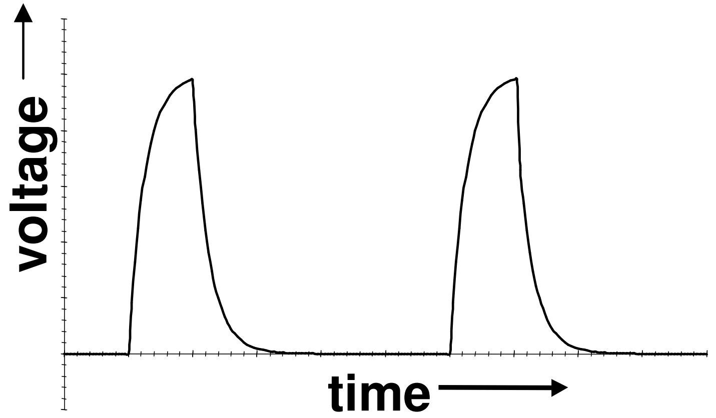
K is determined using initial condition

Digital Signals to a RC circuit

- Every node in a real circuit has capacitances

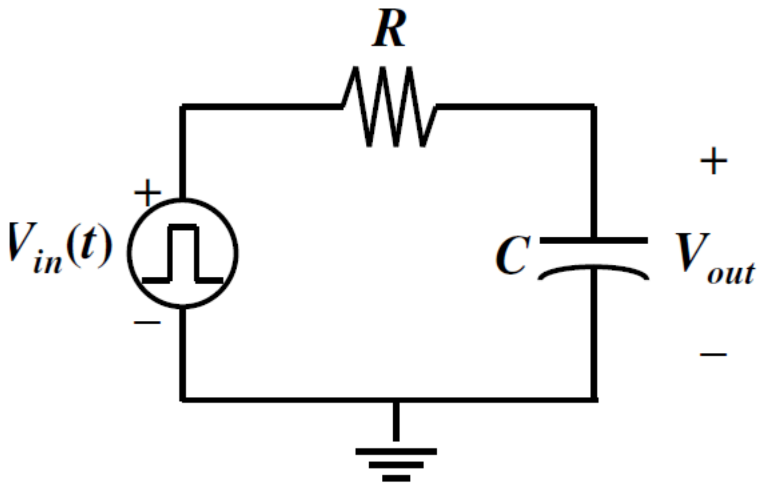


- Even if we send in very 'pure' square looking pulses what we actually get are distorted pulsed due to capacitor charging and discharging. <u>If we switch very very slowly, we may not even notice this behavior, but it's always there.</u>

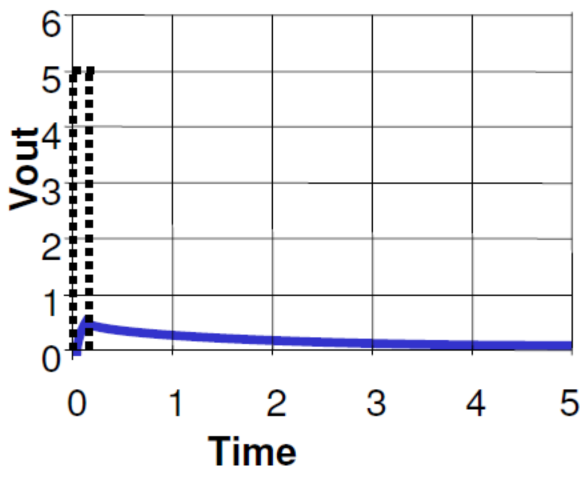


Pulse Distortion

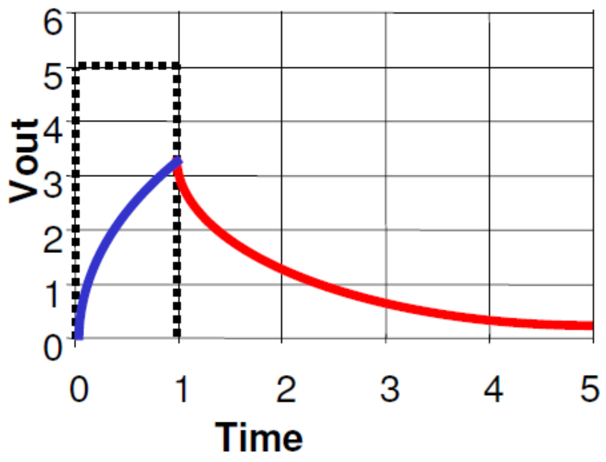
The input voltage pulse width must be large enough; otherwise the pulse is distorted



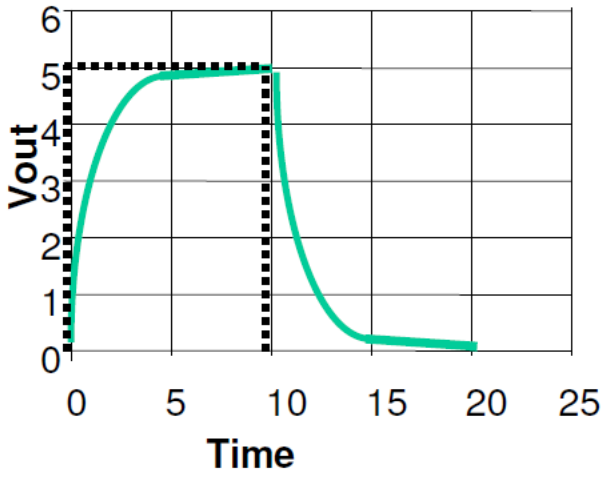
Pulse width = 0.1*RC*



Pulse width = *RC*

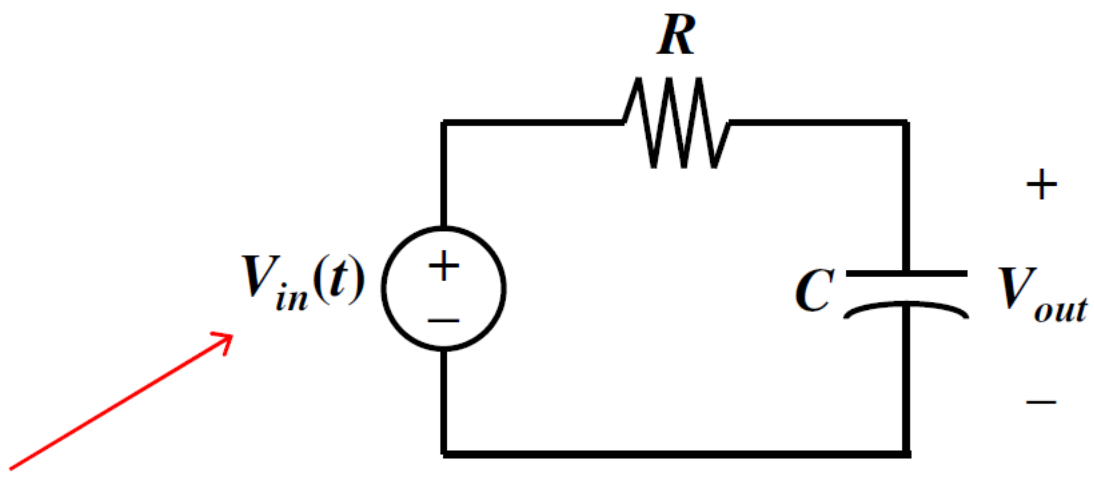


Pulse width = 10*RC*



Computers are RC circuits (almost)

- Digital circuits are predominantly RC circuits (other than the communication part)
- Simplistically a logic gate can be model as a RC circuit
- The speed of the computer is limited by the RC time constant



switches between "low" (logic 0)
and "high" (logic 1) voltage states

R-L Circuits

KVL $t \geq 0$

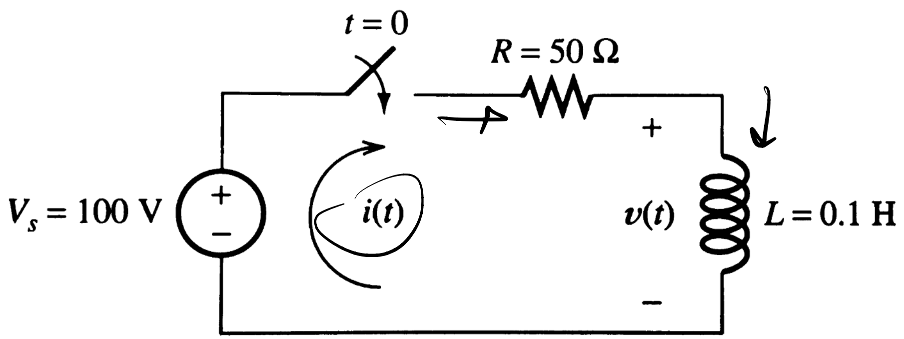
$$V = L\frac{di}{dt}$$

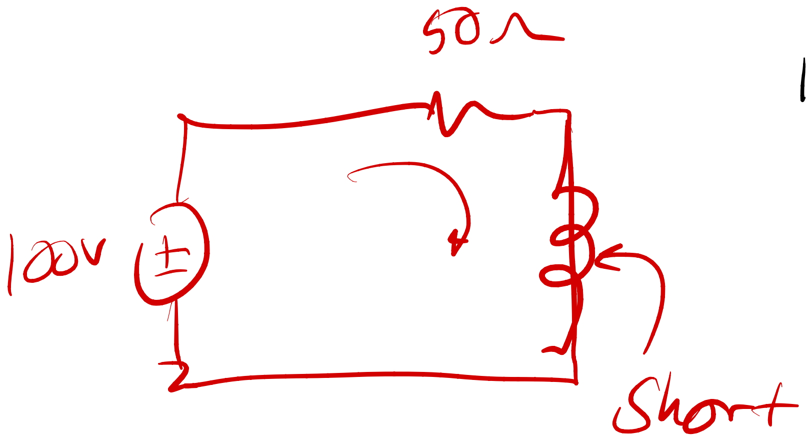
$$V_s = iR + V(t)$$

$$= i \cdot R + L\frac{di}{dt}$$

$$\frac{V_s}{R} = i + \left(\frac{L}{R}\right)\frac{di}{dt}$$

$$\boxed{\frac{V_s}{R} = i + \tau\frac{di}{dt}}$$





$$i(0) = 0A$$

$$i(\infty) = \frac{100V}{50\Omega} = 2A$$

DC Steady State

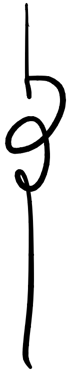
Capacitors:

$$i = C\frac{dv}{dt} = 0$$

$$\frac{d}{dt} \to 0$$

OPEN CIRCUIT

Inductors:

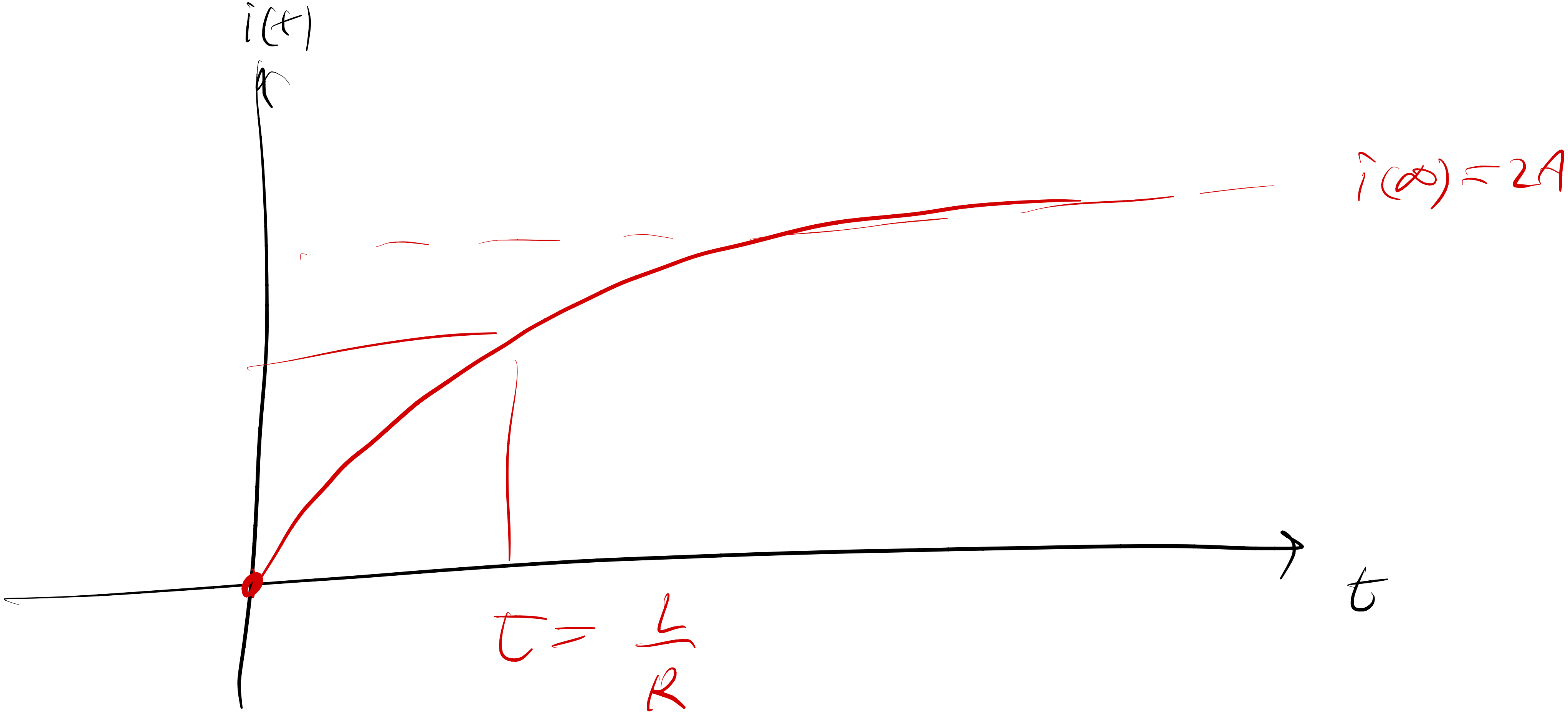


$$v = L\frac{di}{dt} = 0$$

$$\frac{d}{dt} \to 0$$

SHORT CIRCUIT

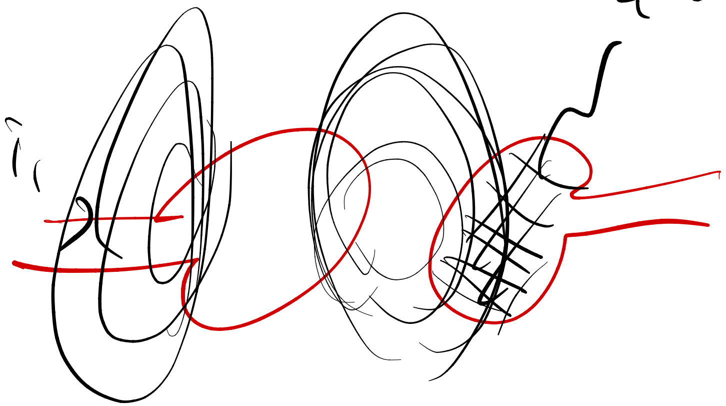
Same Equations → Same Solution

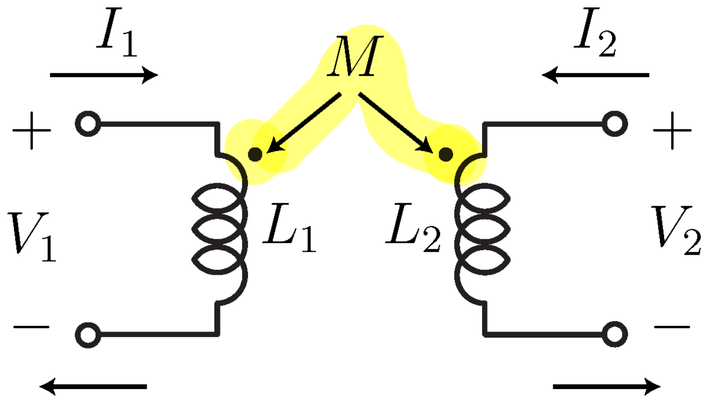


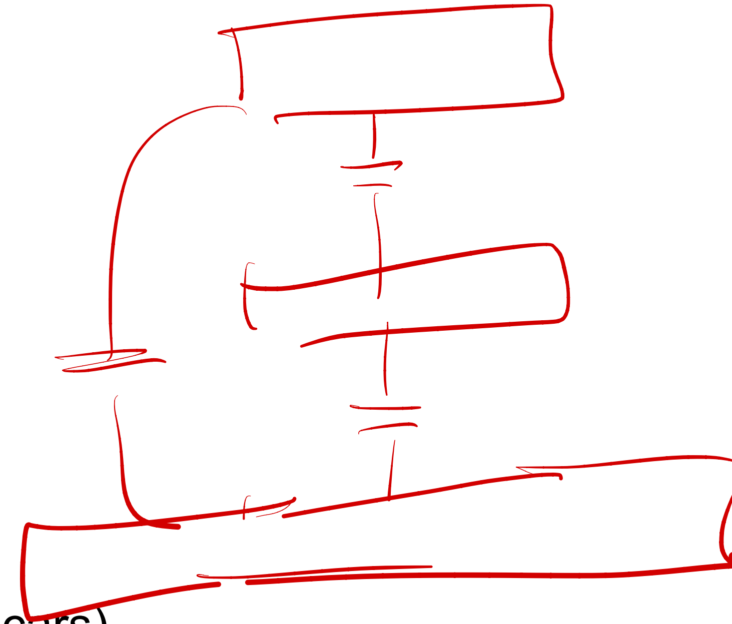
Mutual Inductance

- Mutual inductance occurs when two windings are arranged so that they have a mutual flux linkage
- The change in current in one winding causes a voltage drop to be induced in the other

$$\phi_2 = M_{21} i_1 + M_{22} i_2$$





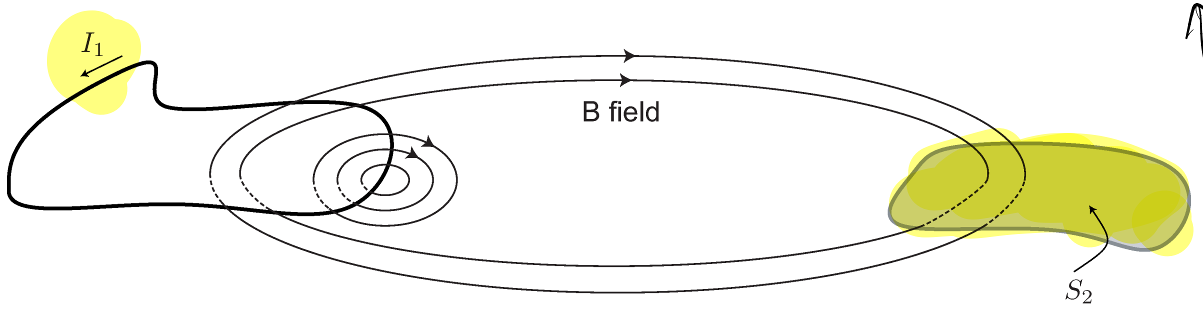


Transformers (adapters), motors, generators (electric cars)

Flux Linkage

$$V_2 = M_{21} \frac{di_1}{dt}$$

mutual inductance



- Magnetic fields vary in time and space. Circuits that "cut" into flux will experience electromagnetic induction.
- Note: These are not intentional transformers !

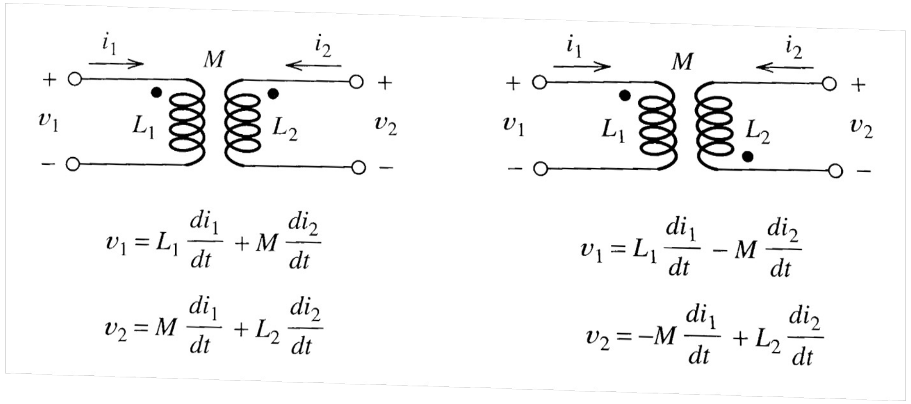
$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

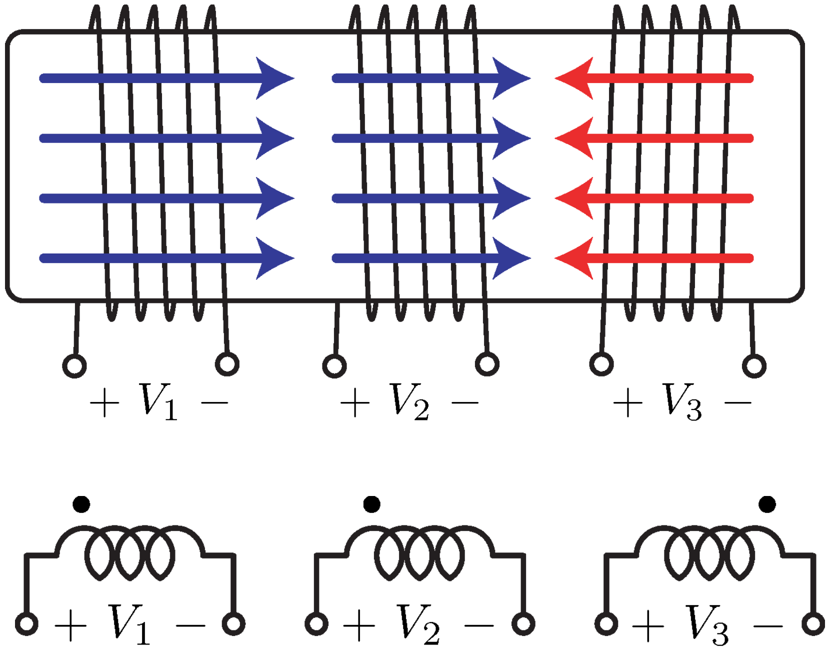
$$V_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$$M_{12} = M_{21}$$

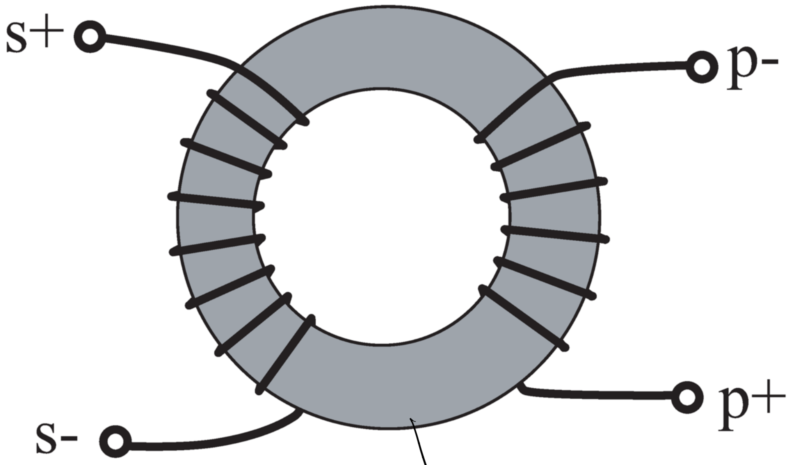
The Dot Convention

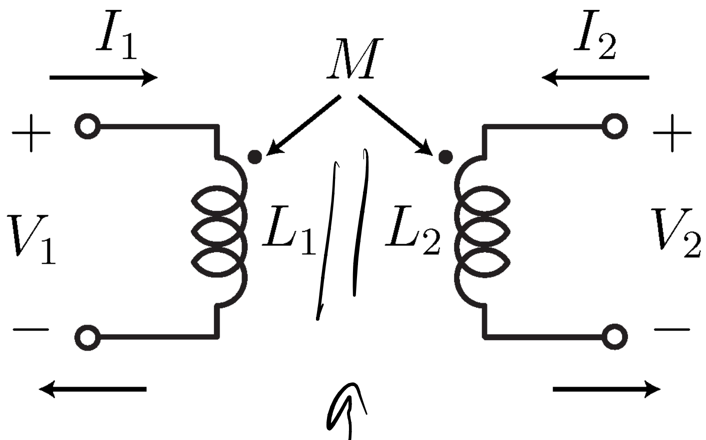
- If a current enters the dotted terminal of a coil, the reference polarity of the voltage induced in the other coil is positive at its dotted terminal.
- If a current leaves the dotted terminal of a coil, the reference polarity of the voltage induced in the other coil is negative at its dotted terminal.
- Total voltage induced in a coil is a summation of its own induced voltage and the mutually induced voltage

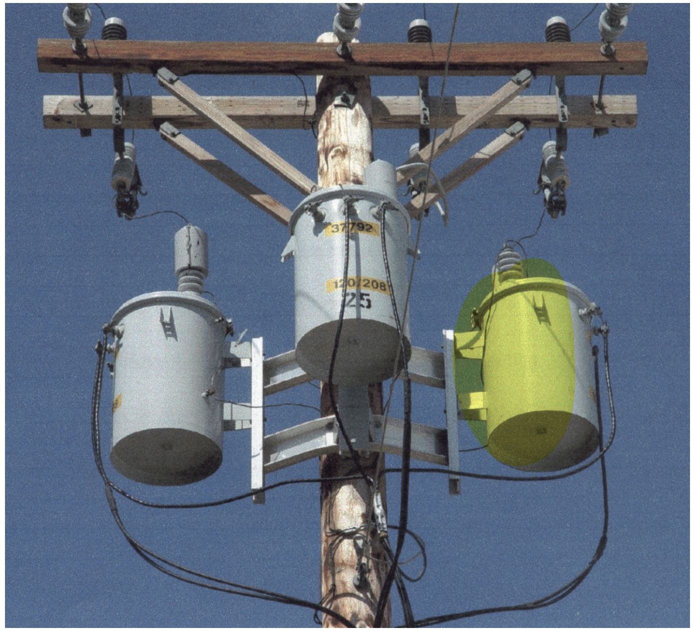




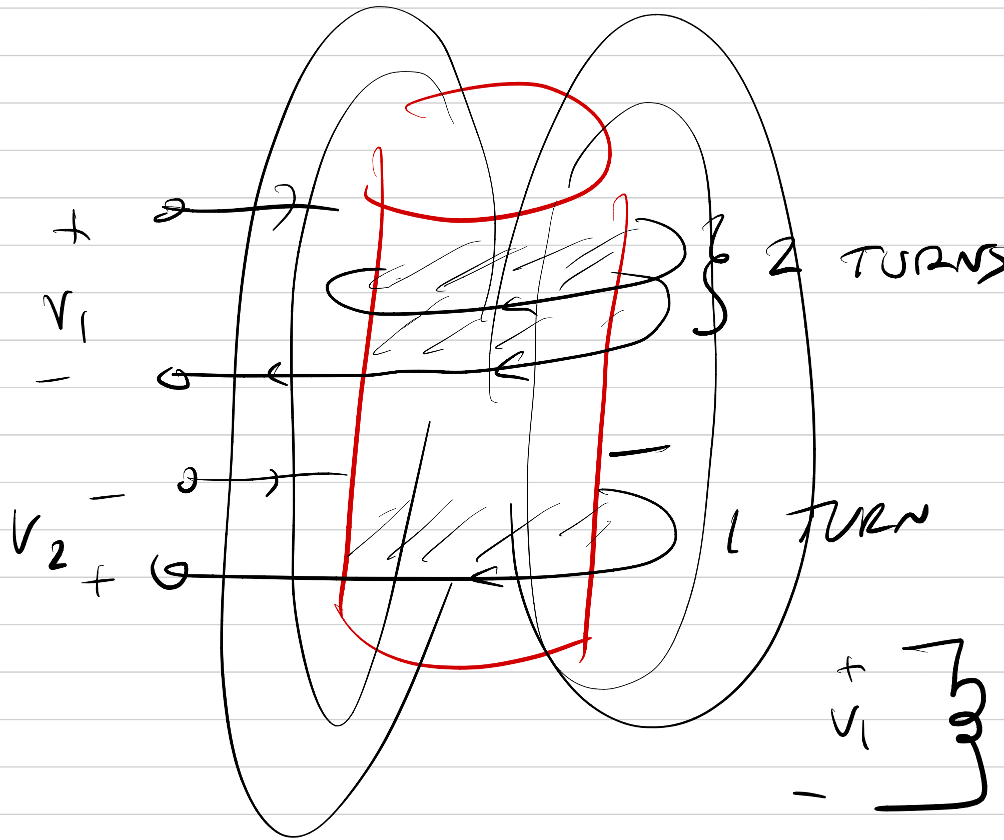
Transformers

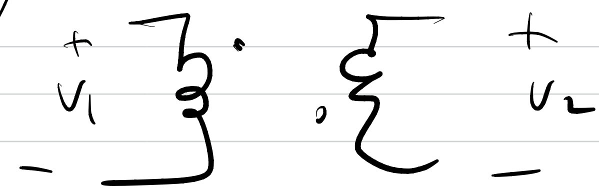


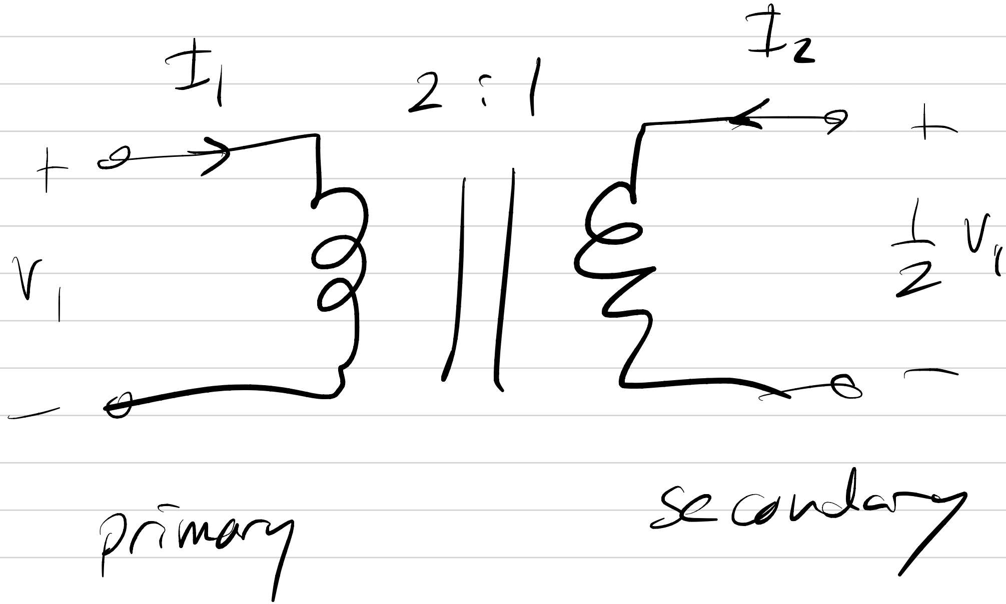




- A common magnetic core is used to boost inductance
- By varying the turns ratio, we can boost the voltage or current


+
V_1
−
2 TURNS
−
V_2
+
1 TURN

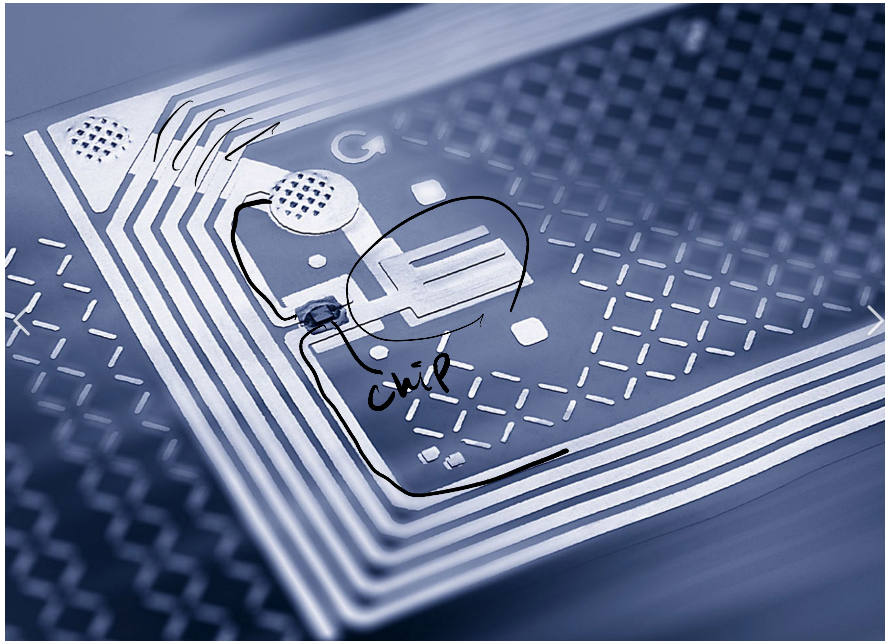

+
V_1
−
+
V_2
−

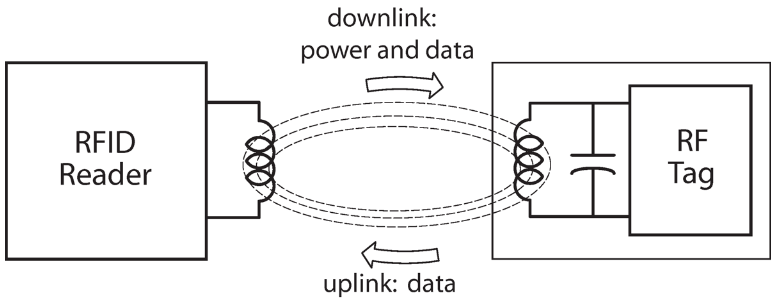


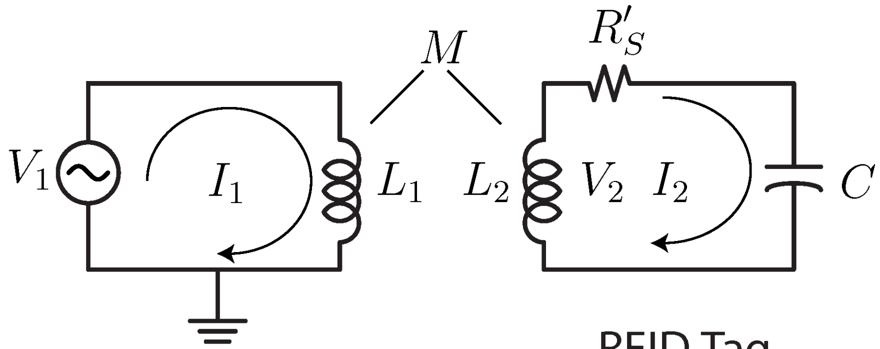
$$V_1 = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt}$$

$$V_2 = M \frac{dI_1}{dt} + L_2 \frac{dI_2}{dt}$$

RFID : Transformer at a Distance !



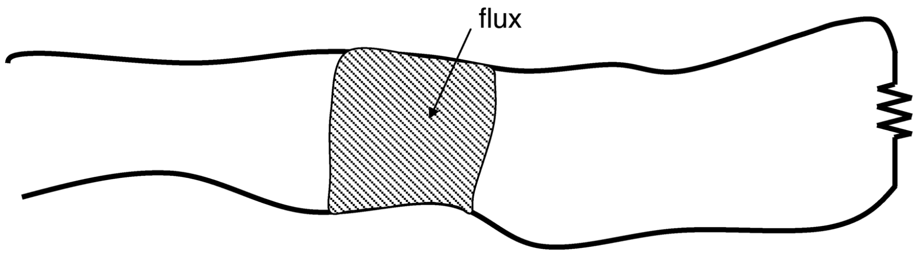


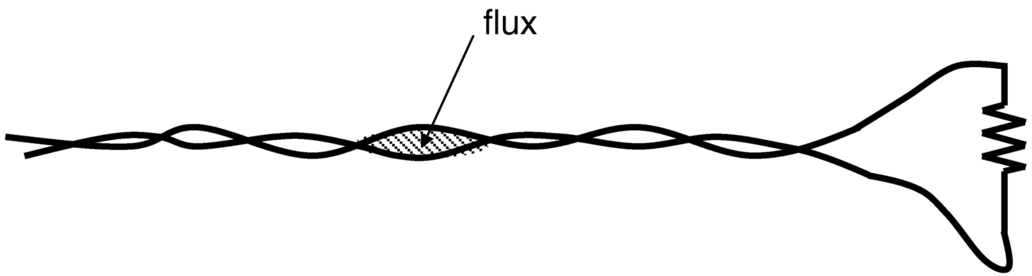


RFID Tag

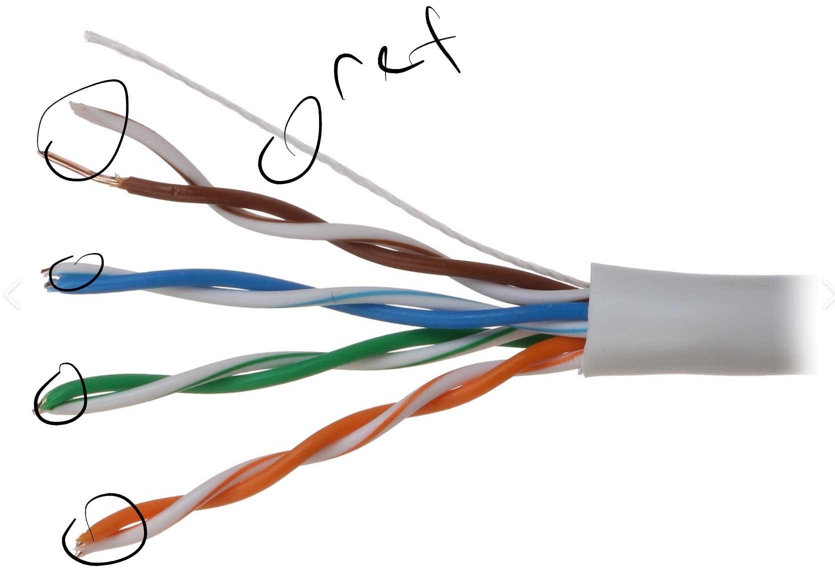
- Card keys, contactless payment, inductive charging

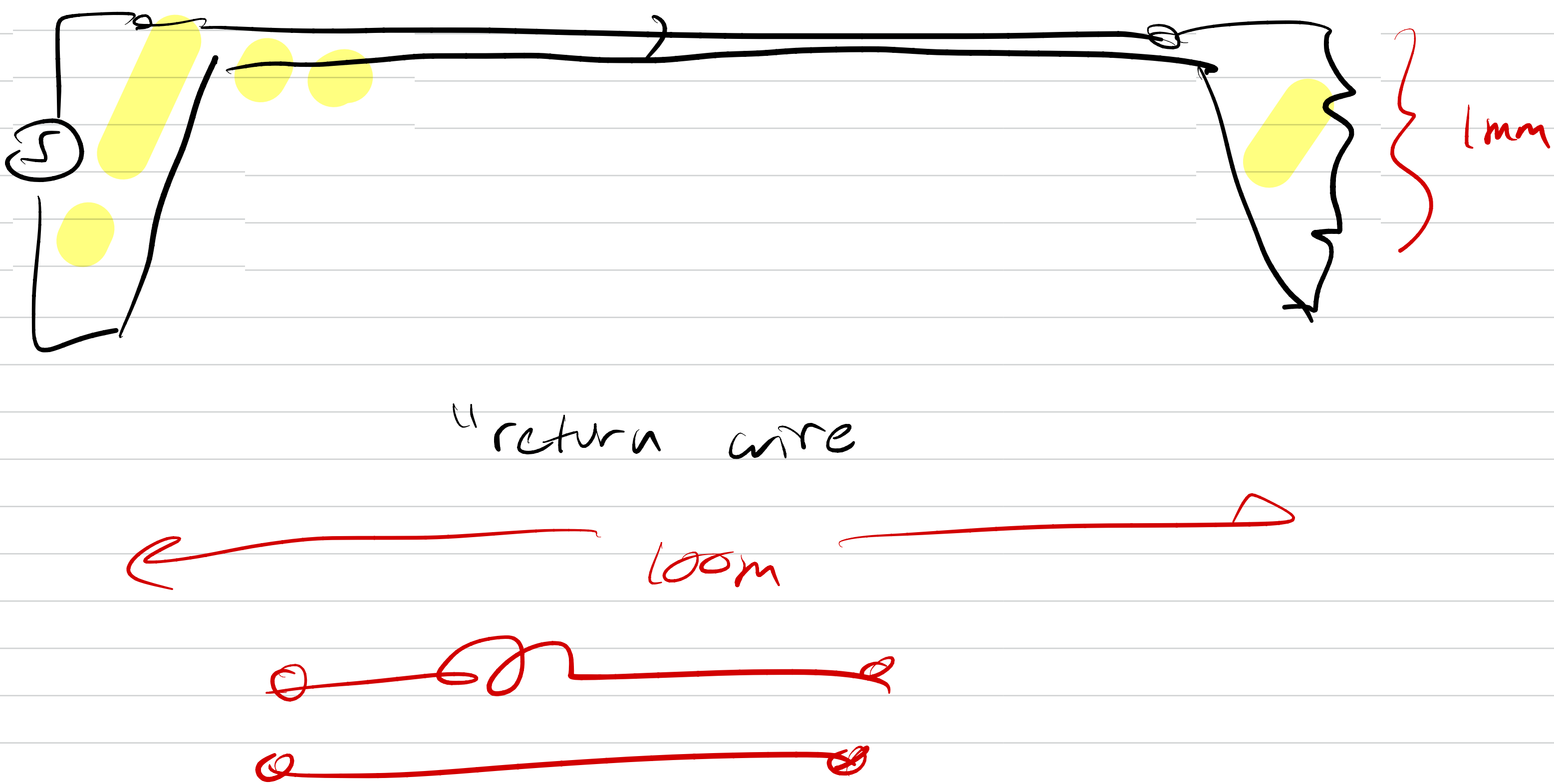
Why We Twist Wires !



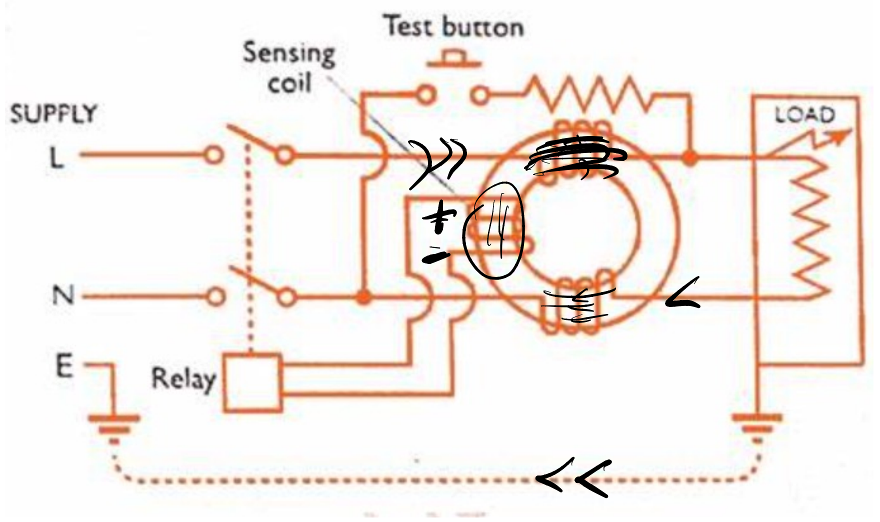


- Twisted pair has a spatially varying magnetic flux that cancels (it flips orientation). Many such twisted pairs can be bundled together and used to send signals over long distances. This minimizes interference.




1mm
"return wire
100m

GFI Circuits (Berkeley Invention !)



- If a device is operating under normal conditions, the + and – currents are balanced and the GFI coil does not detect a signal. If these currents are imbalanced, it’s possible current is flowing through another object (person), so the current is interrupted.