Lecture 5 Key Concepts

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January 31, 2023

KC: Key Concepts.

1 Lecture 5, Module 3

1.1 Slide 17

We reviewed how we use NMOS and PMOS transistors to build gates. Gates are the compution building blocks of computers and digital circuits. We also need memory to store data. How do we store data?

We started out by reviewing how "DRAM" or Dynamic RAM works. Essentially, you need to understand that a DRAM memory bit is basically a capacitor. Using an "access" transistor as a switch, one connects the capacitor to a 0V or 1V to store a 0 or 1. But the access transistor in the "off" state actually leaks current, nanoamps, but it looks like a resistor to ground. The resistor is large, gigaohms or higher, but because the DRAM memory cell is so small, the RC time constant of the memory cell is very small, at best storing information for milliseconds, which means DRAM needs to be refreshed. **KC:** DRAM cells are capacitors and they charge and discharge like RC circuit.

1.2 Slide 18

This motivates us to use "SRAM" or Static RAM. An SRAM cell is basically two inverters hooked up in a loop, with the output of the second connected to the input of the first. **KC:** An SRAM cell has two stable states, it a "bi-stable" circuit. You can use a transistor to store a 0 or 1 into the SRAM cell and the state is maintained. Unlike the DRAM capacitor, the information stored in an SRAM cell is actively refreshed by the inverters.

1.3 Slide 19

Ring Oscillators. If we put an even number of inverters into a loop, we see the circuit is self-consistent[1]. But what if you connect three, or any odd number of stages? **KC:** With an odd number of stages in the ring, the circuit is no longer stable, rather it oscillates. If you

[1] For SRAM we use two inverters which is the smallest size since we want to make the memory cell as small as possible.

assume or pre-charge the input to 0, then the odd number of inverters will invert the signal and feed it back to the input. So the input flips state and cannot remain stable at 0. The same thing happens when the input is a 1. The delay through the inverter chain is due to the RC time constant of the switches (NMOS/PMOS gates) driving the input capacitance of the inverter (NMOS and PMOS gate capacitance). The oscillation period is determined by the RC delay of the inverters driving each other. Note that we analyze inverters chains and ring oscillators in the discussion and in the notes.

1.4 Slide 21

KC: The I-V curve of a transistor can be split into two regions. Note that V is the drain-to-source voltage and I is the current flowing through the transistor from drain-to-source.

The first region is when the transistor behaves as a switch. Here we show an NMOS transistor. Why is there more than one curve? Because the gate voltage (not V but V_{GS}) determines the conductivity of the channel, so the higher the gate voltage, the larger the slope. But eventually, if we continue increasing the drain-to-source voltage on the NMOS transistor, we find that the current saturates. We didn't explain the physics, but given that it saturates, then we can think of the transistor as a current source (flat line on I-V plane). But it's not an independent current source, but a dependent one. That's because the current level is determined by the gate voltage V_{GS}, so it's voltage-controlled current source.

1.5 Slide 22

KC: *:* The op-amp model we learned into 16A originates from the transistor model we just learned. An op-amp has many transistors inside, but effectively we model the amplifier like one dependent source. This is related to Thevenin/Norton, something we'll cover in EE 105. We get voltage gain from a transistor by connecting a large resistance to the drain of an NMOS. The model includes capacitance which means op-amps have an RC time constant just like all the digital circuits we've been studying.

We performed a detailed example of an amplifier current charging the output. **KC:** *: Solution by inspection.* This is super important. We discussed that in DC steady-state, all time derivatives are zero, so the current in a capacitor is zero, so we can replace it with an open circuit. Thus we can predict the DC steady-state solution by inspection of the circuit, simply solving for the output voltage/current by replacing all capacitors with open circuits. Then since we know the general solution of a first-order differential equation driven by constant inputs is given by

$$y(t) = K_1 + K_2 e^{-t/\tau}$$

we can actually quickly calculate K_1 and K_2 by the initial value $y(0)$ and final value $y(\infty)$. With experience, you'll do this effortlessly. Also, τ is the RC time-constant of the circuit. For a single capacitor circuit (or parallel/series combined circuit with one capacitor), R is the Thevenin equivalent resistance seen by the capacitor.

1.6 Slide 23

Slides 23 onward are covered by discussion section, homework, and the lab.

2 Lecture 5, Module 4

This module is an introduction to inductors, inductance, mutual inductance, and transformers.

2.1 Slide 5

Magnetic flux ϕ is analogous to charge stored in a capacitor. **KC:** An inductor is essentially defined by the equation $\phi = LI$ where I is the current in an inductor L and ϕ is the calculated or measured magnetic flux in the circuit. Faraday's Law tells us time varying magnetic flux changes induces a voltage in the circuit. Consequently, the voltage at the terminals of an inductor is given by

$$V = \frac{d\phi}{dt} = \frac{d(LI)}{dt} = l\frac{dI}{dt}$$

2.2 Slide 6

We give a mechanical analog for an inductor as the analogous angular momentum of a waterwheel. The waterwheel can sustain current flow even when the pressure is negative, meaning it can both store and deliver power. **KC:** Inductors store energy and can deliver the energy back to the circuit.

2.3 Slide 8-9

Inductors are everywhere ! We use them in radios (think of your cell phone or WiFi chip), in DC-DC converters, and countless other system.

2.4 Slide 10

KC: The I-V relationship in an inductor.

2.5 Slide 11

KC: The energy stored in an inductor is given by

$$E = \frac{1}{2}LI^2$$

compare this equation with a capacitor.

KC: Also, if we look at the instantaneous power in an inductor, $p(t)$ can be positive or negative depending on the sign of dI/dt, meaning that if we increase the current in the inductor, we are storing energy. If we try to decrease the current, the inductor delivers energy.

2.6 Slide 12

Learn how to simplify series/parallel inductors.

2.7 Slide 13

IC inductors are "flat" and usually spirals or loops.

2.8 Slide 14

KC: Inductance is a property of circuits. Even if your circuit doesn't have a physical inductor connected, there's an inductance associated with the shape of the circuit.

2.9 Slide 15-19

Summary.

2.10 Slide 20

KC: The differential equation of RL circuits is the same as RC circuits, so use the same procedure to solve RL circuits.

2.11 Slide 21

KC: This is a super important slide. In DC steady-state, inductors are short circuits, so we can solve problems by inspection by replacing inductors with short circuits. This allows us to predict the DC steady-state and to find the constants K_1 and K_2 just as before:

$$y(t) = K_1 + K_2 e^{-t/\tau}$$

KC: Like RC time constants, RL circuits have a time constant $\tau = L/R$. How to remember this given that the units of inductance are fairly abstract ? Let's look at the IV relationship and the units of L, denoted $[L]$:

$$[L] = [\frac{V}{\frac{dI}{dt}}] = [V/dI][dt] = \Omega \times \mathrm{s}$$

This means the time constant is L/R.

2.12 Slide 23

KC: Mutual inductance is how we describe magnetic flux from one loop leaking and appearing in another loop. Magnetic flux is mangetic flux, whether it's due to the self-current or another remote current. So we can generalize and say:

$$V = \frac{d\phi}{dt} = \frac{d}{dt}\left(\phi_{11} + \phi_{12} + \cdots\right)$$

where ϕ_{11} is the magnetic flux in circuit 1 due to current in circuit 1, and ϕ_{12} is magnetic flux in circuit 1 due to current in circuit 2, and so on. Using the definition of inductance and introducing the concept of mutual inductance, we write this as follows:

$$\phi_{1,k} = M_{1,k} I_k$$

where $M_{1,1} = L_1$. Putting this all together, we have

$$V = \frac{d\phi}{dt} = L_1 \frac{dI_1}{dt} + M_{1,2} \frac{dI_2}{dt} + \cdots$$