EECS 16B

Designing Information Devices and Systems II

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Module 6: Vector Differential Equations and State-Space Representation

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Summary

- Systematic method to setup VDE
- How to solve vector differential equation (VDE)
- Solution of VDE in eigenspace with sources
- Steady-state solution of VDE
- Example

Example

- Suppose we have the following circuit.

- We can by “inspection” (KCL/KVL) setup a VDE. The key point is the write all voltages and currents only in terms of the “state” variables
- The state variables are the voltages of the capacitors and the currents of the inductors.

Systematic Way of Setting Up VDE

- There are efficient systematic ways of setting up the VDE but that's beyond what we've learned in 16AB (see **Basic Circuit Theory by** Desoer and Kuh)
- We'll use repeated application of Thevenin's theorem

Replace State Variables

- For each capacitor C or inductor L,
  - Replace it with a source that has the same voltage / current as the state variable
  - Inductors become independent current sources
  - Capacitors become independent voltage sources
- Note that everything about the circuit is known for time *t*
- How does the system evolve?

Evolution of a Particular State

- For each state variable L/C
  - Look into the circuit from the terminals of the L/C using the modified schematic
  - Find the Thevenin equivalent voltage and Thevenin equivalent resistance seen by this terminal (or Norton)
  - Note these sources are going to be a function of only the actual sources and the state variables
  - Write a simple first-order differential equation for the state transition
- You now have the VDE

Example Circuit

Complete VDE

- Key points: For the matrix *A*, each row is a linear combination of the sources … the sources are the true external stimuli and the other state variables

Solve VDE Using Diagonalization

VDE Solution (cont)

Eigenvalues of *A*

- Recall how we find the eigenvalues. Essentially, we solve an algebraic equation and find n roots for a system with n state variables
- Each root is a possible “mode” of the overall system
- These eigenvalues only depend on the values of *LCR* in the circuit and not on the sources. They are a “natural” part of the circuit

Solution for Eigenvalue *k*

- Consider the homogeneous solution. Each equation is simply a first-order system that we've met before (RC circuit)
- The time constant is NOT related to "RC"

Stable Solution

- The only difference is that the eigenvalue is actually in general a complex quantity since it's the eigenvalues of the matrix A
- For the system to be stable, we need to ensure that all the eigenvalues have a negative real part !

What are the Initial Conditions?

- Note that the initial conditions are a linear combination of the circuit initial conditions (state variables)

Forced Response

- Since we've reduced the system to simple constant coefficient linear differential equations, we know the forced response !
- Note that the sources are also transformed into the eigenspace through the matrix Q^{-1}

Steady-State DC Response

Forced Response in Steady-State

- What happens to all the natural modes of the circuit?

Detailed Example

Steady-State Sinusoidal Response

AC Analysis

- Can we get here directly without setting up all the VDE and solving it ?