EECS 16B

Designing Information Devices and Systems II

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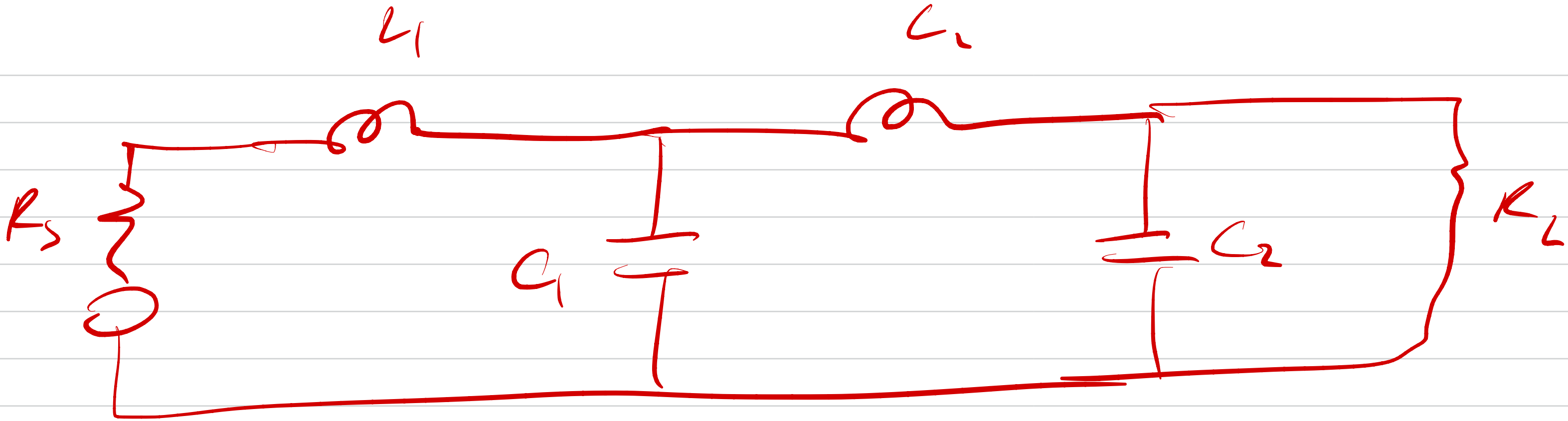
Module 6: Vector Differential Equations and State-Space Representation

EECS 16B

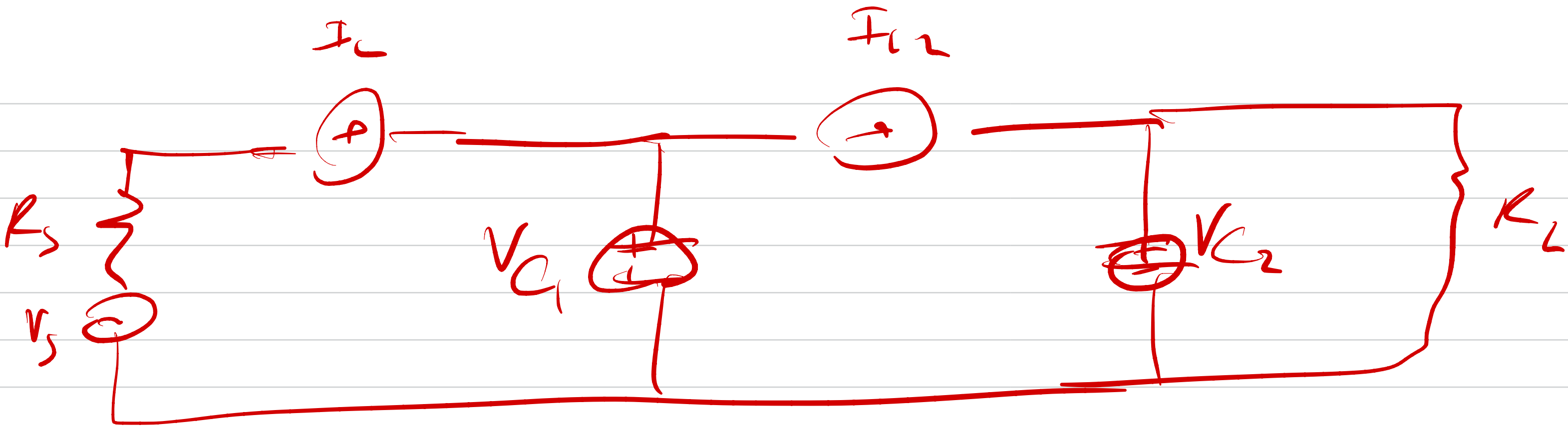
Summary

- Systematic method to setup VDE
- How to solve vector differential equation (VDE)
- Solution of VDE in eigenspace with sources
- Steady-state solution of VDE
- Example



$$\vec{x} = \begin{pmatrix} V_{C1} \\ V_{C2} \\ I_{L1} \\ I_{L2} \end{pmatrix}$$

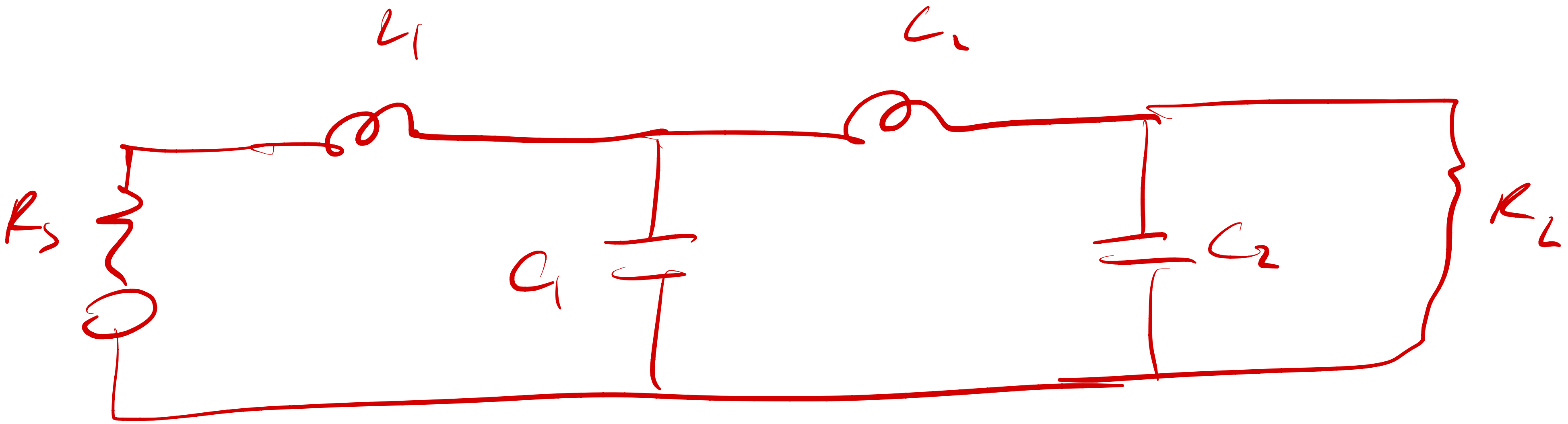
$\vec{x}$ is the state of the system



$$\dot{\vec{x}} = A\vec{x} + \vec{b}_s$$

Example

- Suppose we have the following circuit.



- We can by “inspection” (KCL/KVL) setup a VDE. The key point is the write all voltages and currents only in terms of the “state” variables
- The state variables are the voltages of the capacitors and the currents of the inductors.

Systematic Way of Setting Up VDE

- There are efficient systematic ways of setting up the VDE but that's beyond what we've learned in 16AB (see **Basic Circuit Theory by** Desoer and Kuh) + Chua
- We'll use repeated application of Thevenin's theorem

OR Norton

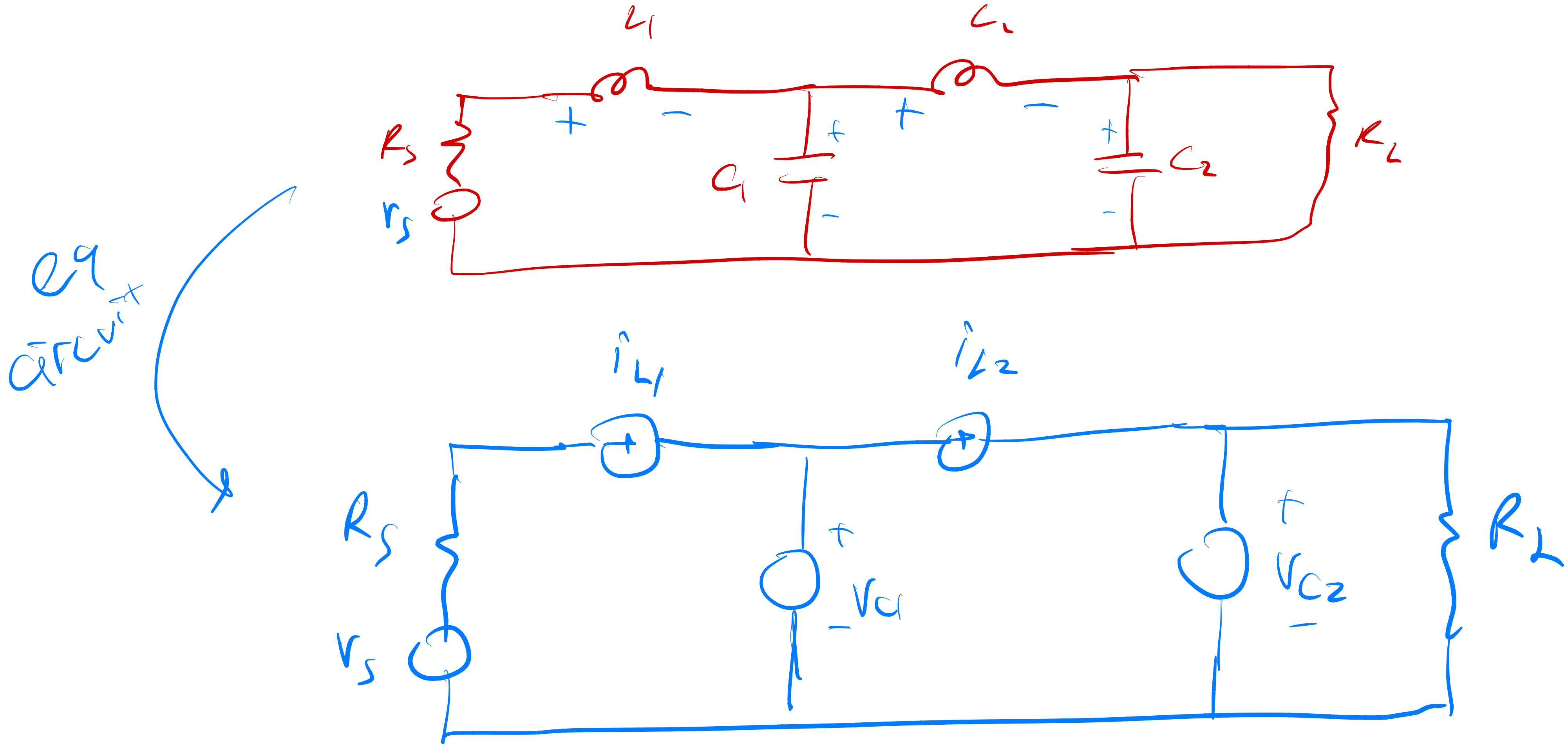
Replace State Variables

- For each capacitor C or inductor L,
  - Replace it with a source that has the same voltage / current as the state variable
  - Inductors become independent current sources
  - Capacitors become independent voltage sources
- Note that everything about the circuit is known for time t
- How does the system evolve?

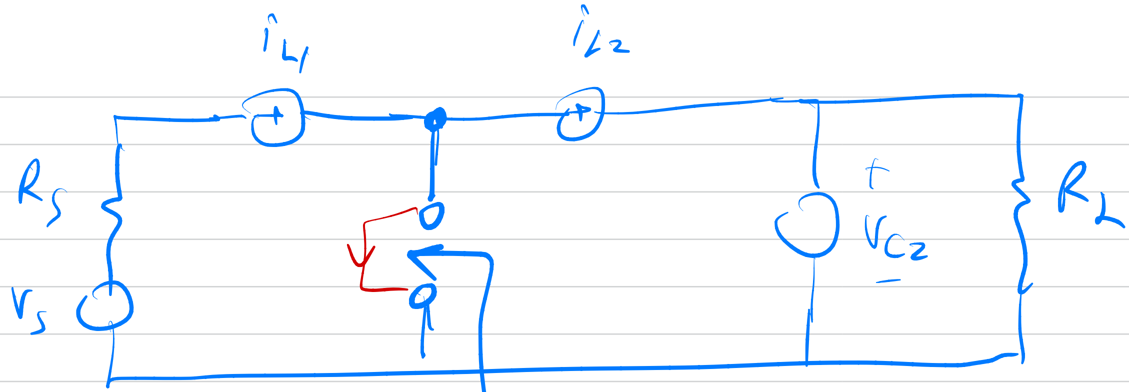
Evolution of a Particular State

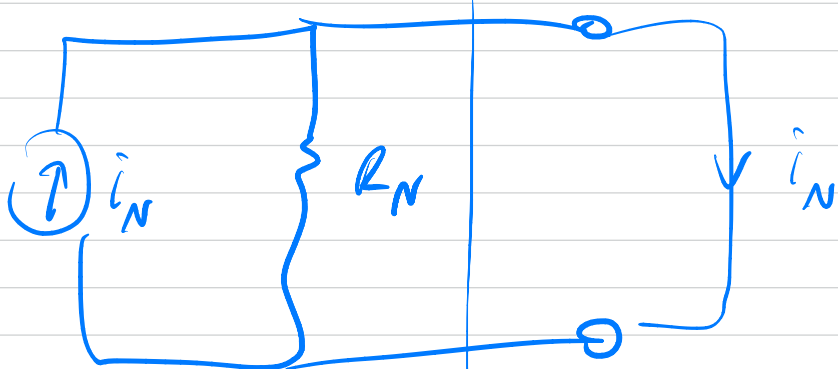
- For each state variable L/C
  - Look into the circuit from the terminals of the L/C using the modified schematic
  - Find the Thevenin equivalent voltage and Thevenin equivalent resistance seen by this terminal (or Norton)
  - Note these sources are going to be a function of only the actual sources and the state variables
  - Write a simple first-order differential equation for the state transition
- You now have the VDE

Example Circuit



C_1

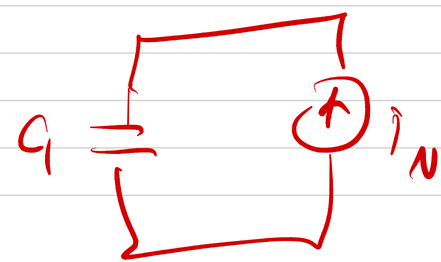




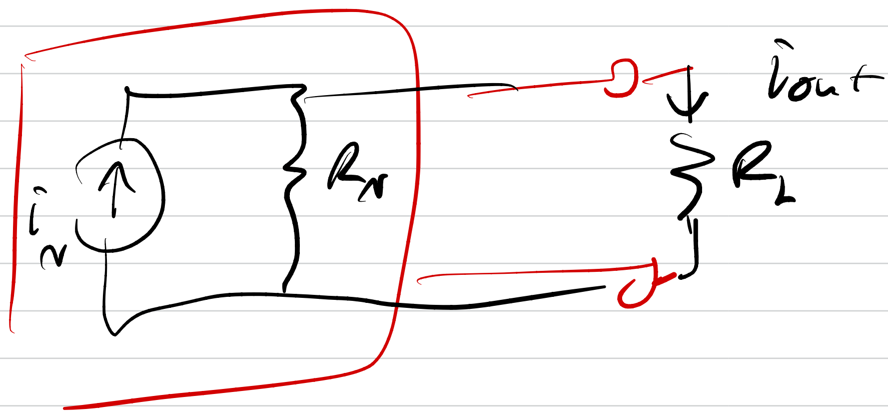
$$i_N = i_{L1} - i_{L2}$$

$$R_N = \infty\,\Omega$$

$$\equiv$$



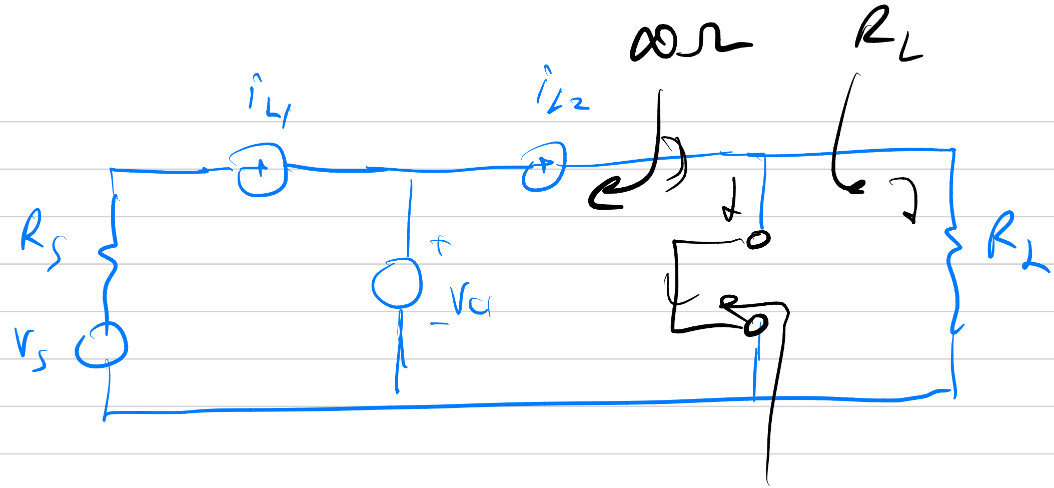
$$\boxed{C_1 \frac{dv_{C1}}{dt} = i_{L1} - i_{L2}}$$



$$i_{out} = \frac{R_N}{R_N + R_L} \cdot i_N \quad \Big\} \text{ current divider}$$

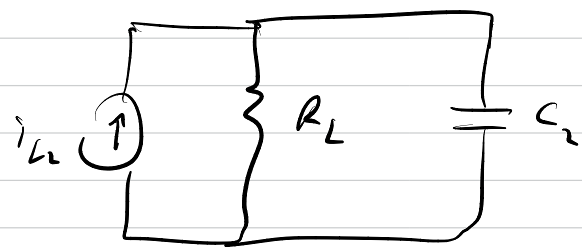
$$\approx i_N \quad \text{As } R_N \to \infty\,\Omega$$

$\underline{C_2}$



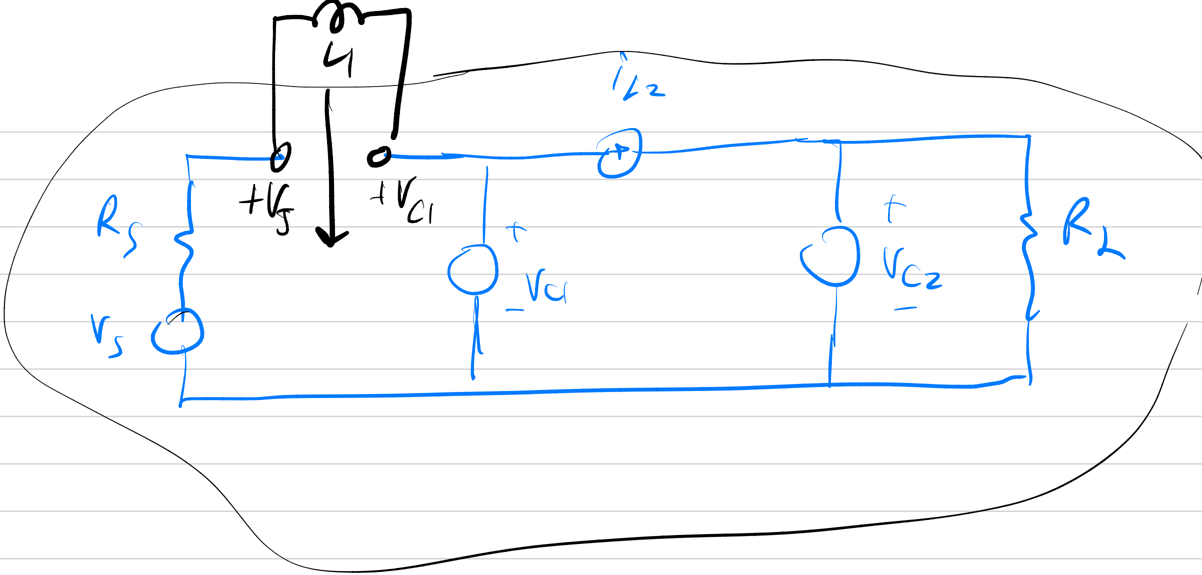
$i_N = i_{L2}$

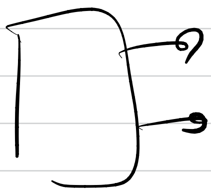
$R_N = R_L$



$$\underbrace{C_2 \frac{dv_{C2}}{dt}} = \underbrace{i_{L2} - \frac{v_{C2}}{R_L}}_{\text{state variables}}$$

L_1

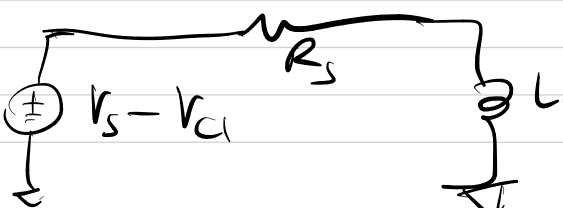


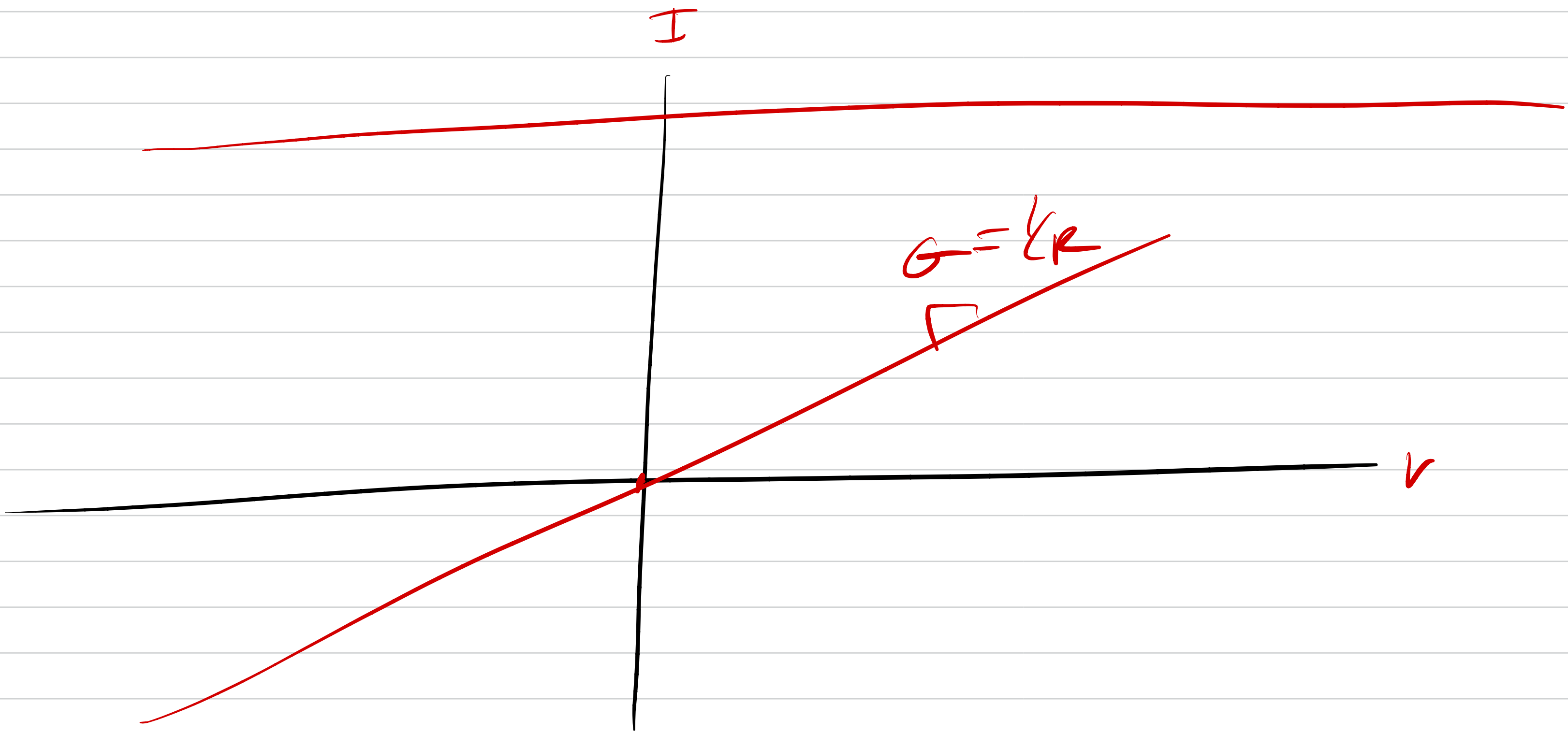


$$V_{oc} = V_{TH} = v_s - v_{C1}$$

$$R_{TH} = R_s$$

$$\boxed{L_1 \frac{di_{L1}}{dt} = v_s - v_{C1} - i_{L1} \cdot R_s}$$




I
G=1/R
V

L_2

$$L_2 \frac{di_2}{dt} = V_{C1} - V_{C2}$$

do at home

$$\frac{d\vec{x}}{dt} = \frac{d}{dt}\underbrace{\begin{pmatrix} V_{C1} \\ V_{C2} \\ i_{L1} \\ i_{L2} \end{pmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 0 & 0 & \frac{1}{C_1} & -\frac{1}{C_1} \\ 0 & -\frac{1}{R_L C_2} & 0 & \frac{1}{C_2} \\ -1 & 0 & \frac{R_s}{L_1} & 0 \\ \frac{1}{L_2} & -\frac{1}{L_2} & 0 & 0 \end{bmatrix}}_{A} \vec{x} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{L_1} \\ 0 \end{pmatrix} v_s$$

Complete VDE

- Key points: For the matrix *A*, each row is a linear combination of the sources … the sources are the true external stimuli and the other state variables

$$\frac{d\vec{x}}{dt} = A\vec{x} + \vec{b}_s$$

$$\frac{d\vec{x}}{dt} = Q\Lambda\left[\underbrace{Q^{-1}\vec{x}}_{\vec{u}}\right] + \vec{b}_s$$

$$\underbrace{Q^{-1}\dot{\vec{x}}}_{\frac{d\vec{u}}{dt}} = Q^{-1}Q\Lambda\vec{u} + Q^{-1}\vec{b}_s = \Lambda\vec{u} + \tilde{\vec{b}}_s$$

Solve VDE Using Diagonalization

$$Q^{-1}\dot{\vec{x}} = \Lambda \vec{u} + \vec{\tilde{b}}_s$$

$$Q^{-1}\dot{x} = Q^{-1}\frac{d}{dt}\vec{x} = \frac{d}{dt}(Q^{-1}\vec{x}) = \frac{d}{dt}\vec{u} = \vec{\dot{u}}$$

$$\vec{\dot{u}} = \Lambda \vec{u} + \vec{\tilde{b}}_s$$

$$\boxed{\frac{d}{dt}\vec{u} = \Lambda \vec{u} + \vec{\tilde{b}}_s}$$

VDE Solution (cont)

$$\frac{d}{dt}u_1 = \lambda_1 u_1 + \hat{b}_{s1}$$

1st order constant coefficient diff eq.

$$\frac{du_1}{dt} = \lambda_1 u_1 \qquad u_1 = A e^{\lambda_1 t}$$

$$\cancel{A}\lambda_1 \cancel{e^{\lambda_1 t}} = \lambda_1 \cancel{A} \cancel{e^{\lambda_1 t}}$$

$1 = 1$ ✓