EECS 16B

Designing Information Devices and Systems II

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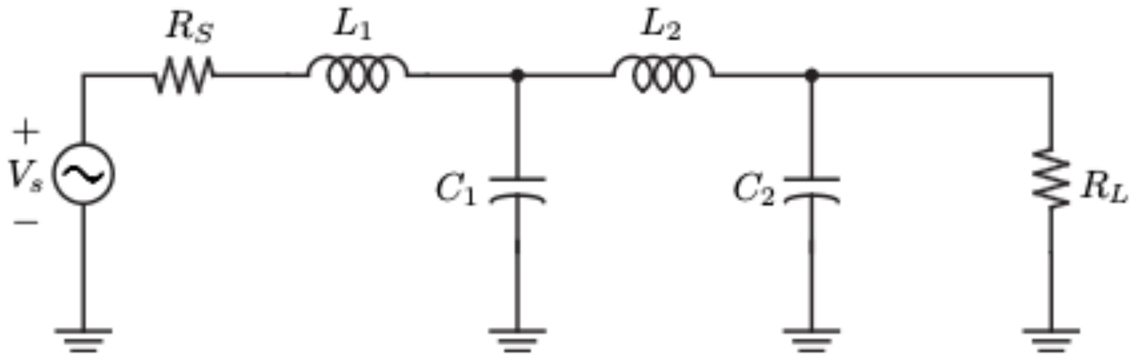
Module 6: Sinusoidal Steady-State Solution to Vector Differential Equations

EECS 16B

Summary

- Solution of VDE in eigenspace with sources
- Steady-state solution of VDE
- Concept of Impedance
- AC Circuits
- Examples

Example Circuit

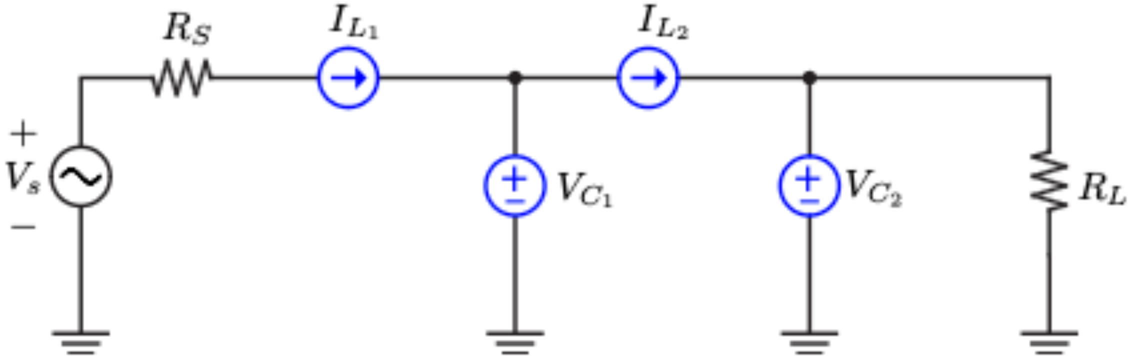


$$\vec{x} = (V_{C_1}\, V_{C_2}\, I_{L_1}\, I_{L_2})^T$$

$$\frac{d}{dt}\vec{x} = A\vec{x} + B\vec{b_s}$$

$$A = \begin{pmatrix} 0 & 0 & 1/C_1 & -1/C_1 \\ 0 & -1/R_L C_2 & 0 & 1/C_2 \\ -1 & 0 & -R_S/L_1 & 0 \\ 1/L_2 & -1/L_2 & 0 & 0 \end{pmatrix}$$

$$B = (0\;0\;1\;0)^T V_s.$$



Numerical Example

L1 = 7410^ − 9; L2 = 114.210^ − 9; C1 = 45.61

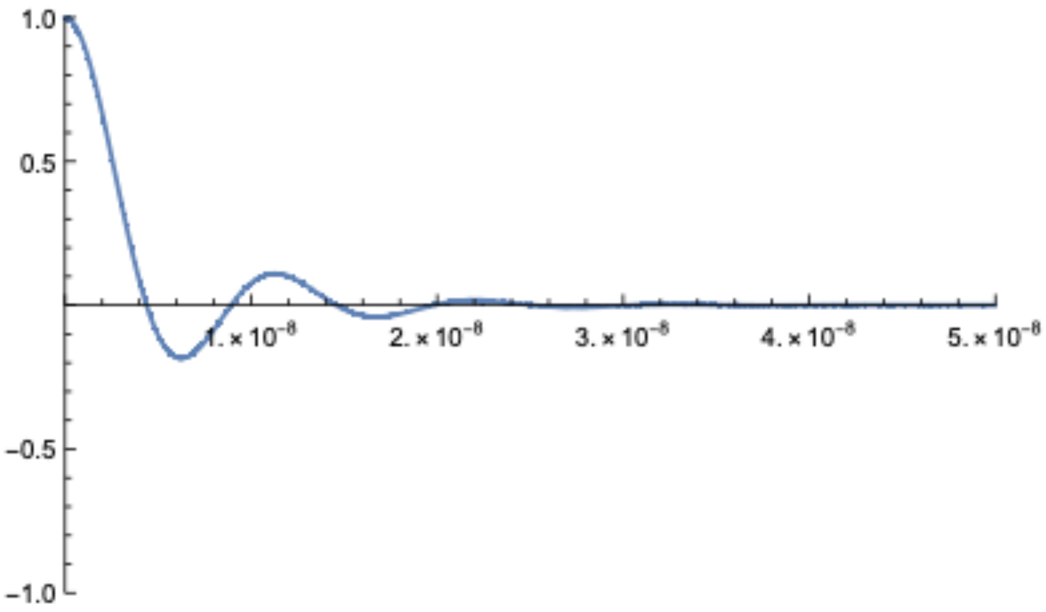
Let's see the eigenvalues for this matrix:

Eigenvalues[A]//MatrixForm

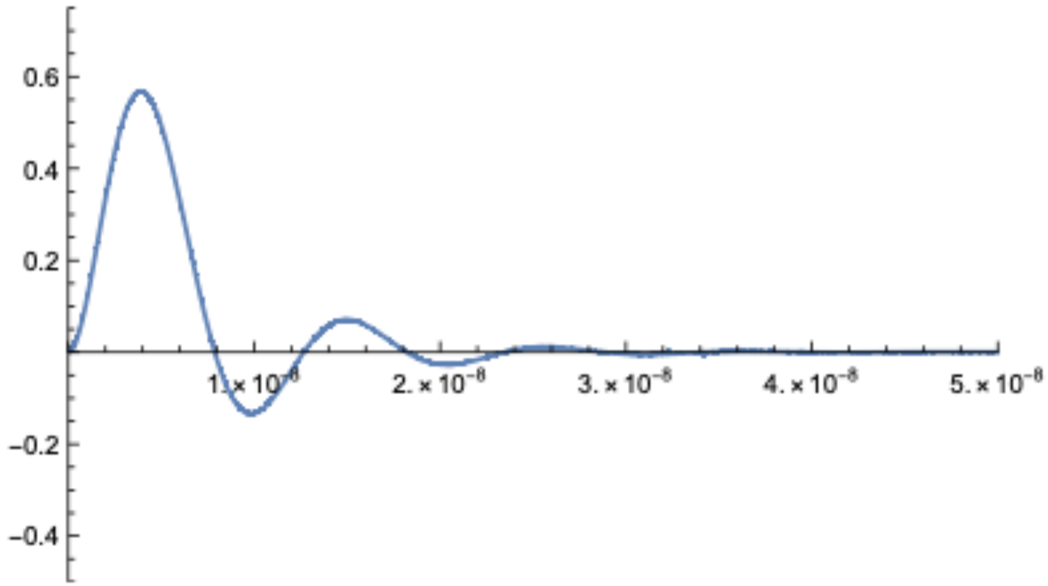
$$\begin{pmatrix} -6.75676 \times 10^8 + 0.i \\ -1.64354 \times 10^8 + 5.88916 \times 10^8 i \\ -1.64354 \times 10^8 - 5.88916 \times 10^8 i \\ -3.47197 \times 10^8 + 0.i \end{pmatrix}$$

Stability

Homogeneous Solution



State 1



State 2

Forced Response

- Since we've reduced the system to simple constant coefficient linear differential equations, we know the forced response !
- Note that the sources are also transformed into the eigenspace through the matrix Q^{-1}

Steady-State DC Response

Review: General Solution to 1st–order ODE

- With constant coefficients, we derived

Apply Solution to VDE

Elegant Notation

- Let’s define the matrix exponential
- While for now you can accept this as merely a convenient notation, it turns out to be a more general idea ! (https://youtu.be/O85OWBJ2ayo)

Forced Response in Steady-State

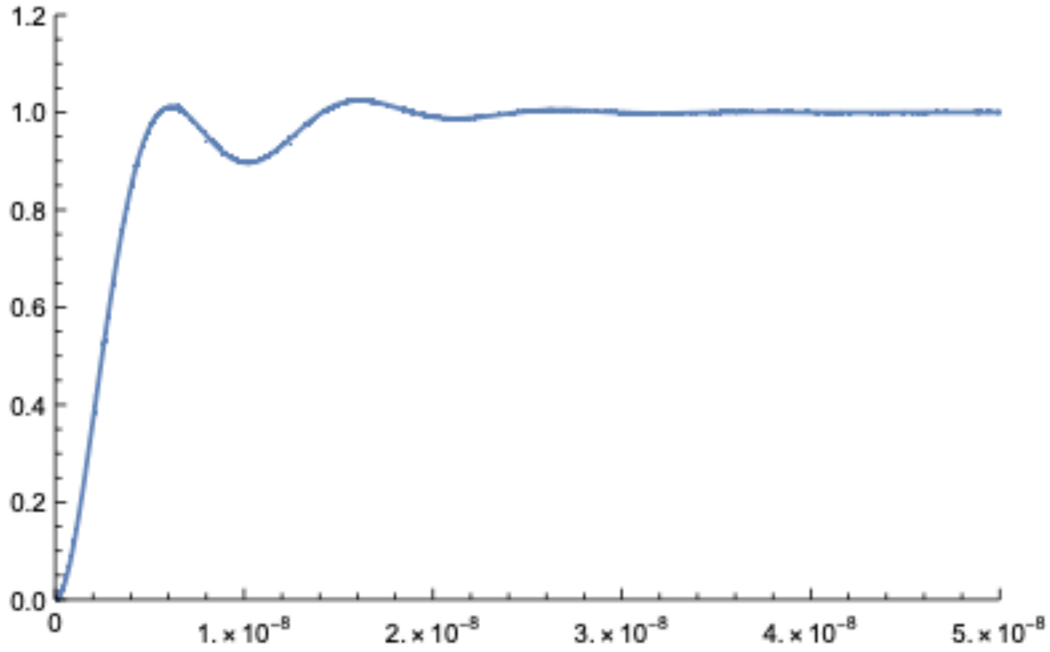
- Assume source is sinusoidal (very common in practice)
  - RF signals essentially sinusoidal on short time scales
  - Musical notes are superpositions of tones (frequencies)
  - Any general waveform can be decomposed into sinusoids ! (Fourier Series and Transform … EE120)

Complex Exponential (again)

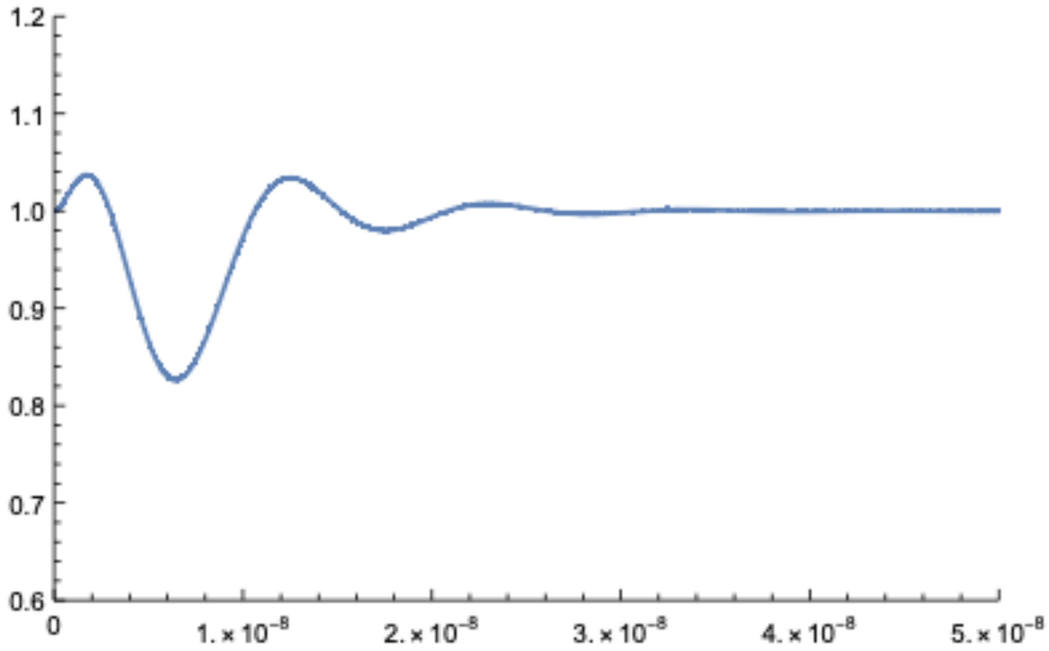
- Sine and Cosine are merely shadows of a circle
- Your new best friend !

Steady-State Continued

Example: Forced Solution



State 1



State 2

Steady-State Sinusoidal Response

- Weighted integral perspective

Summing Complex Exponentials

- Recall animation

AC Analysis

- Can we get here directly without setting up all the VDE and solving it ?

“Phasors”

- A complex number by another name

Concept of Impedance

Reactance

Capacitors in AC Circuits

Inductors in AC Circuits

Direct Solution (no VDE required)

Phasor Algebra

- Same as any complex number