# EECS 16B

# Designing Information Devices and Systems II

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# Module 9:  AC Analysis

EECS 16B

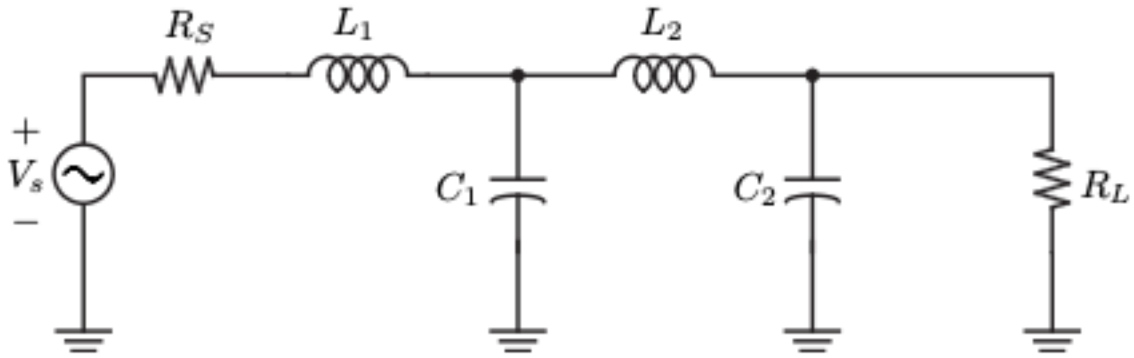
# Summary

- Steady-state solution of VDE
- Concept of Impedance
- AC Circuits
- Examples

# Review:  Forced Solution

# Forced Sinusoidal Solution

# Example Circuit

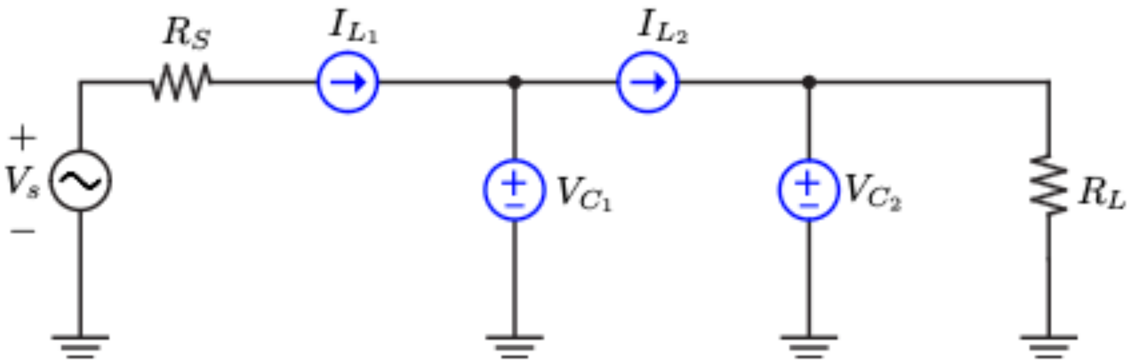



$$\vec{x} = (V_{C_1}\ V_{C_2}\ I_{L_1}\ I_{L_2})^T$$

$$\frac{d}{dt}\vec{x} = A\vec{x} + B\vec{b_s}$$

$$A = \begin{pmatrix} 0 & 0 & 1/C_1 & -1/C_1 \\ 0 & -1/R_L C_2 & 0 & 1/C_2 \\ -1/L_1 & 0 & -R_S/L_1 & 0 \\ 1/L_2 & -1/L_2 & 0 & 0 \end{pmatrix}$$

$B = (0\ 0\ 1/L_1\ 0)^T V_s.$



# Numerical Example

```
In[120]:= EV = Eigenvalues[A] ; EV // MatrixForm
```

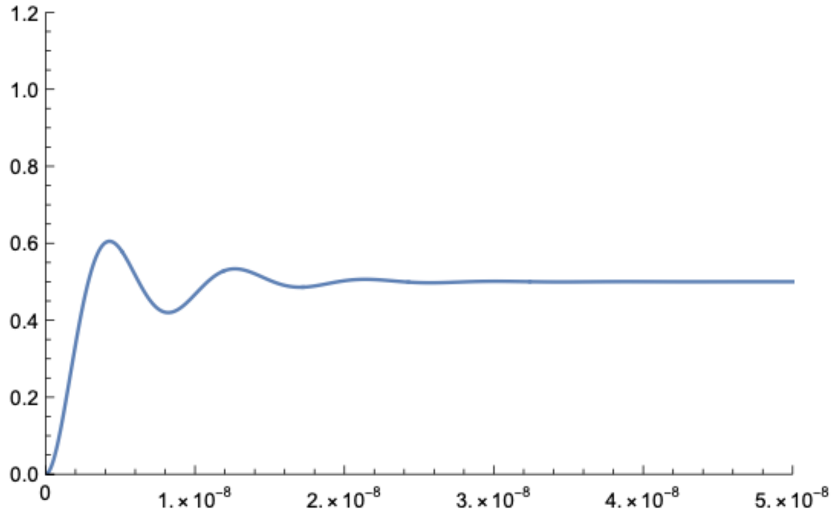
Out[120]//MatrixForm=

$$\begin{pmatrix} -1.9034\times10^{8} + 7.2085\times10^{8}\, i \\ -1.9034\times10^{8} - 7.2085\times10^{8}\, i \\ -4.8545\times10^{8} + 2.82637\times10^{8}\, i \\ -4.8545\times10^{8} - 2.82637\times10^{8}\, i \end{pmatrix}$$

Note that the eigenvalues have an imaginary part close to the cut-off frequency of the filter (100 MHz):

```
In[121]:= Im[EV] / (2 Pi 100 × 10 ^ 6) // N
Out[121]= {1.14727, -1.14727, 0.449831, -0.449831}
```

Plot[Re[xforce[t][[2]]], {t, 0, 5010^ − 9}, PlotRange->{{0, 5010^ − 9}, {0, 1.2}}]




bs = {{0}, {0}, {1/L1}, {0}};

# Forced Response

- Since we've reduced the system to simple constant coefficient linear differential equations, we know the forced response !
- Note that the sources are also transformed into the eigenspace through the matrix $Q^{-1}$

```
xforce[t_] = Q.MatrixExp[Dlam t].Qi.xi + Q.MatrixExp[Dlam t].Qi .Integrate[Q.MatrixExp[-Dlam sigma ].Qi . bs , {sigma, 0, t}];
```

# Steady-State DC Response

```
In[140]:= Limit[Q.MatrixExp[Dlam t].Qi .Integrate[Q.MatrixExp[-Dlam sigma ].Qi . bs , {sigma, 0, t}], t → Infinity]

Out[140]= {{0.5}, {0.5 + 0. i}, {0.01 + 0. i}, {0.01}}
```

The final values are also easily derived from the basic vector equation by setting all derivatives to zero:

```
In[141]:= -Inverse[A] . bs

Out[141]= {{0.5}, {0.5}, {0.01}, {0.01}}
```

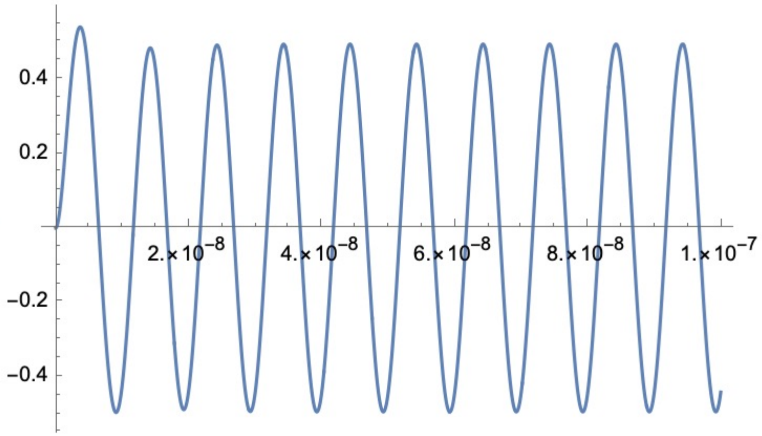
# Sinusoidal Steady-State

```
w1 = 2 Pi 100×10^6;

In[143]:= xac1[t_] = Q.MatrixExp[Dlam t].Qi.xi + Q.MatrixExp[Dlam t].Qi .Integrate[Q.MatrixExp[-Dlam sigma ].Qi . bs Cos[w1 sigma] , {sigma, 0, t}];

In[145]:= Plot[Re[xac1[t][[2]]], {t, 0, 100×10^-9}]
```

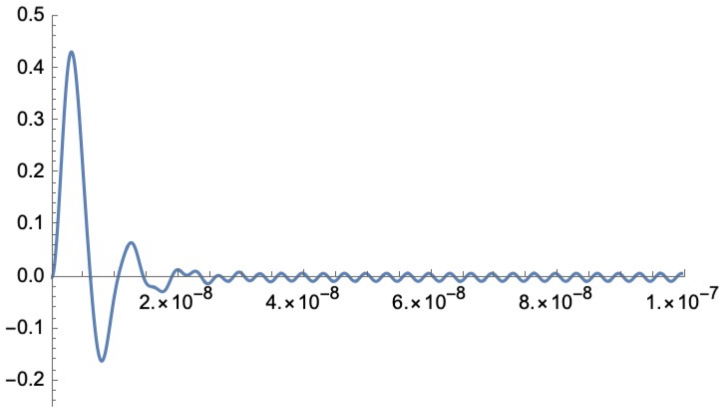
Out[145]=



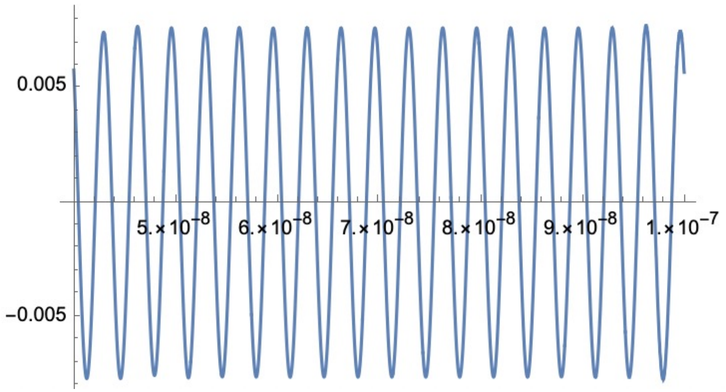
# Higher Frequency Response

```
xac2[t_] = Q.MatrixExp[Dlam t].Qi.xi + Q.MatrixExp[Dlam t].Qi .Integrate[Q.MatrixExp[-Dlam sigma ].Qi . bs Cos[3 w1 sigma] , {sigma, 0, t}];

Plot[Re[xac2[t][[2]]], {t, 0, 100×10^-9}, PlotRange → {{0, 100×10^-9}, {-.25, .5}}]
```



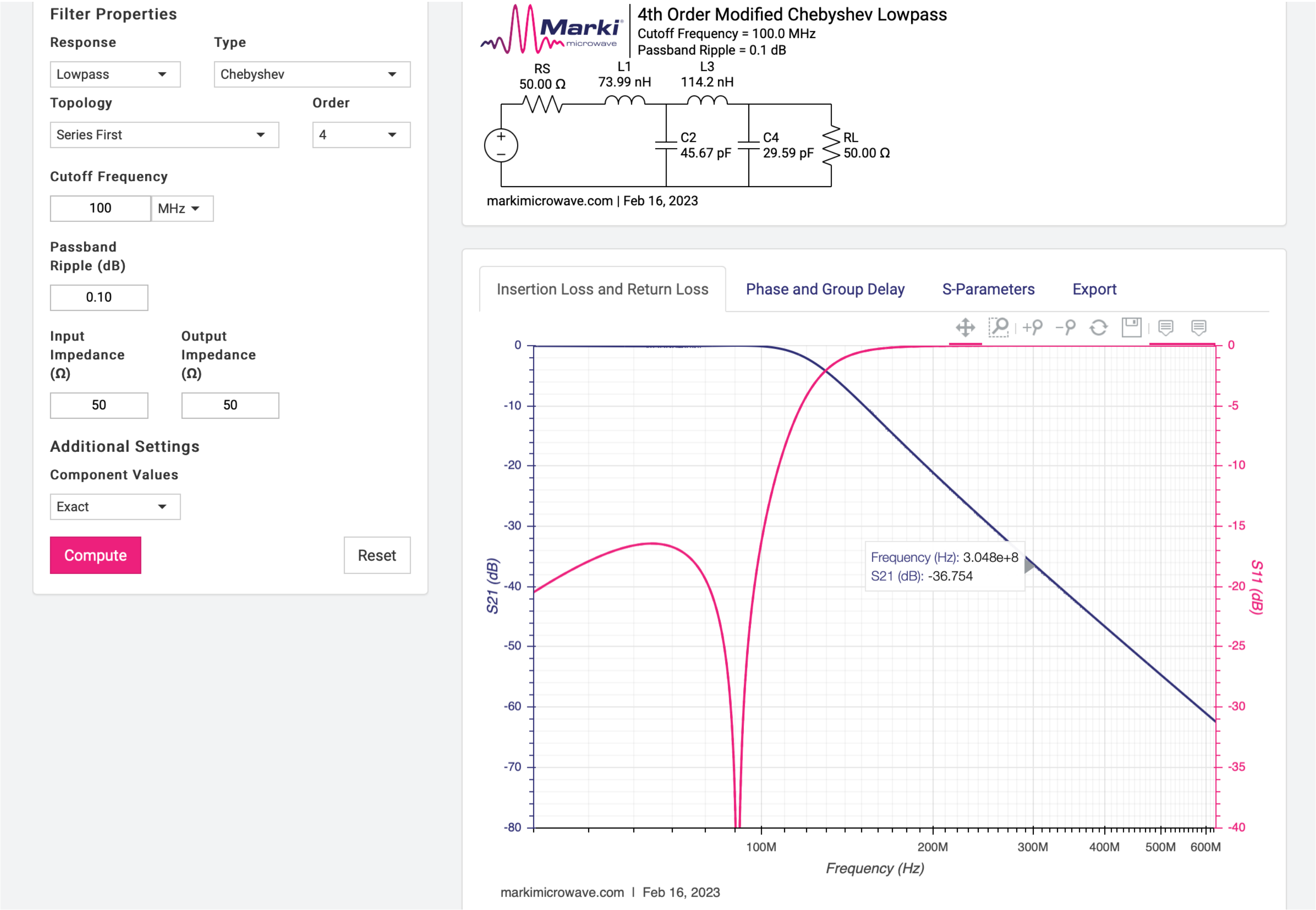

```
Plot[Re[xac2[t][[2]]], {t, 40×10^-9, 100×10^-9}]
```



# Steady-State Sinusoidal Response

- Weighted integral perspective

# Filter Design



# AC Analysis

- Can we get here directly without setting up all the VDE and solving it ?

# “Phasors”

- A complex number by another name

# Concept of Impedance

# Reactance

# Capacitors in AC Circuits

# Inductors in AC Circuits

# Direct Solution (no VDE required)

# Phasor Algebra

- Same as any complex number

# $2^{nd}$ Order AC Circuits