EECS 16B

Designing Information Devices and Systems II

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Module 10: AC Networks and AC Power

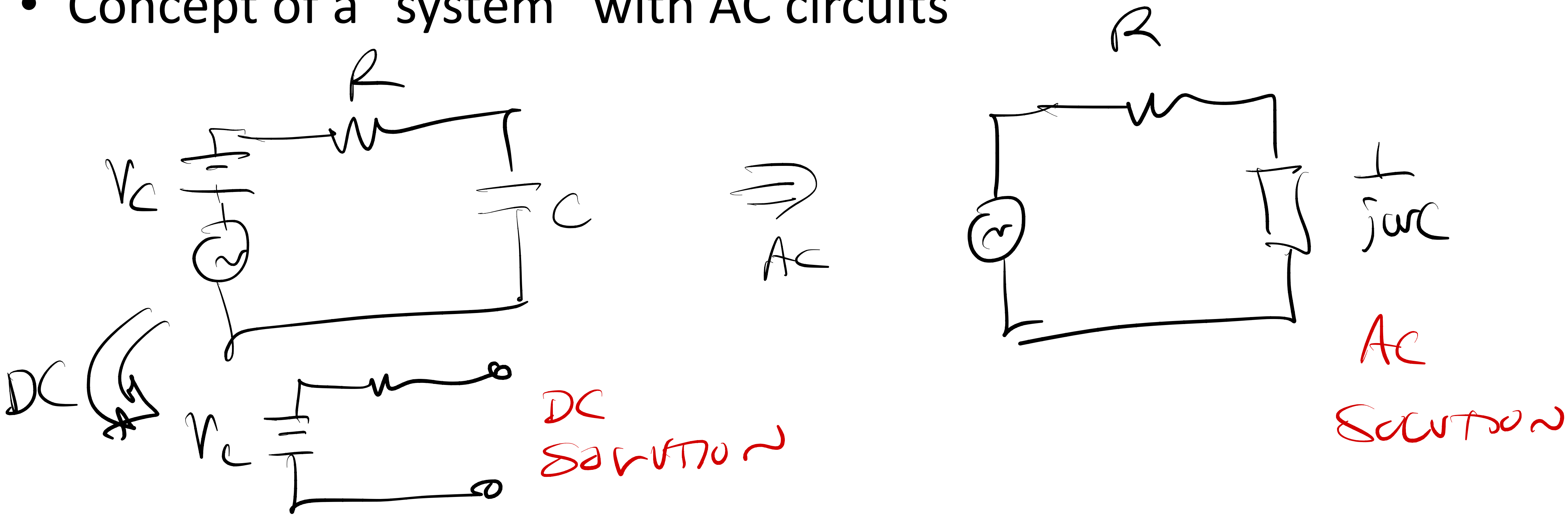
EECS 16B

Summary

- AC Circuit Network Theorems
  - Voltage / current dividers
  - Source superposition
  - Thevenin/Norton
- AC Power
  - Average Power
  - Reactive Power
- Maximum power transfer theorem

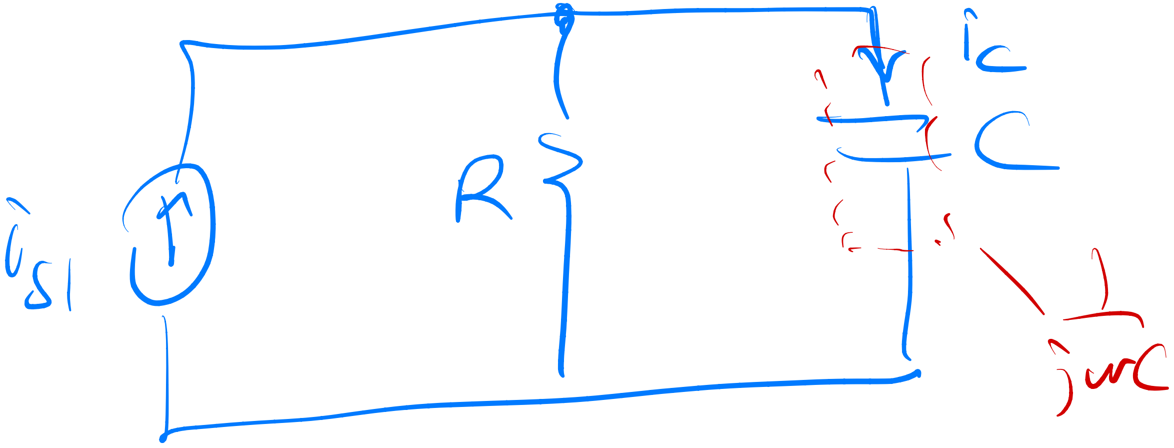
Review AC Circuits

- Concept of Impedance
- AC equivalent circuit
- Concept of a “system” with AC circuits



AC ~~Voltage~~ Divider

current



$$\hat{i}_C = \frac{Y_C}{Y_C + G}\,\hat{i}_{s1}$$

$$\boxed{G = 1/R}$$

$$\hat{i}_c = \frac{j\omega C}{j\omega C + G}\,\hat{i}_{s1} = \frac{j\omega C}{G + j\omega C}\,\hat{i}_{s1}$$

$$= \frac{j\omega RC}{1 + j\omega RC}\,\hat{i}_{s1}$$

$$H(j\omega) = \frac{\hat{i}_c}{\hat{i}_{s1}} = \frac{j\omega RC}{1 + j\omega RC}$$

Reminder

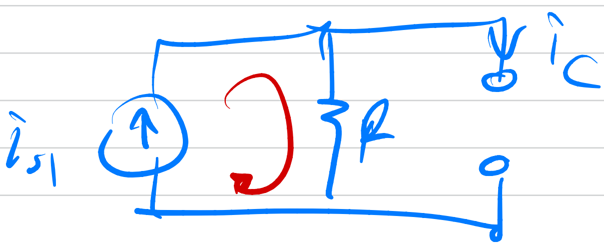
$$\frac{i_2}{i_s} = \frac{G_2}{G_1 + G_2}$$

Sanity Checks

- Check Units $\quad A/A \quad \checkmark$

- DC solution $\quad \omega = 0$

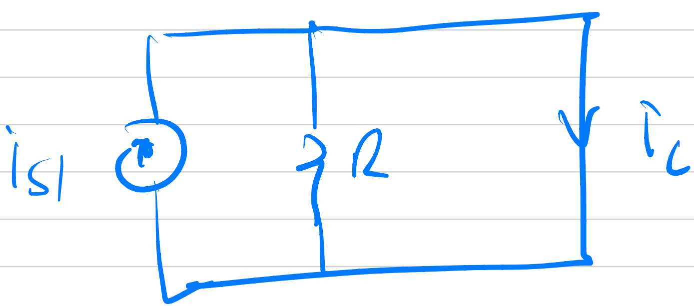
$$H(j\omega)\Big|_{\omega=0} = \frac{j\omega RC}{1 + j\omega RC}\Bigg|_{\omega=0} = 0 \quad \checkmark$$



o $\omega \to \infty$ VERY HIGH FREQ

$\frac{1}{\omega C} \to$ SHORTS

$\omega L \to$ OPENS



$$H(j\omega)\Big|_{\omega\to\infty} = 1$$ ✓

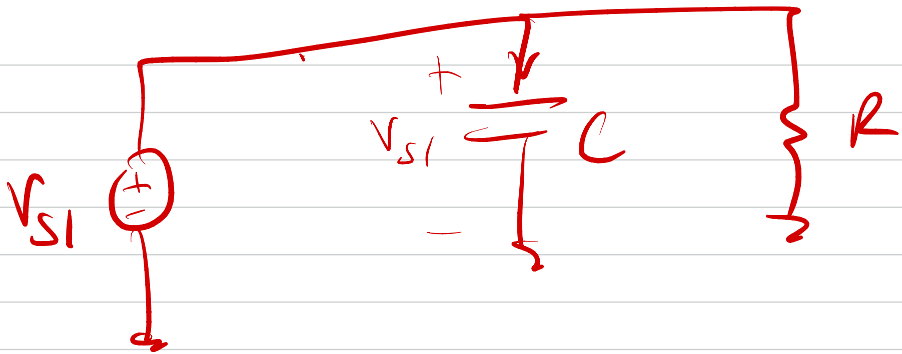
$$H(j\omega)\Big|_{\omega=\infty} = \frac{\cancel{j\omega RC}}{1+\cancel{j\omega RC}}\Bigg|_{\omega=\infty} = 1$$

AC Source Superposition

$$\cancel{H_1 = \frac{j\omega RC}{1+j\omega RC}}$$

$$\frac{i_C}{i_{s2}} = 0$$

$$H_1 = 0$$

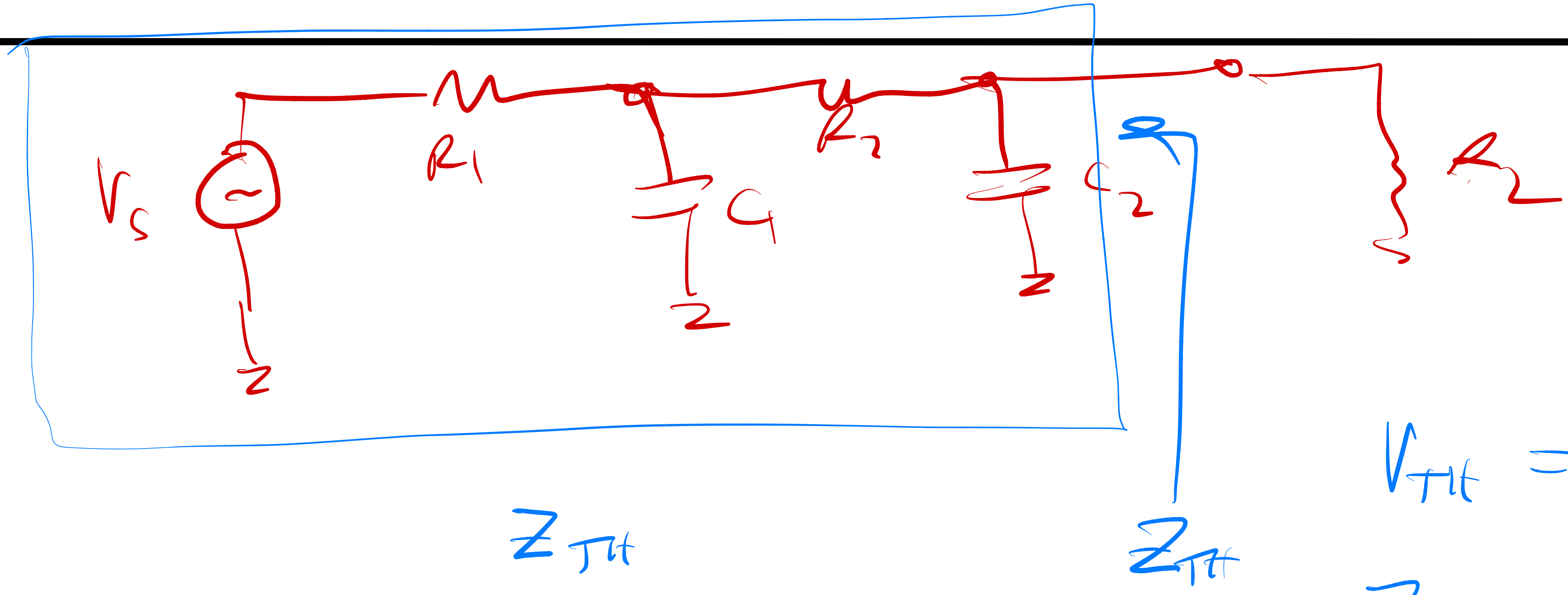


$$\hat{i}_{c1} = \frac{v_{s1}}{1/j\omega C} = j\omega C \; v_{s1}$$

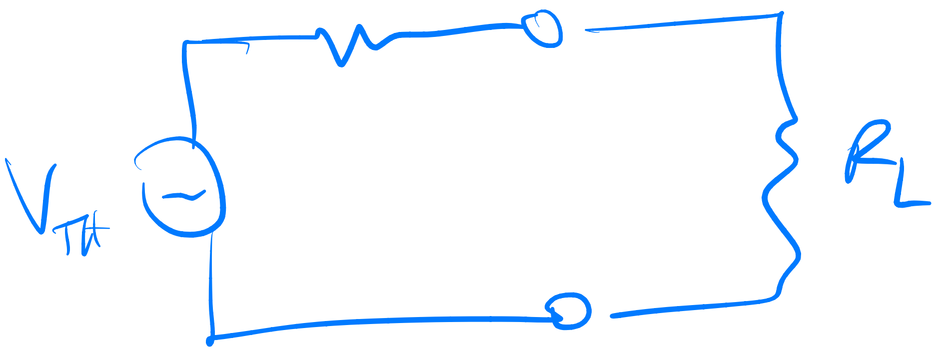
$$\frac{\hat{i}_{c1}}{v_{s1}} = j\omega C$$

$$\left\{ \text{TURN OFF} \right\} \begin{cases} v_s \rightarrow \text{SHORT} \\ i_s \rightarrow \text{OPEN} \end{cases}$$

AC Thevenin/Norton

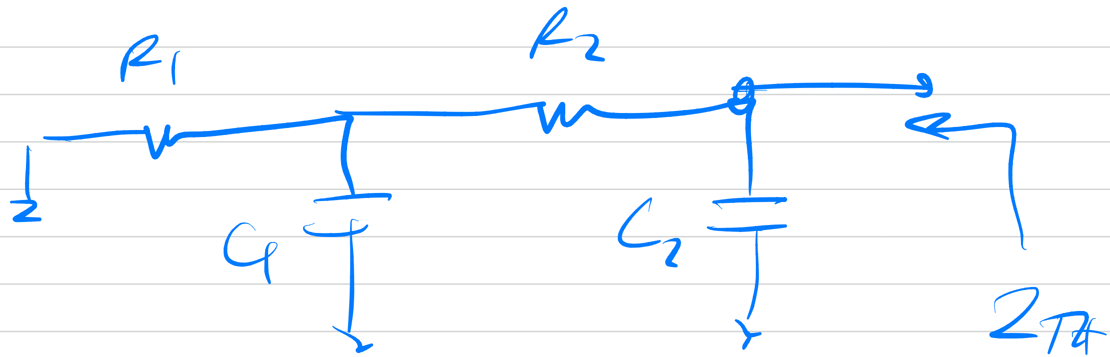


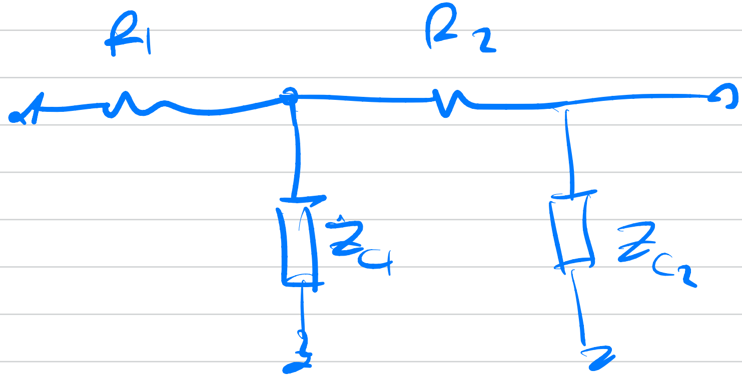
Z_{TH}



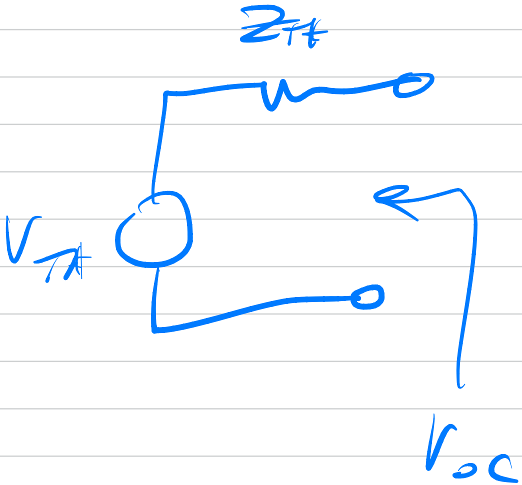
$V_{TH} = V_{oc}$

Z_{TH} = impedance looking back w/ $V_s = 0V$

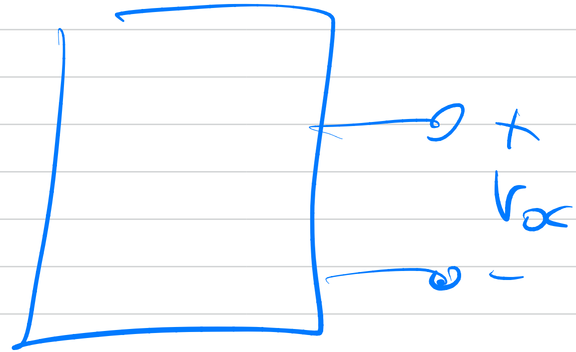




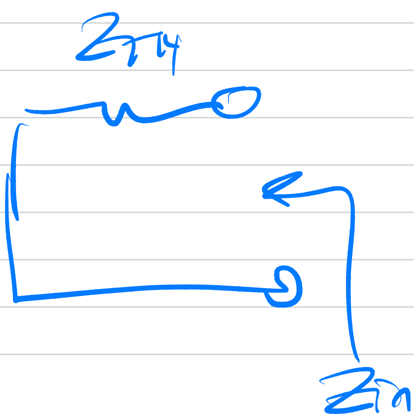
$$Z_{TH} = \left(R_1 \,\|\, Z_{C1} + R_2\right) \| \, Z_{C2}$$



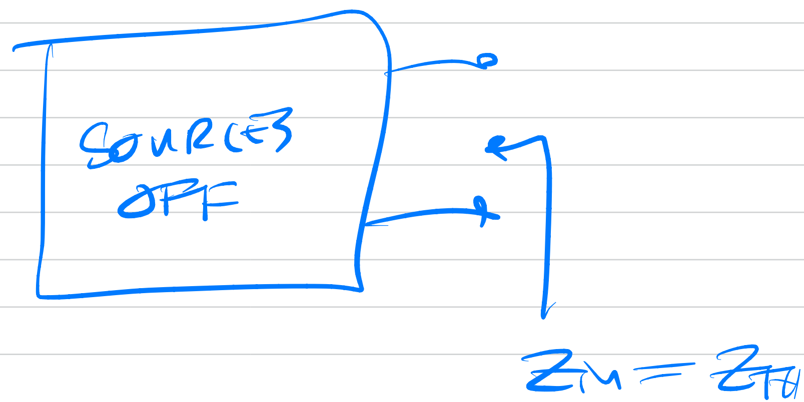
Z_TH
V_TH
V_oc



+
V_oc
–



Z_TH
Z_in



SOURCES
OFF
Z_in = Z_TH

$$Z_{TH} = \left(R_1 \parallel Z_{C1} + R_2\right) \parallel Z_{C_2}$$

$$\left(R_1 \parallel \frac{1}{j\omega C_1} + R_2\right) \parallel \frac{1}{j\omega C_2}$$

$$= \frac{\left(\dfrac{R_1}{1+j\omega R_1C_1} + \dfrac{R_2(1+j\omega R_1C_1)}{1+j\omega R_1C_1}\right) \cdot \dfrac{1}{j\omega C_2}}{\dfrac{R_1}{1+j\omega R_1C_1} + \dfrac{R_2(1+j\omega R_1C_1)}{1+j\omega R_1C_1} + \dfrac{1}{j\omega C_2}}$$

$$= \frac{R_1 + R_2(1+j\omega R_1C_1)}{\left(R_1 + R_2(1+j\omega R_1C_1)\right)j\omega C_2 + 1+j\omega R_1C_1}$$

$$Z_{TH}(j\omega) = \frac{N(j\omega)}{D(j\omega)}$$

$$N(j\omega) = R_1 + R_2(1 + j\omega C_1 R_1) = b_1 + b_2 j\omega$$

$$D(j\omega) = (\underbrace{R_1}_{\text{linear}} + R_2(1 + j\underbrace{\omega R_1 C_1))}_{\text{quadratic}} j\omega C_2 + 1 + j\underbrace{\omega R_1 C_1}_{\text{linear}}$$

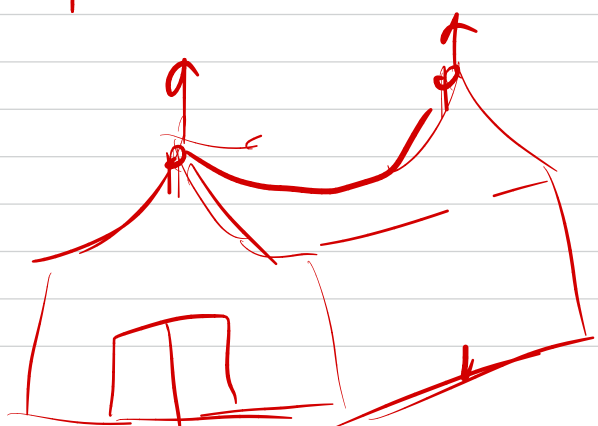
$$= a_1 + a_2 j\omega + a_3 (j\omega)^2$$

Roots of $N(j\omega) = 0$

zeros of Transfer func

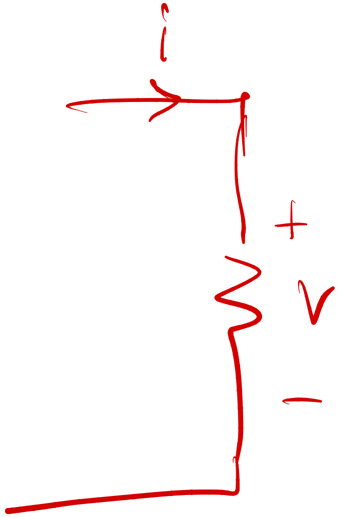
Roots of $D(j\omega) = 0$

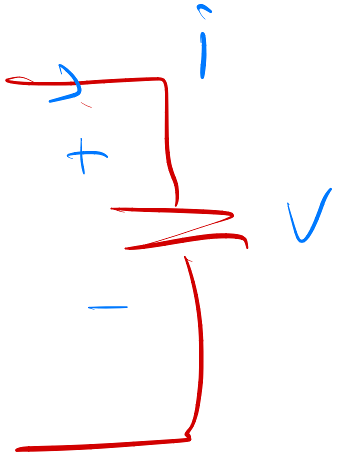
poles of transfer func

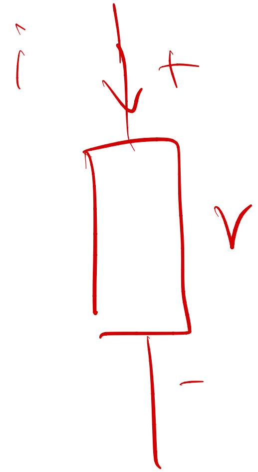


AC Power Flow

- Review passive sign convention







$$p(t) = i(t) \cdot v(t)$$

$$i = C \frac{dv}{dt} \geq 0$$

$$\Rightarrow i(t)\, v(t) = p(t) \geq 0$$

$$v > 0 \qquad \frac{dv}{dt} > 0$$

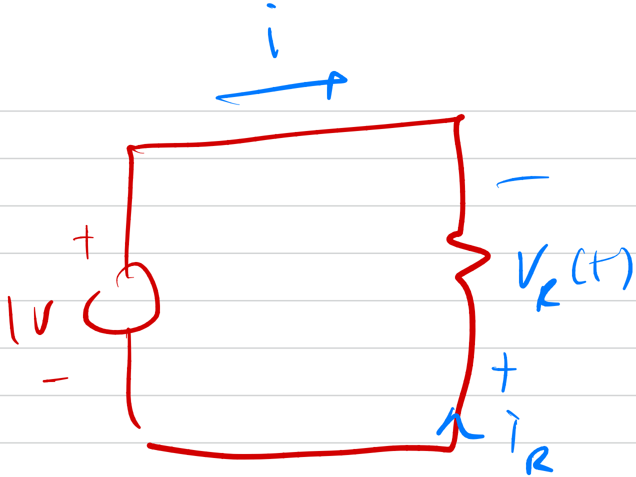
if $\frac{dv}{dt} < 0 \Rightarrow$

$p(t) = i(t)\ v(t) < 0$

negative since $\frac{dv}{dt} < 0$

positive

Supplying Energy

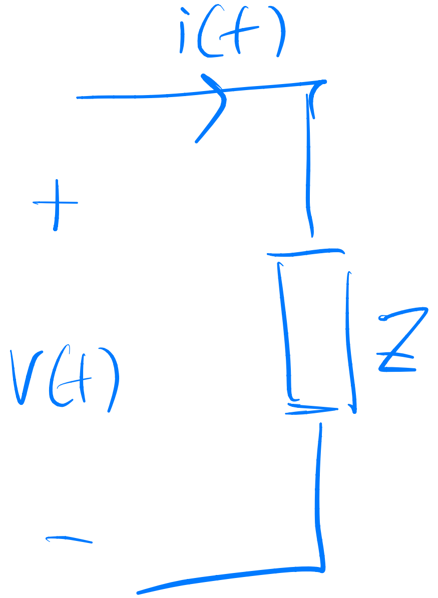


$$P_R(t) = V_R(t) \cdot i_R(t) > 0 \Rightarrow$$ dissipating power

$$= (-1V)(-i)$$

$$= 1V \times i$$

Instantaneous Power



In sinusoidal steady state:

$$v(t) = V_0 \cos \omega t$$

$$i(t) = I_0 \cos(\omega t + \phi)$$

$$I_0 = V_0 / |Z|$$

$$\phi = -\angle Z$$

$$v(t) = V_0 \cos \omega t$$

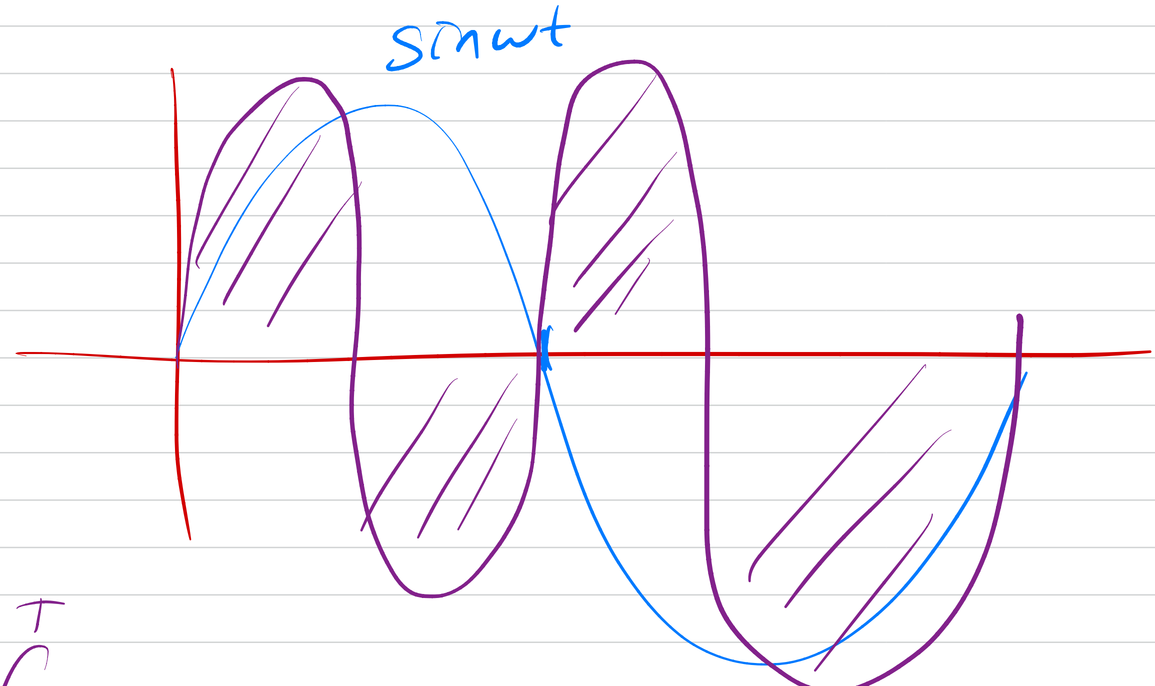
$$i(t) = I_0 \cos(\omega t + \phi)$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$p(t) = v(t) \cdot i(t)$$

$$= V_0 \cos \omega t \cdot I_0 \left(\cos \omega t \cos \phi - \sin \omega t \sin \phi \right)$$

$$= \underbrace{V_0 I_0 \cos \phi}_{\text{constant}} \underbrace{\cos^2 \omega t}_{\frac{1+\cos 2\omega t}{2}} - \underbrace{V_0 I_0 \sin \phi}_{\text{constant}} \underbrace{\cos \omega t \cdot \sin \omega t}_{\sin 2\omega t}$$



$$\int_0^T \sin 2\omega t \, dt = 0$$

Average Power

$$P_{av} = \frac{1}{T}\int_0^T p(t)\,dt$$

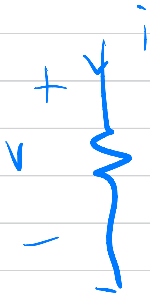
T : one cycle of $\cos\omega t$

$$\omega = \frac{2\pi}{T}$$

$$P_{av} = \frac{1}{\cancel{T}}\cos\phi \times V_0 \times I_0 \times \frac{1}{2}\cancel{T}$$

$$= \frac{1}{2} V_0 \cdot I_0 \cos\phi$$ — Average Power Dissipated by "Z"

Average Power Dissipated By R



$$P_{av} = \frac{1}{2} V_0 I_0 \cos\phi$$

$$\phi \equiv 0 \quad \text{for resistor}$$

$$P_{av} = \frac{1}{2} V_0 I_0 = \frac{1}{2} I_0 \cdot R \cdot I_0$$

$$= \frac{1}{2} I_0^2 R = I_{rms}^2 R$$

$$I_{rms} \triangleq \frac{I_0}{\sqrt{2}}$$

RMS : ROOT MEAN SQUARED

$$P_{av} = \frac{1}{2} V_0 I_0 \cos\phi$$

$$= V_{rms} \cdot I_{rms} \cdot \cos\phi$$

$$V_{rms} \triangleq \frac{V_0}{\sqrt{2}}$$

$$I_{rms} \triangleq \frac{I_0}{\sqrt{2}}$$

$$p(t) = v(t)\, i(t)$$

$$= i(t) \cdot R \times i(t) = i^2(t) \cdot R$$

$$P_{av} = \frac{1}{T} \int_T p(t)\, dt = R \int_T i^2(t)\, dt = I_{rms}^2 R$$

$$I_{rms} = \sqrt{\int_T i^2(t)\, dt}$$

R M S

AC Power in Phasors

$p \stackrel{?}{=} \tilde{V} \cdot \tilde{I}$

$\tilde{V}$ is a phasor

$\tilde{I}$ is a phasor

$\tilde{V} \cdot \tilde{I} \neq \frac{1}{2} V_0 I_0 \cos\phi$

$P_{av} = \frac{1}{2} V_0 I_0 \cos(\underbrace{\phi_v - \phi_i}_{\phi})$

$\tilde{P}_{av} \triangleq \frac{1}{2} \tilde{V} \tilde{I}^*$

$Re\{\tilde{P}_{av}\} = P_{av}$

$$\tilde{P}_{av} = \frac{1}{2} \tilde{V} \tilde{I}^* \qquad \tilde{I} = I_o e^{j\phi_i}$$

$$= \frac{1}{2} \tilde{V} I_o e^{-j\phi_i}$$

$$= \frac{1}{2} V_o e^{j\phi_v} \cdot I_o e^{-j\phi_i}$$

$$= \frac{1}{2} V_o I_o e^{j\underbrace{(\phi_v - \phi_i)}_{=\phi}}$$

$$Re\{\tilde{P}_{av}\} = \frac{1}{2} V_o I_o \cos\phi \quad \checkmark$$

$$\tilde{P}_{av} = \frac{\tilde{V}^* \cdot \tilde{I}}{2} = \frac{e^{-j\phi_v} V_o \quad I_o e^{j\phi_i}}{2}$$

$$= \frac{1}{2} V_o I_o \, e^{j(\phi_i - \phi_v)}$$

$$Re\{\tilde{P}_{av}\} = \frac{1}{2} V_o I_o \cos \phi_i - \phi_v$$

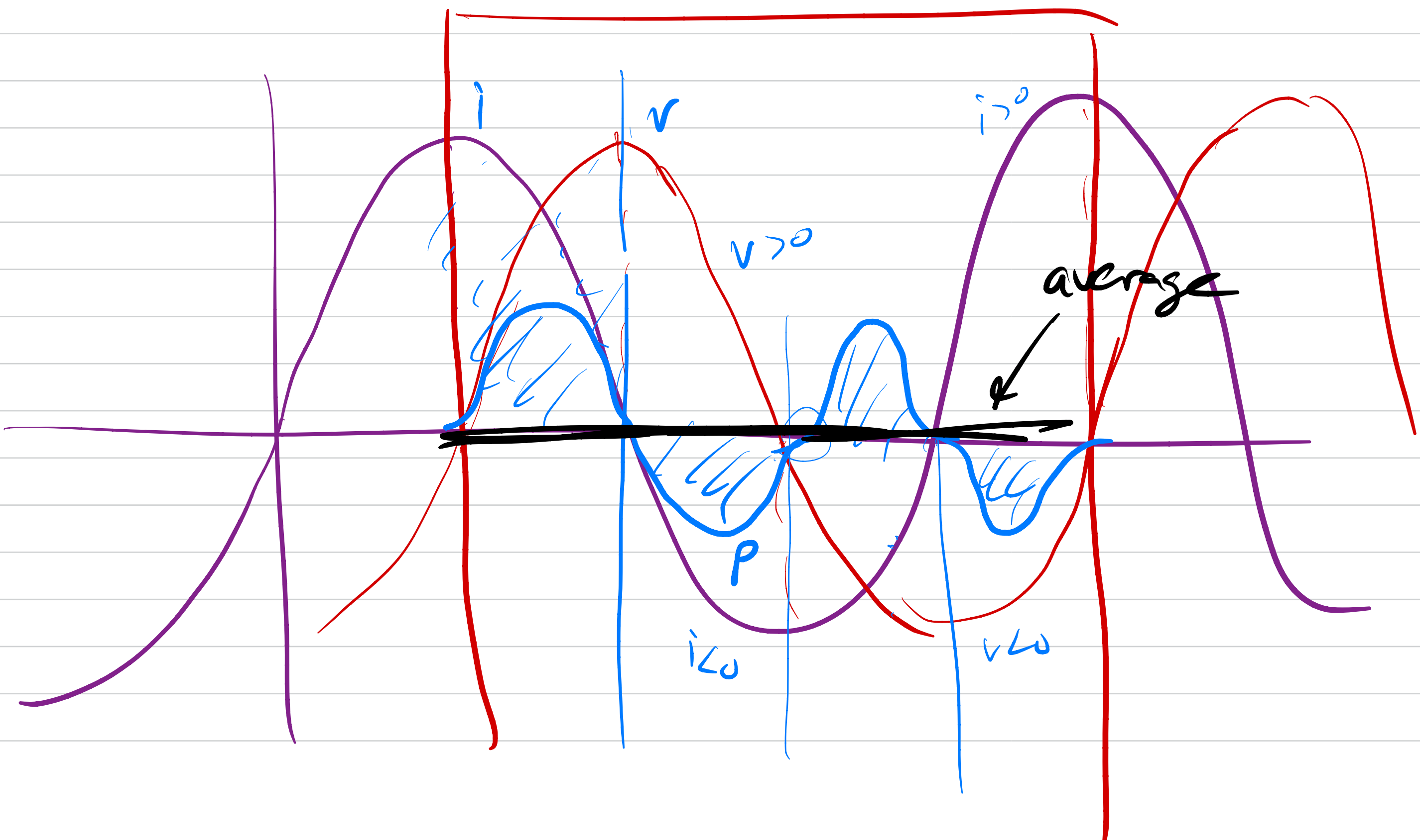
$$= \frac{1}{2} V_o I_o \cos \underbrace{\phi_v - \phi_i}_{\phi}$$

$$P_{av} = Re\left\{ \frac{\tilde{V}^* \tilde{I} + \tilde{I}^* V^*}{2} \right\}$$

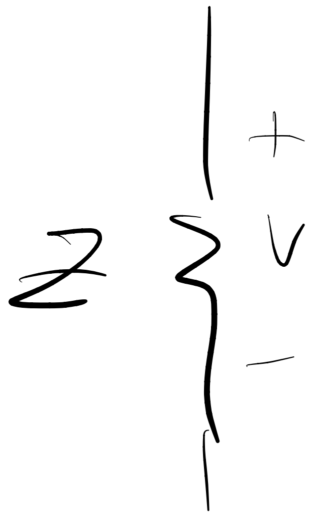
Ex 1

$$V = V_0 \cos \omega t \qquad \tilde{I} = j\omega C \cdot V$$

$$I = \omega C V_0 \cos(\omega t + 90^\circ)$$



Alternate Forms



$$P_{av} = \frac{I \cdot V^*}{2}$$

$$= \frac{V}{Z} \cdot \frac{V^*}{2} = \frac{|V|^2}{2Z}$$

$$Re(P_{av}) = Re\left\{ \frac{|V|^2}{2 Z Z^*} Z^* \right\}$$

$$= \frac{|V|^2}{2|Z|^2} Re\{Z\}$$

Inductors/Capacitors Store Energy

- Inductors and capacitors cannot dissipate energy.
- But charging a capacitor requires work, which is stored (not dissipated)
- It seems that we should account for this stored power in AC components

$$p(t) = \underbrace{\frac{1}{2} V_0 I_0 \cos\phi}_{P_{av}} (1+\cos 2\omega t)) + \underbrace{\left\{\frac{1}{2} V_0 I_0 \sin\phi \sin 2\omega t\right\}}_{\text{Reactive power}}$$

$$\phi = \phi_v - \phi_i$$

Reactive Power

$$P_{reac}(t) = \frac{1}{2} V_0 I_0 \sin\phi \sin 2\omega t$$

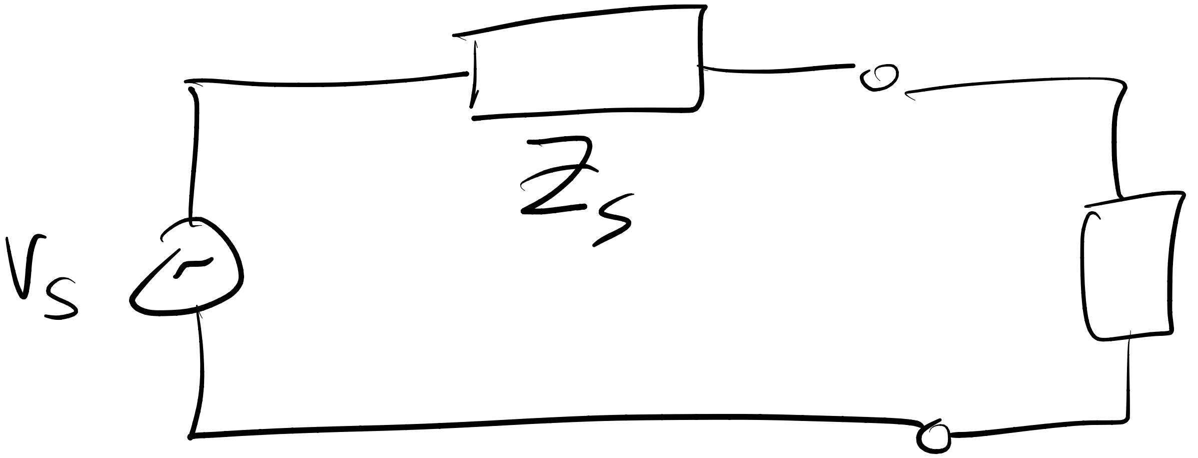
$$\int_T P_{reac}(t)\, dt = 0$$

$\pm 90^\circ$ maximize reactive power

$+90^\circ$ stores power when -90° ~~or~~

dissipates power

Maximum Power Transfer Theorem



$Z_L = R_L + jX_L$

$P_{av} = \frac{1}{2} \mathrm{Re}\{V \cdot I^*\}$

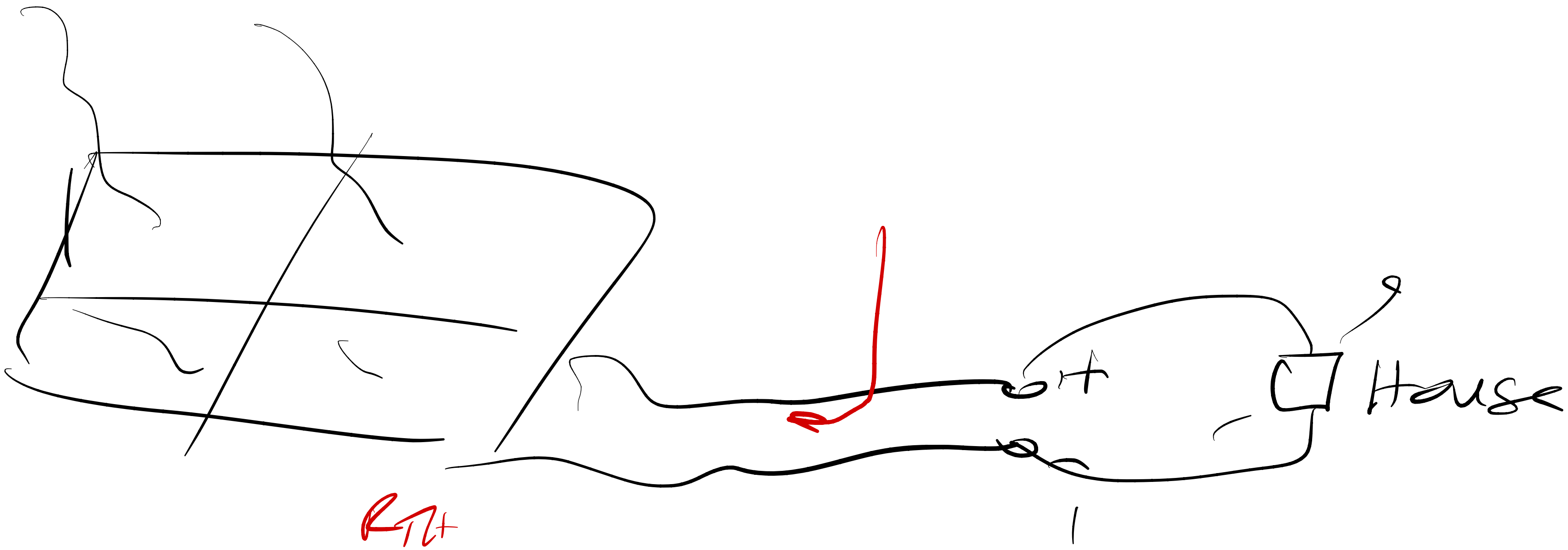
$\frac{dP_{av}}{dX_L} = 0 \qquad \frac{dP_{av}}{dR_L} = 0$

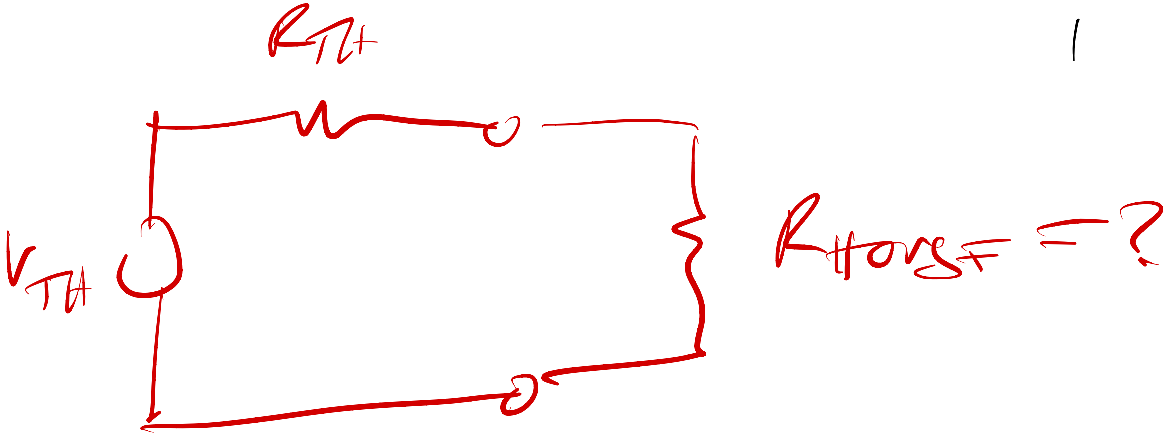
$\Rightarrow$ solve

$R_L = \mathrm{Re}(Z_s)$

$X_L = -\mathrm{Im}(Z_s)$

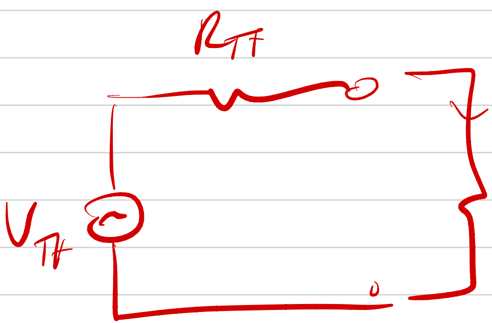
Applications: Solar Panel





$$P_{house} = v \cdot i$$

$$= \frac{V \cdot V}{R_H} = i^2 R_H$$

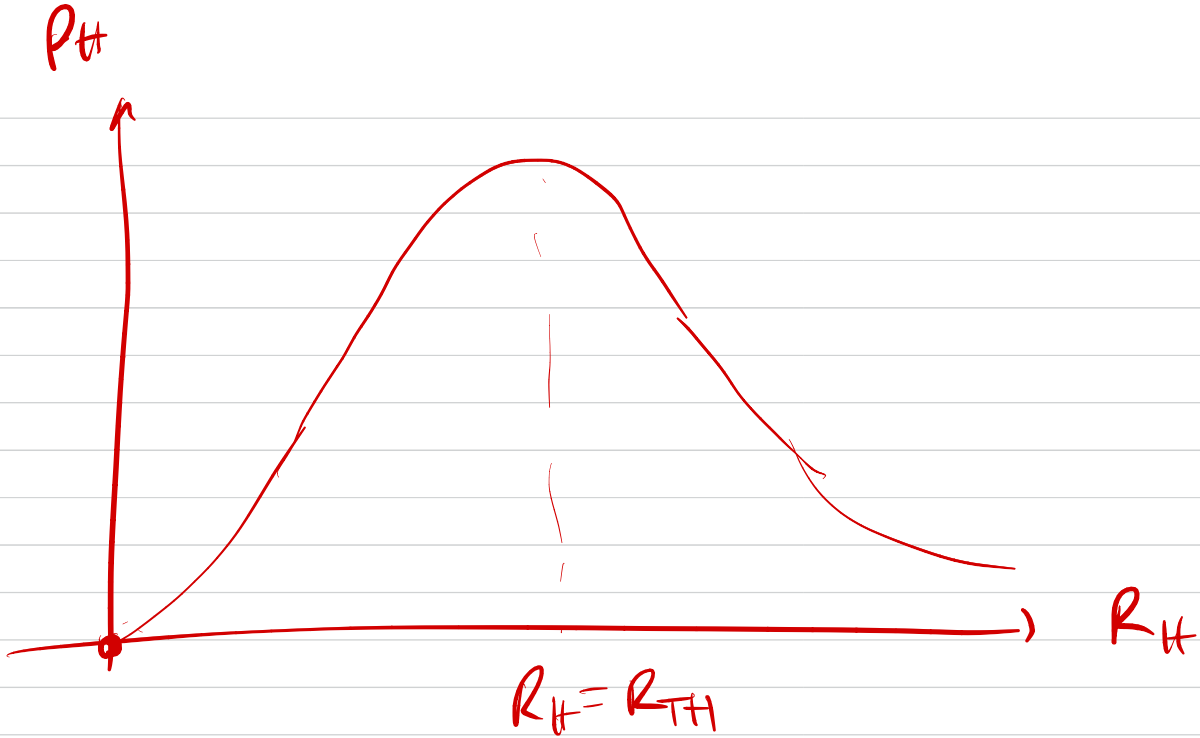


$R_H = R_{TH}$

power match

$$\frac{dP_{house}}{dR} = 0 = \frac{d}{dR}\left(\frac{V_{TH}}{R_{TH}+R_H} \cdot R_H\right)$$

$\Rightarrow$ SOLVE $R_H = R_{TH}$


ρ_H
R_H
$R_H = R_{TH}$