EECS 16B

Designing Information Devices and Systems II

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Module 10: AC Networks and AC Power

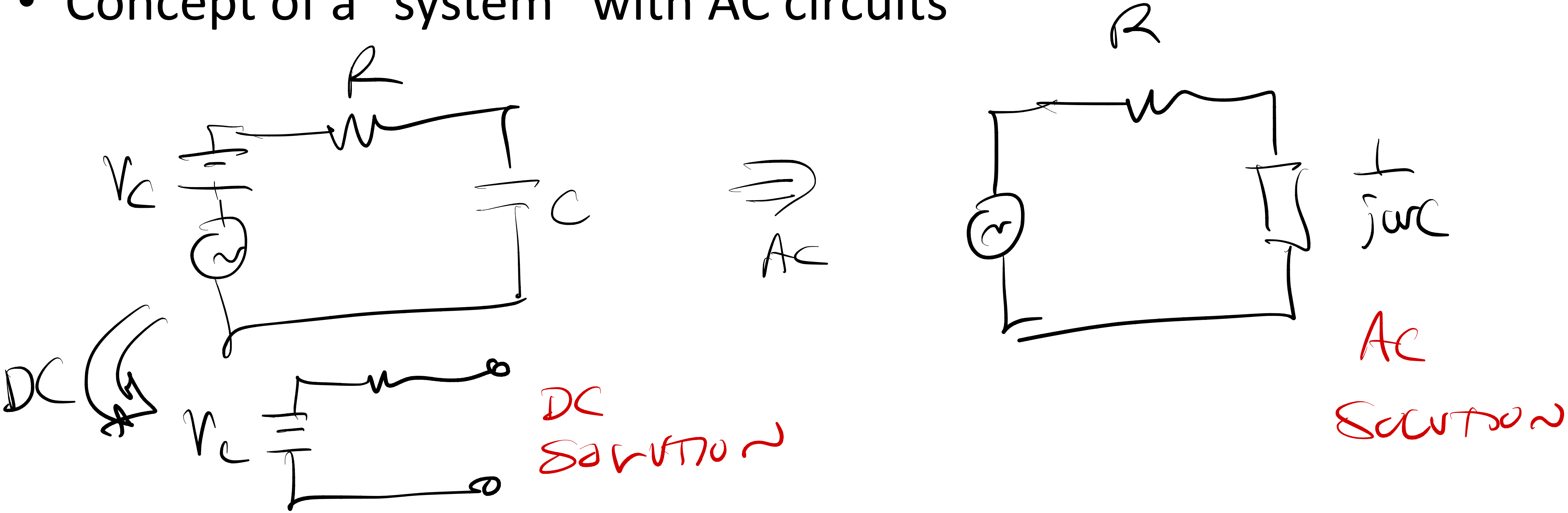
EECS 16B

Summary

- AC Circuit Network Theorems
  - Voltage / current dividers
  - Source superposition
  - Thevenin/Norton
- AC Power
  - Average Power
  - Reactive Power
- Maximum power transfer theorem

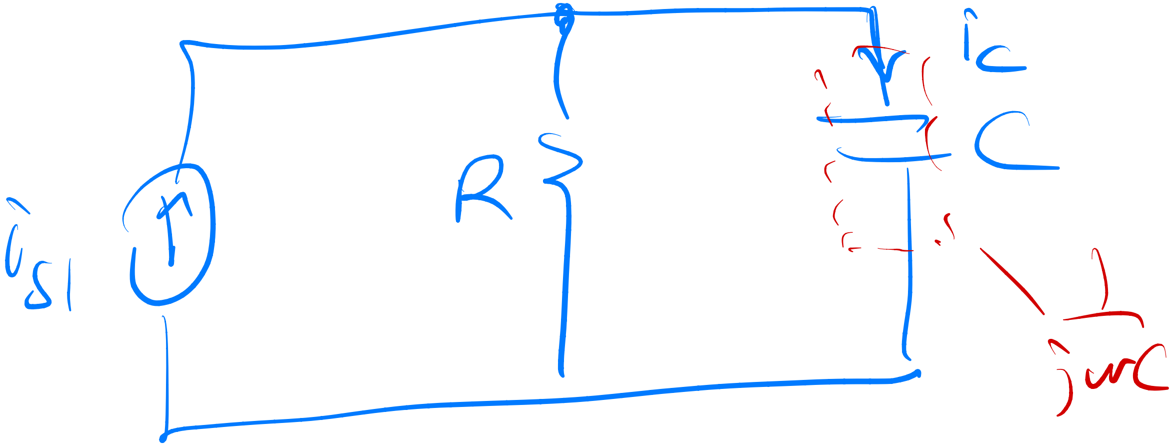
Review AC Circuits

- Concept of Impedance
- AC equivalent circuit
- Concept of a “system” with AC circuits



AC ~~Voltage~~ Divider

current



$$\hat{i}_c = \frac{Y_C}{Y_C + G}\,\hat{i}_{s1}$$

$$\boxed{G = 1/R}$$

$$\hat{i}_c = \frac{j\omega C}{j\omega C + G}\,\hat{i}_{s1} = \frac{j\omega C}{G + j\omega C}\,\hat{i}_{s1} = \frac{j\omega RC}{1 + j\omega RC}\,\hat{i}_{s1}$$

$$H(j\omega) = \frac{\hat{i}_c}{\hat{i}_{s1}} = \frac{j\omega RC}{1 + j\omega RC}$$

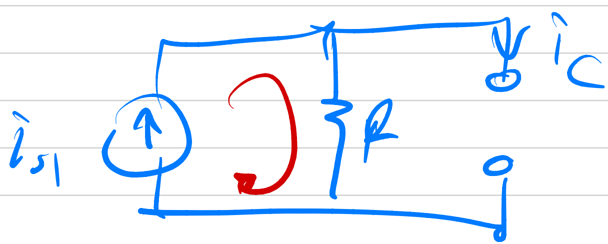
Reminder

$$\frac{i_2}{i_s} = \frac{G_2}{G_1 + G_2}$$

Sanity Checks

- Check Units A/A ✓
- DC solution $\omega = 0$

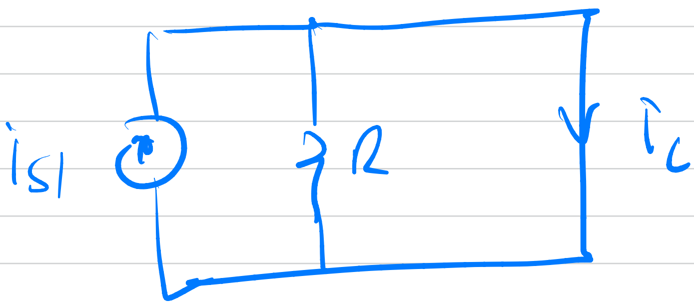
$$H(j\omega)\Big|_{\omega=0} = \frac{j\omega RC}{1 + j\omega RC}\Bigg|_{\omega=0} = 0 \checkmark$$



- $\omega \to \infty$ VERY HIGH FREQ

$\frac{1}{\omega C} \to$ SHORTS

$\omega L \to$ OPENS



$$H(j\omega)\Big|_{\omega\to\infty} = 1$$

$$H(j\omega)\Big|_{\omega=\infty} = \frac{\cancel{j\omega RC}}{1+\cancel{j\omega RC}}\Bigg|_{\omega=\infty} = 1$$

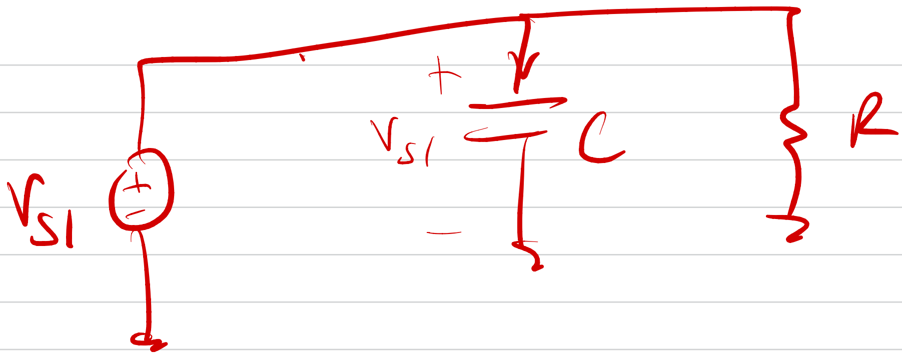
AC Current Divider

AC Source Superposition

$$\cancel{H_1 = \frac{j\omega RC}{1 + j\omega RC}}$$

$$\frac{i_C}{i_{s2}} = 0$$

$$H_1 = 0$$

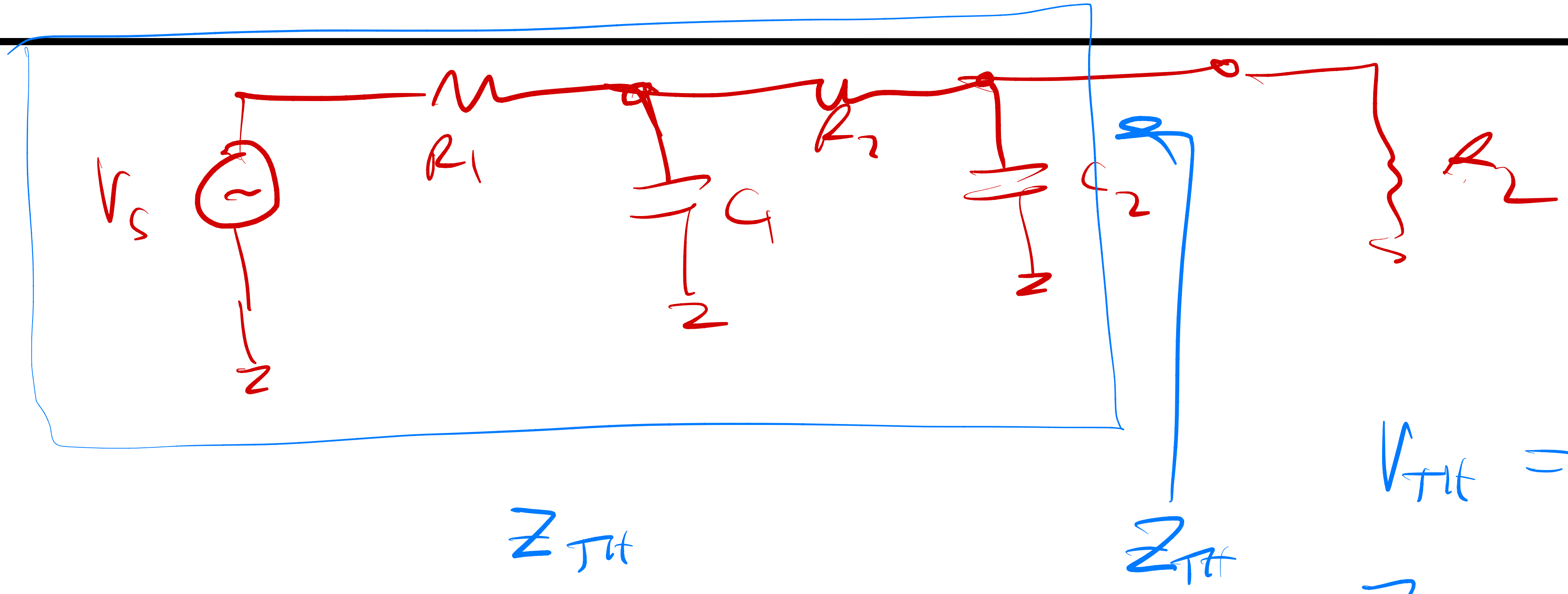


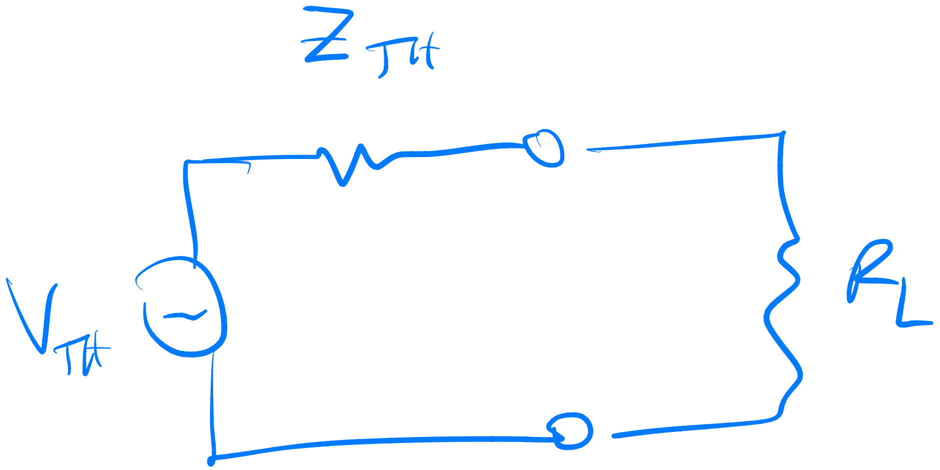
$$\hat{i}_{C1} = \frac{V_{S1}}{1/j\omega C} = j\omega C \, V_{S1}$$

$$\frac{\hat{i}_{C1}}{V_{S1}} = j\omega C$$

TURN OFF $\begin{cases} V_s \rightarrow \text{SHORT} \\ \hat{i}_s \rightarrow \text{OPEN} \end{cases}$

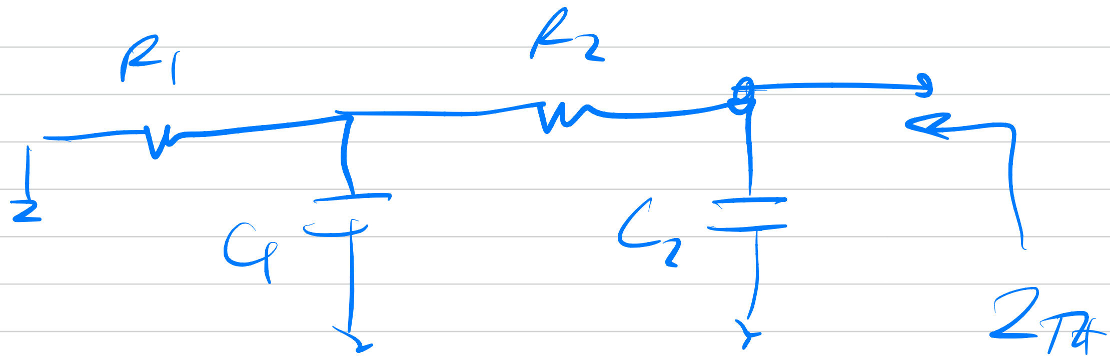
AC Thevenin/Norton

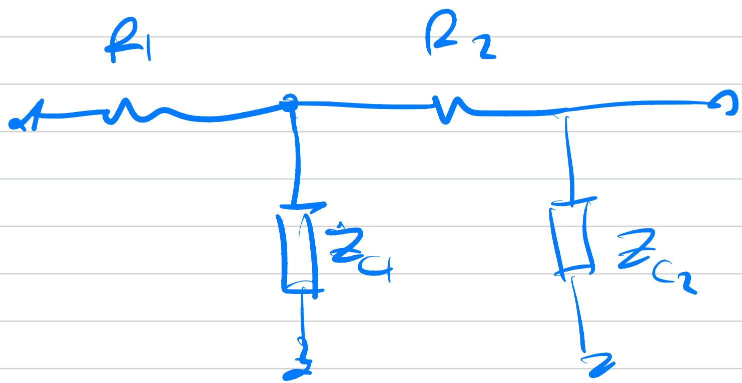




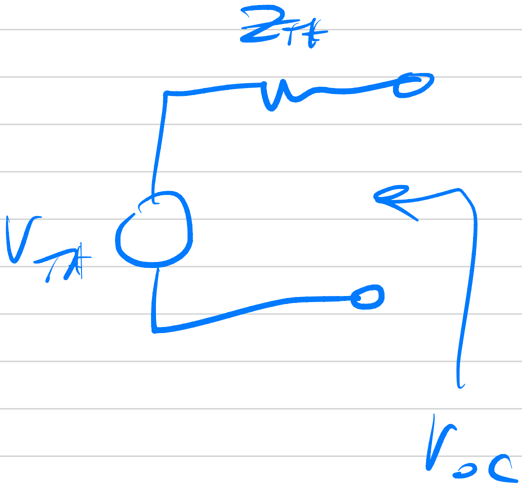
$V_{TH} = V_{oc}$

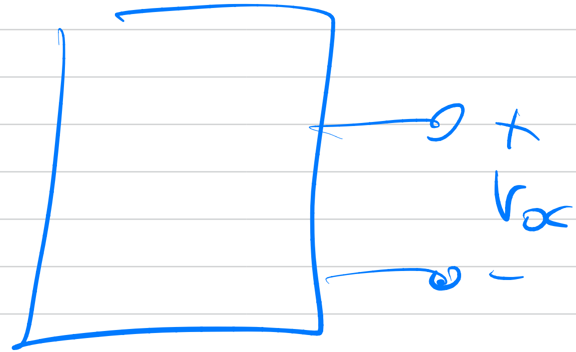
Z_{TH} = impedance looking back w/ $V_S = 0V$

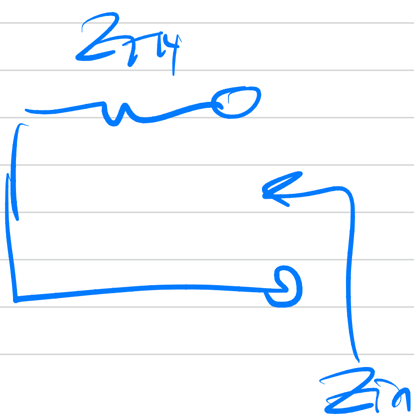


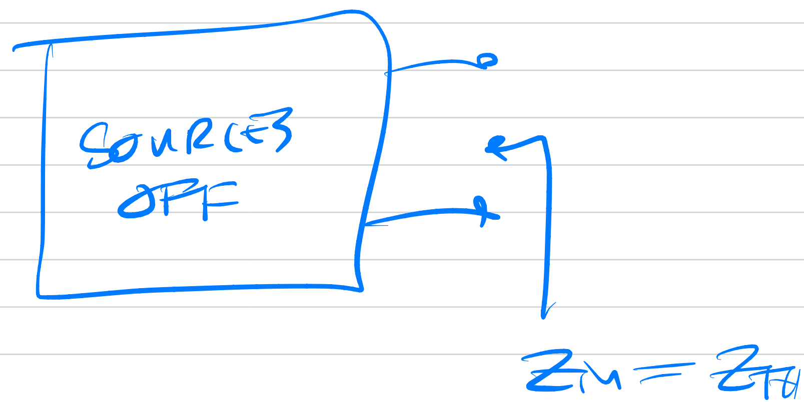


$$Z_{TH} = (R_1 \| Z_{C1} + R_2) \| Z_{C_2}$$


Z_{Th}
V_{Th}
V_{oc}


+
V_{oc}
−


Z_{Th}
Z_{in}


SOURCES OFF
$Z_{in} = Z_{Th}$

$$Z_{TH} = \left(R_1 \parallel Z_{C_1} + R_2\right) \parallel Z_{C_2}$$

$$\left(R_1 \parallel \frac{1}{j\omega C_1} + R_2\right) \parallel \frac{1}{j\omega C_2}$$

$$= \frac{\dfrac{R_1 + R_2(1+j\omega R_1 C_1)}{1+j\omega R_1 C_1} \cdot \dfrac{1}{j\omega C_2}}{\dfrac{R_1 + R_2(1+j\omega R_1 C_1)}{1+j\omega R_1 C_1} + \dfrac{1}{j\omega C_2}}$$

$$= \frac{R_1 + R_2(1+j\omega R_1 C_1)}{\left(R_1 + R_2(1+j\omega R_1 C_1)\right) j\omega C_2 + 1 + j\omega R_1 C_1}$$

$$Z_{TH}(j\omega) = \frac{N(j\omega)}{D(j\omega)}$$

$$N(j\omega) = R_1 + R_2(1 + j\omega C_1 R_1) = b_1 + b_2 j\omega$$

$$D(j\omega) = \Big(\underset{\text{linear}}{R_1} + R_2(1 + \underbrace{j\omega R_1 C_1}_{\text{quadratic}})\Big) j\omega C_2 + 1 + \underbrace{j\omega R_1 C_1}_{\text{linear}}$$

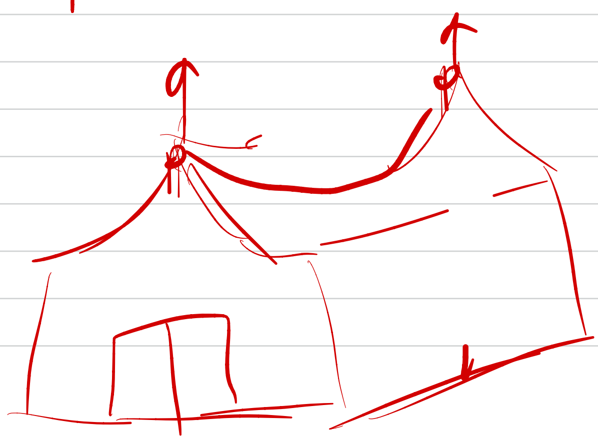
$$= a_1 + a_2 j\omega + a_3 (j\omega)^2$$

Roots of $N(j\omega) = 0$

zeros of Transfer func

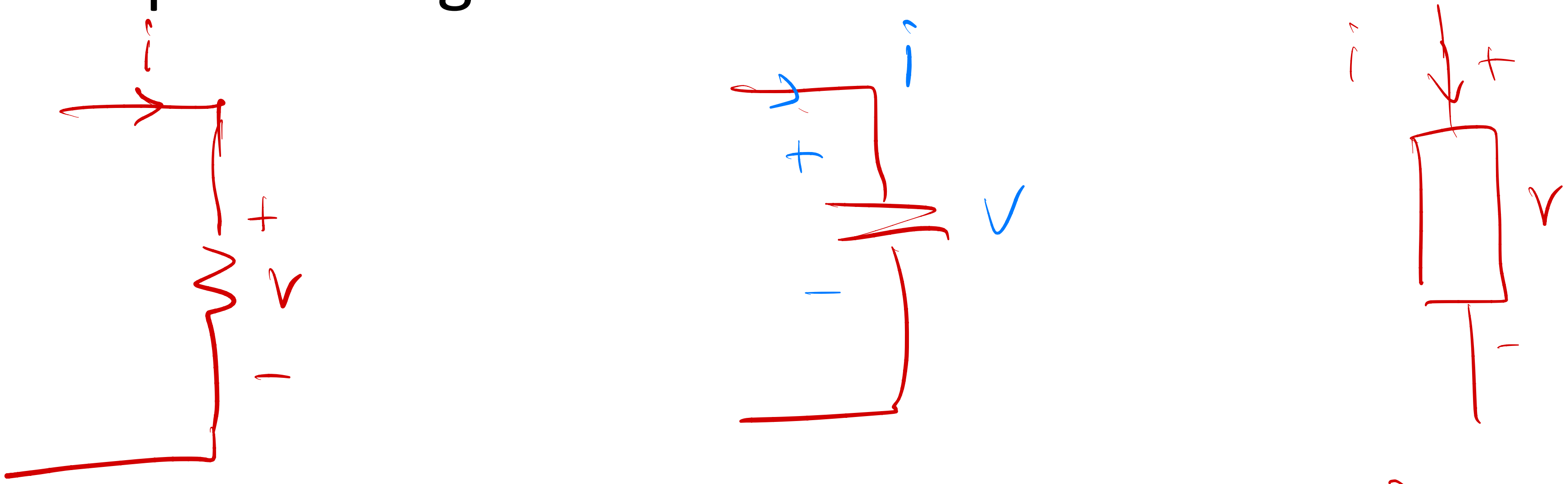
Roots of $D(j\omega) = 0$

poles of transfer func



AC Power Flow

- Review passive sign convention



$$p(t) = i(t) \cdot v(t)$$

$$i = C \frac{dv}{dt} \geq 0$$

$$v > 0 \qquad \frac{dv}{dt} > 0 \qquad \Rightarrow \quad i(t)\, v(t) = p(t) \geq 0$$

if $\frac{dv}{dt} < 0 \Rightarrow$

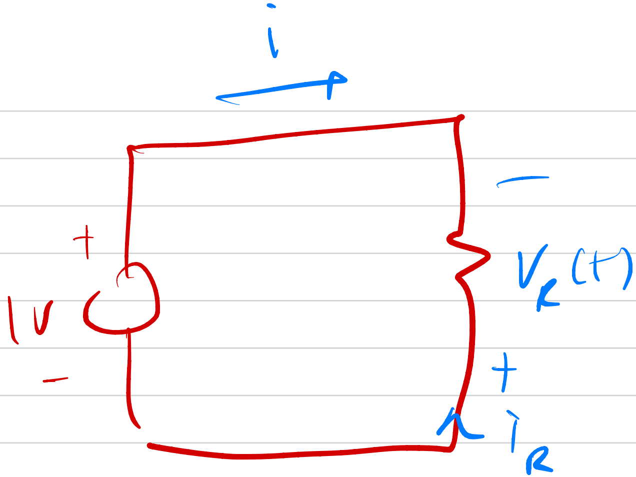
$p(t) = i(t)\ v(t) < 0$

negative

positive

since $\frac{dv}{dt} < 0$

Supplying Energy

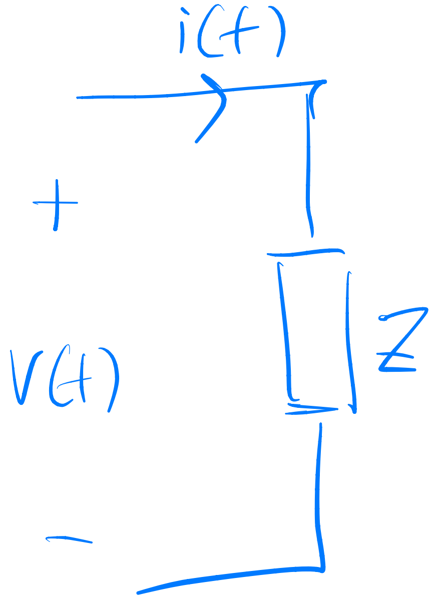


$$p_R(t) = V_R(t) \cdot i_R(t) \geq 0 \Rightarrow$$ dissipating power

$$= (-1V)(-i)$$

$$= 1V \times i$$

Instantaneous Power



In sinusoidal steady state:

$$v(t) = V_0 \cos \omega t$$

$$i(t) = I_0 \cos(\omega t + \phi)$$

$$I_0 = V_0 / |Z|$$

$$\phi = -\angle Z$$

$$v(t) = V_0 \cos \omega t$$

$$i(t) = I_0 \cos(\omega t + \phi)$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

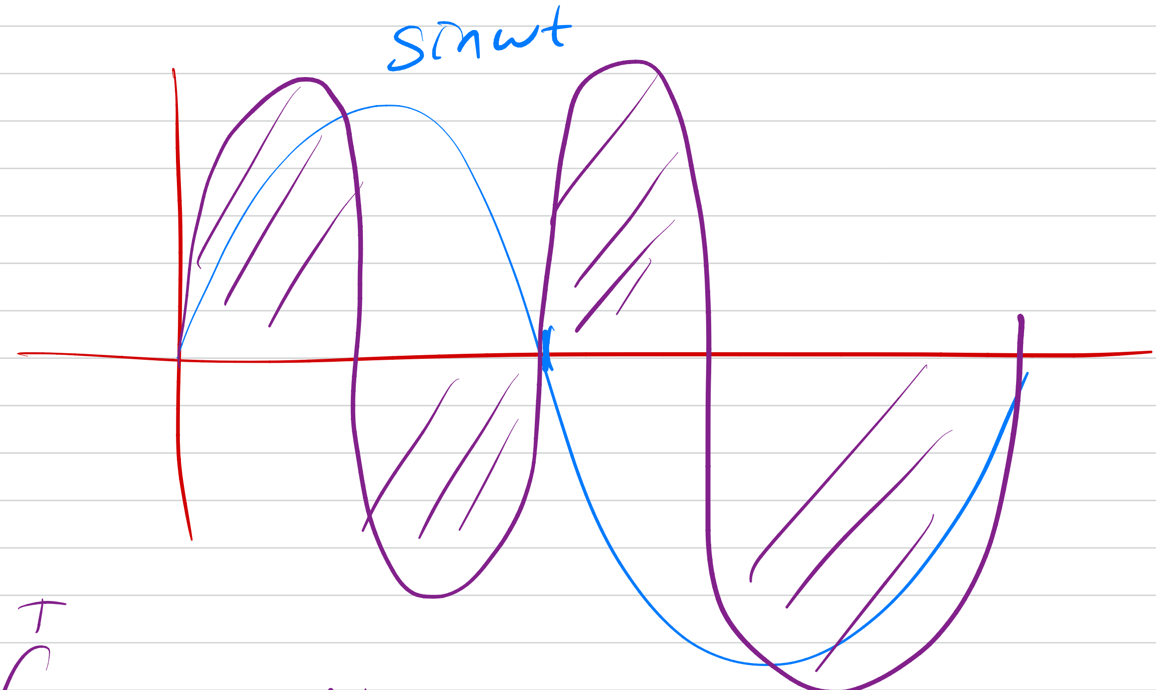
$$p(t) = v(t) \cdot i(t)$$

$$= V_0 \cos \omega t \cdot I_0 (\cos \omega t \cos \phi - \sin \omega t \sin \phi)$$

$$= \underbrace{V_0 I_0 \cos \phi}_{\text{constant}} \cos^2 \omega t - \underbrace{V_0 I_0 \sin \phi}_{\text{constant}} \cos \omega t \cdot \sin \omega t$$

$$\cos^2 \omega t \rightarrow \frac{1 + \cos 2\omega t}{2}$$

$$\cos \omega t \cdot \sin \omega t \rightarrow \sin 2\omega t$$



$$\int_0^T \sin 2wt \, dt = 0$$

Average Power

$$P_{av} = \frac{1}{T}\int_0^T p(t)\,dt$$

T : one cycle of $\cos\omega t$

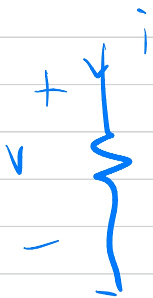
$$\omega = \frac{2\pi}{T}$$

$$P_{av} = \frac{1}{\cancel{T}} \cos\phi \times V_0 \times I_0 \times \frac{1}{2} \cancel{T}$$

$$= \frac{1}{2} V_0 \cdot I_0 \cos\phi$$

— Average Power Dissipated by "Z"

Average Power Dissipated By R



$$P_{av} = \frac{1}{2} V_0 I_0 \cos\phi$$

$\phi \equiv 0$ for resistor

$$P_{av} = \frac{1}{2} V_0 I_0 = \frac{1}{2} I_0 \cdot R \cdot I_0$$

$$= \frac{1}{2} I_0^2 R = I_{rms}^2 R$$

$$I_{rms} \triangleq \frac{I_0}{\sqrt{2}}$$

RMS : ROOT MEAN SQUARED

$$P_{av} = \frac{1}{2} V_0 I_0 \cos\phi$$

$$= V_{rms} \cdot I_{rms} \cdot \cos\phi$$

$$V_{rms} \triangleq \frac{V_0}{\sqrt{2}}$$

$$I_{rms} \triangleq \frac{I_0}{\sqrt{2}}$$

$$p(t) = v(t)\, i(t)$$

$$= i(t) \cdot R \times i(t) = i^2(t) \cdot R$$

$$P_{av} = \frac{1}{T} \int_T p(t)\, dt = R \int_T i^2(t)\, dt = I_{rms}^2 R$$

$$I_{rms} = \sqrt{\int_T i^2(t)\, dt}$$

R M S

AC Power in Phasors

$$p \overset{?}{=} \tilde{V} \cdot \tilde{I}$$

$\tilde{V}$ is a phasor

$\tilde{I}$ is a phasor

$$\tilde{V} \cdot \tilde{I} \neq \frac{1}{2} V_0 I_0 \cos\phi$$

$$P_{av} = \frac{1}{2} V_0 I_0 \cos(\underbrace{\phi_v - \phi_i}_{\phi})$$

$$\tilde{P}_{av} \triangleq \frac{1}{2} \tilde{V} \tilde{I}^*$$

$$\mathrm{Re}\{\tilde{P}_{av}\} = P_{av}$$

$$\tilde{P}_{av} = \frac{1}{2} \tilde{V} \tilde{I}^{*} \qquad \tilde{I} = I_o e^{j\phi_i}$$

$$= \frac{1}{2} \tilde{V} I_o e^{-j\phi_i}$$

$$= \frac{1}{2} V_o e^{j\phi_v} \cdot I_o e^{-j\phi_i}$$

$$= \frac{1}{2} V_o I_o e^{j\underbrace{(\phi_v - \phi_i)}_{=\phi}}$$

$$Re\{\tilde{P}_{av}\} = \frac{1}{2} V_o I_o \cos\phi \qquad \checkmark$$

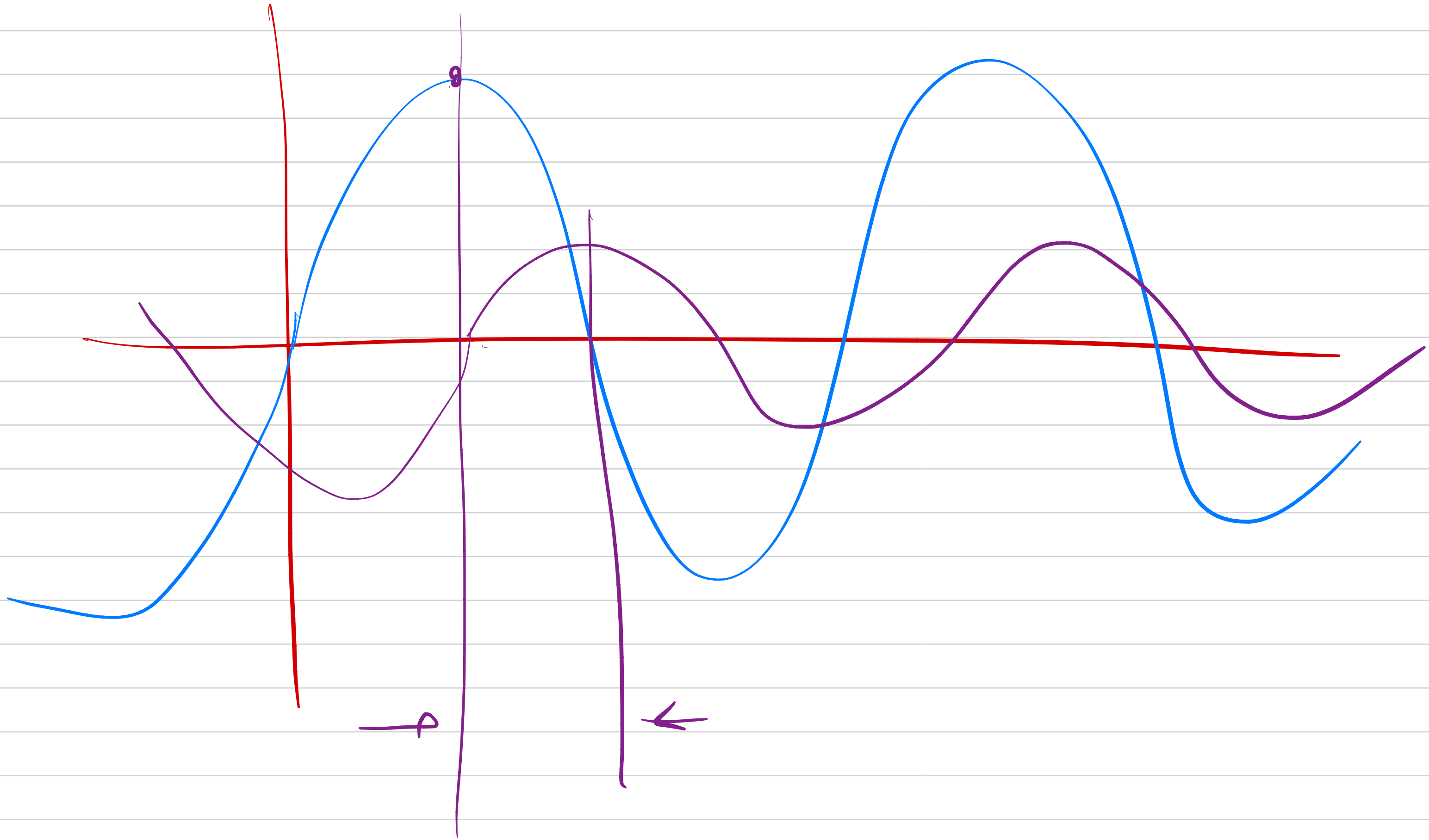
$$\tilde{P}_{av} = \frac{\tilde{V}^* \cdot \tilde{I}}{2} = \frac{e^{-j\phi_v} V_0 \quad I_0 e^{j\phi_i}}{2}$$

$$= \frac{1}{2} V_0 I_0 \; e^{j(\phi_i - \phi_v)}$$

$$Re\{\tilde{P}_{av}\} = \frac{1}{2} V_0 I_0 \cos \phi_i - \phi_v$$

$$= \frac{1}{2} V_0 I_0 \cos \underbrace{\phi_v - \phi_i}_{\phi}$$

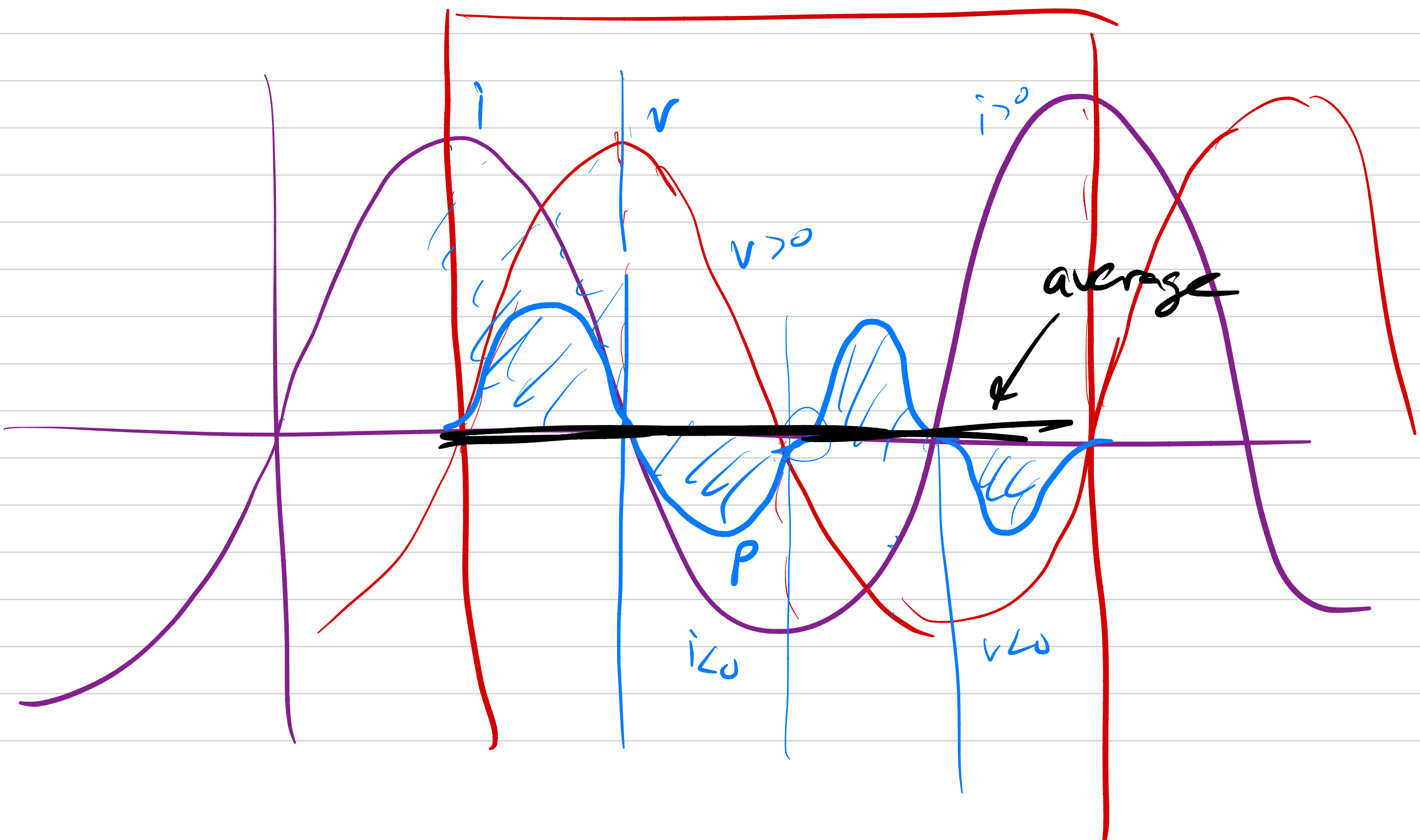
$$P_{av} = Re\left\{\frac{\tilde{V}^* \tilde{I} + \tilde{I}^* V^*}{2}\right\}$$



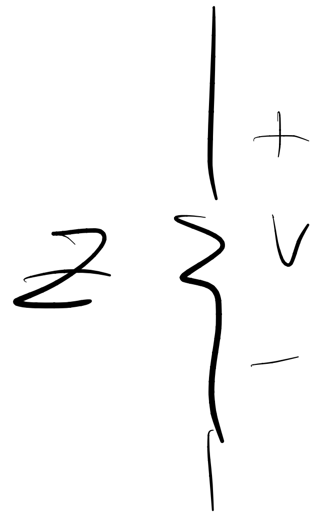
Ex 1

$$V = V_0 \cos \omega t \qquad \tilde{I} = j\omega C \cdot V$$

$$I = \omega C V_0 \cos(\omega t + 90^\circ)$$



Alternate Forms



$$P_{av} = \frac{I \cdot V^*}{2}$$

$$= \frac{V}{Z} \cdot \frac{V^*}{2} = \frac{|V|^2}{2Z}$$

$$Re(P_{av}) = Re\left\{ \frac{|V|^2}{2 Z Z^*} Z^* \right\}$$

$$= \frac{|V|^2}{2|Z|^2} Re\{Z\}$$

Inductors/Capacitors Store Energy

- Inductors and capacitors cannot dissipate energy.
- But charging a capacitor requires work, which is stored (not dissipated)
- It seems that we should account for this stored power in AC components

$$p(t) = \underbrace{\frac{1}{2} V_0 I_0 \cos\phi}_{P_{av}} (1+\cos 2\omega t)) + \underbrace{\left\{\frac{1}{2} V_0 I_0 \sin\phi \sin 2\omega t\right\}}_{\text{Reactive power}}$$

$$\phi = \phi_v - \phi_i$$

Reactive Power

$$P_{reac}(t) = \frac{1}{2} V_0 I_0 \sin\phi \sin 2\omega t$$

$$\int_T P_{reac}(t)\, dt = 0$$

$\pm 90^\circ$ maximize reactive power

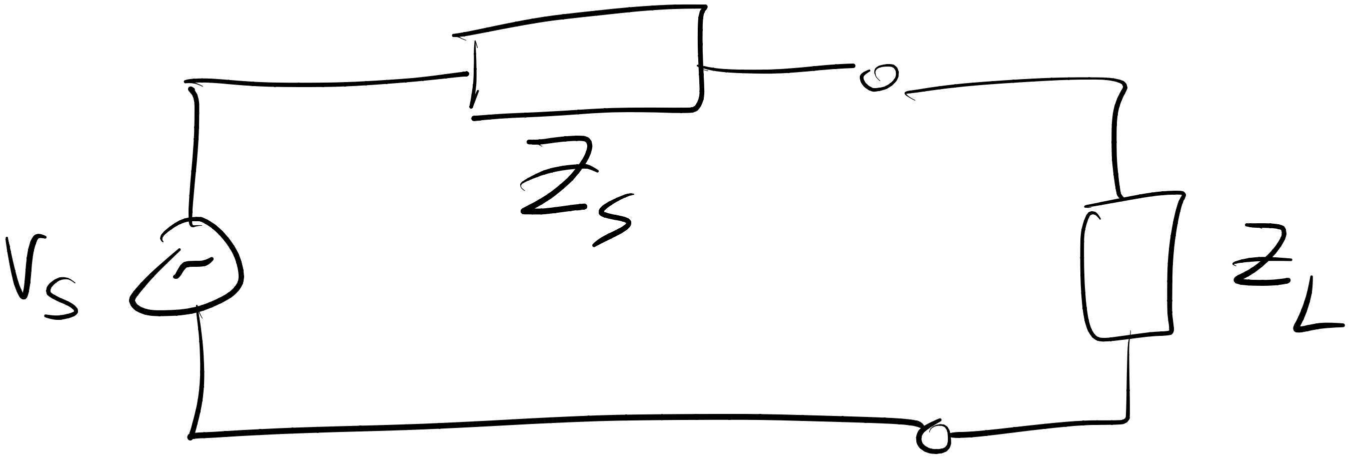
$+90^\circ$ stores power when -90° ~~or~~

dissipates power

Example: Real Inductor

- Concept of quality factor

Maximum Power Transfer Theorem



$Z_L = R_L + jX_L$

$P_{av} = \frac{1}{2} Re\{V \cdot I^*\}$

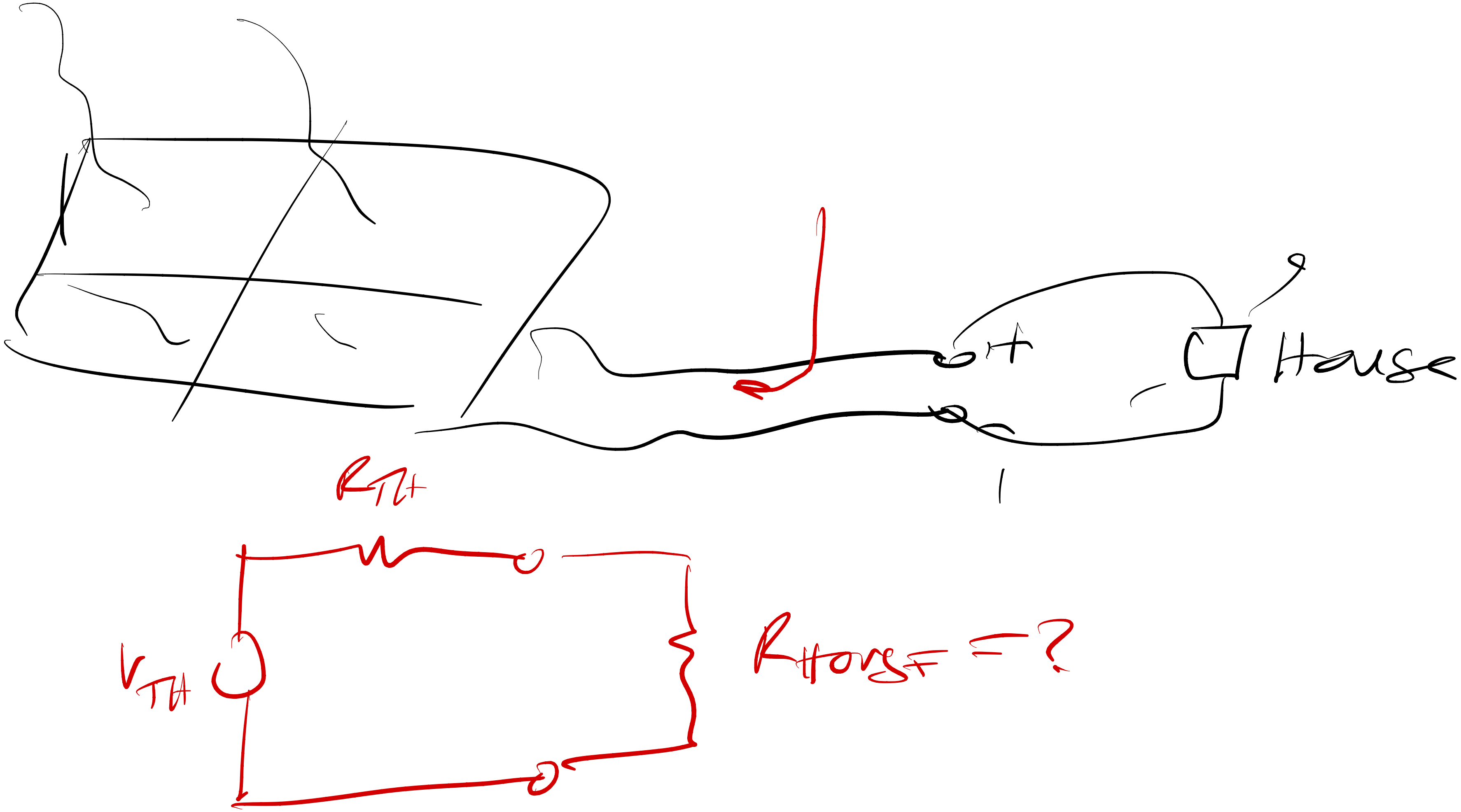
$\frac{dP_{av}}{dX_L} = 0$ $\frac{dP_{av}}{dR_L} = 0$

$\Rightarrow$ solve

$R_L = Re(Z_s)$

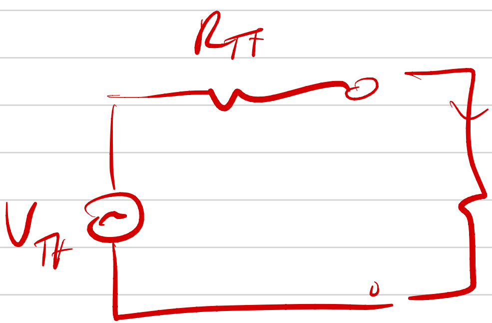
$X_L = -Im(Z_s)$

Applications: Solar Panel



$$P_{house} = v \cdot i$$

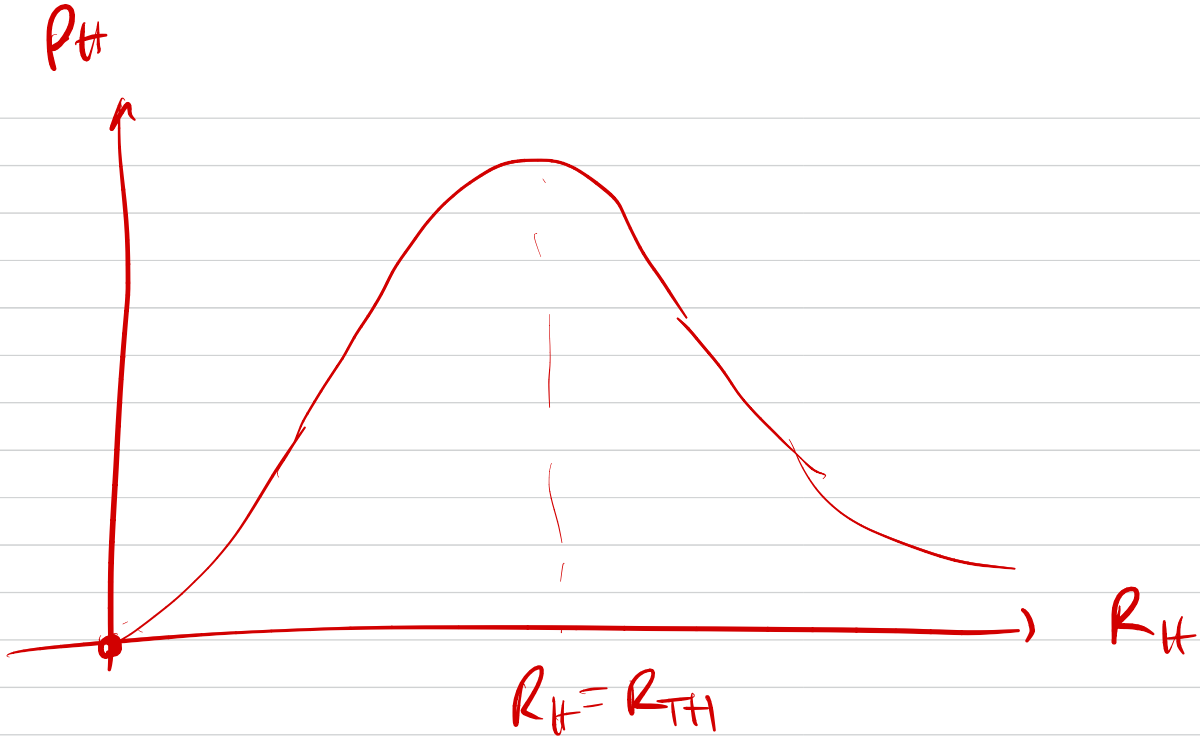
$$= \frac{V \cdot V}{R_H} = i^2 R_H$$



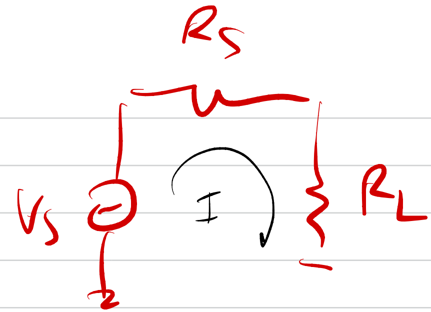
$R_H = R_{TH}$ power match

$$\frac{dP_{house}}{dR} = 0 = \frac{d}{dR}\left(\frac{V_{TH}}{R_{TH}+R_H} \cdot R_H\right)$$

$\Rightarrow$ SOLVE $R_H = R_{TH}$


P_H
R_H
$R_H = R_{TH}$

Power Flowing into Load



$$P_L = \frac{1}{2} I^2 R_L$$

$$= \frac{1}{2}\left(\frac{V_S}{R_S+R_L}\right)^2 R_L$$

$$= \underbrace{\frac{1}{2} V_S^2}_{K} \frac{R_L}{(R_S+R_L)^2}$$

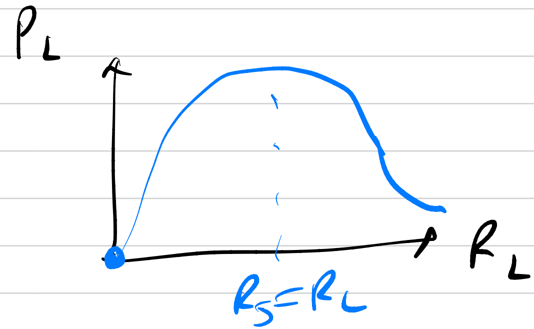
$$\frac{dP_L}{dR_L} = 0 = \frac{\cancel{K}}{(R_S+R_L)^2} + R_L \frac{-2}{(R_S+R_L)^3} \cancel{K}$$

$$\frac{dP_L}{dR_L} = 0 = \frac{\cancel{K}}{\cancel{(R_S+R_L)^2}} + R_L \frac{-2}{(R_S+R_L)^{\cancel{3}}} \cancel{K}$$

$$1 = \frac{2R_L}{R_S+R_L}$$

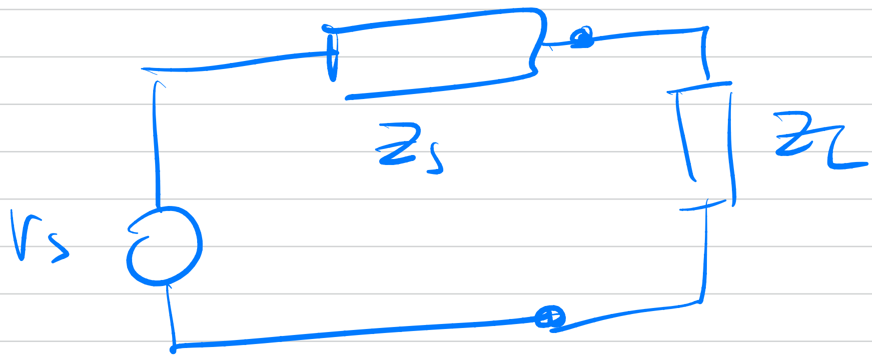
$$R_S + \cancel{R_L} = \cancel{2}R_L$$

$$\boxed{R_S = R_L}$$



Matched Load **Resistance** !

AC Impedance



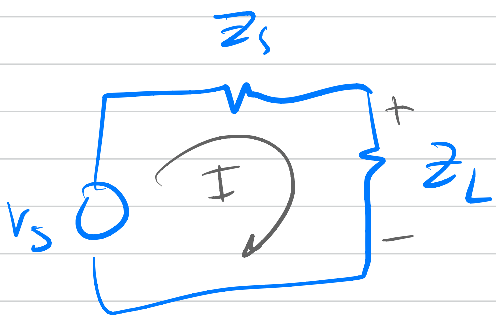
$$Z_s = R_s + jX_s$$

$$Z_L = R_L + jX_L$$

Z_{Lopt} to max power transfer

$$\boxed{R_{Lopt} = R_s}$$

AC Power



$$I = \frac{V_s}{Z_s + Z_L}$$

$$V_L = Z_L I \qquad I_L = I$$

$$P_L = \frac{1}{2} \mathrm{Re}\left\{ V_L I^* \right\} = \frac{1}{2} \mathrm{Re}\left\{ I V_L^* \right\}$$

$$= \frac{1}{2} \mathrm{Re}\left\{ \frac{V_s}{Z_s + Z_L} \cdot \left(\frac{V_s}{Z_s + Z_L} \right)^* Z_L^* \right\}$$

$$= \frac{1}{2} \frac{|V_s|^2}{|Z_s + Z_L|^2} \mathrm{Re}(Z_L)$$

$$P_L = \frac{1}{2} \frac{|V_s|^2}{|Z_s + Z_L|^2} \operatorname{Re}(Z_L)$$

$$R_L = R_s$$

$$\frac{dP_L}{dX_L} = 0$$

$$P_L = \frac{1}{2} \frac{|V_s|^2}{(R_s + R_L)^2 + \underbrace{(X_s + X_L)^2}} R_L$$

$$X_L = -X_s$$

Conjugate Matched Load

$$Z_L = Z_S^*$$

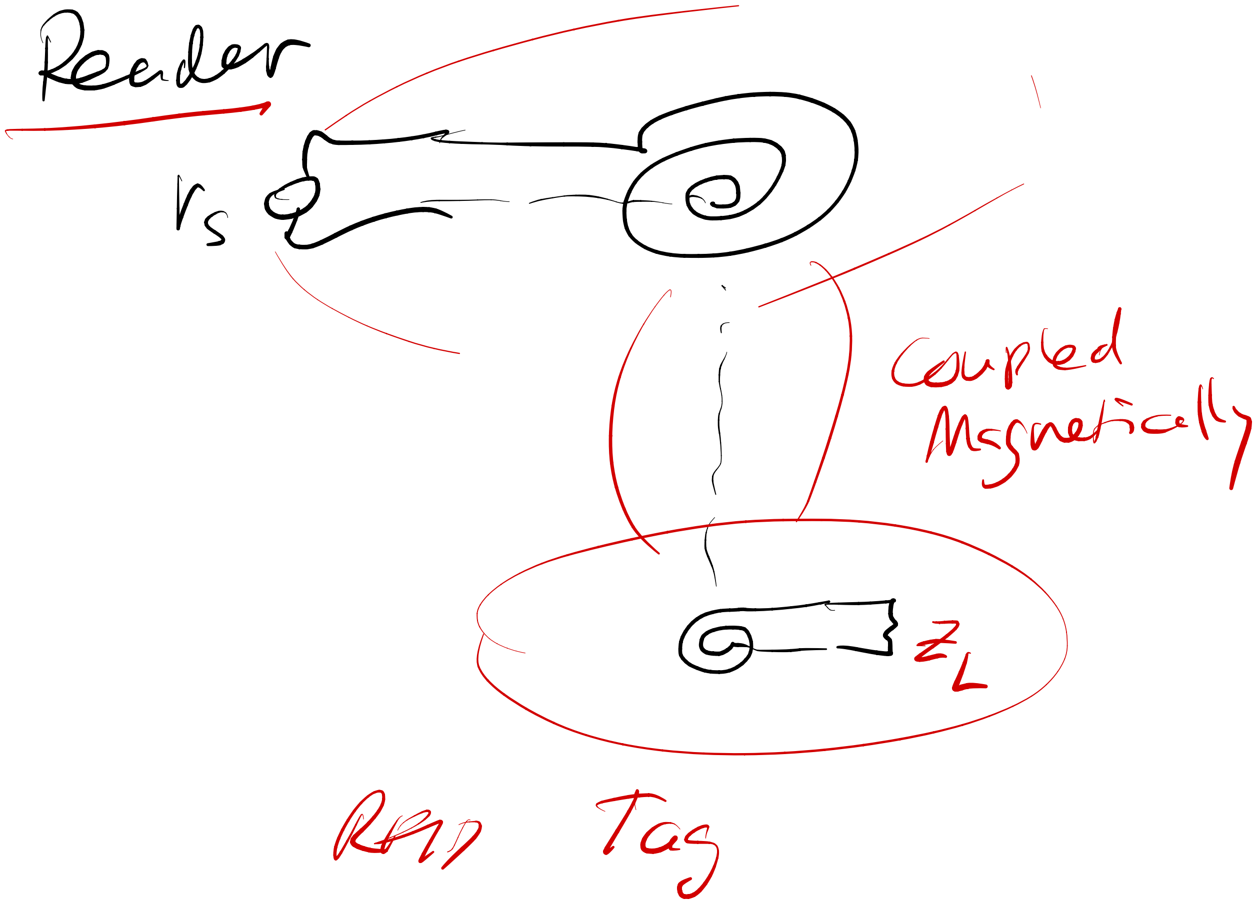
$$R_L = R_S$$

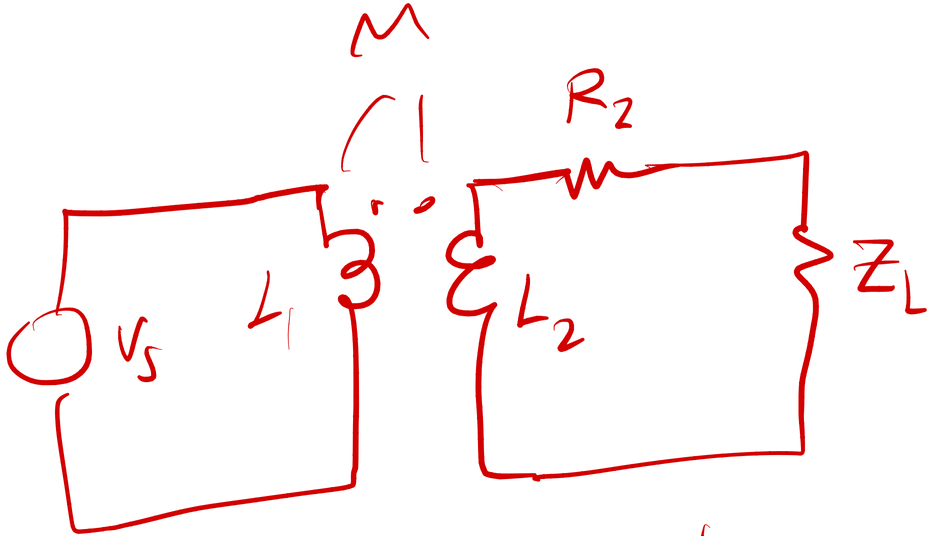
$$X_L = -X_S$$

Max power transfer from the source to the load

Application: Power Matching

- Consider an inductive power link (RFID)



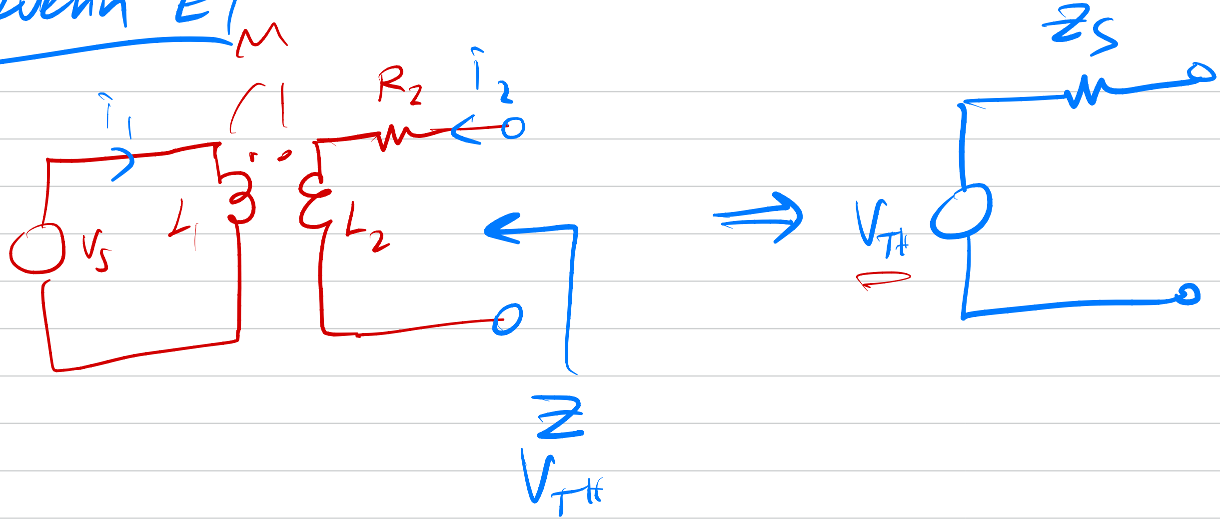


L_1 : reader coil

L_2 : tag coil

M : mutual coupling

Thevenin Eq



$$V_s = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = j\omega L_1 i_1 + j\omega M i_2$$

$$V_2 = j\omega M i_1 + j\omega L_2 i_2$$

$$V_{oc} = V_2 \Big|_{i_2=0} = j\omega M i_1$$

$$V_s = j\omega L_1 i_1 \Big|_{i_2=0}$$

$$i_1 = \frac{V_s}{j\omega L_1}$$

$\underline{V_{TH}}$

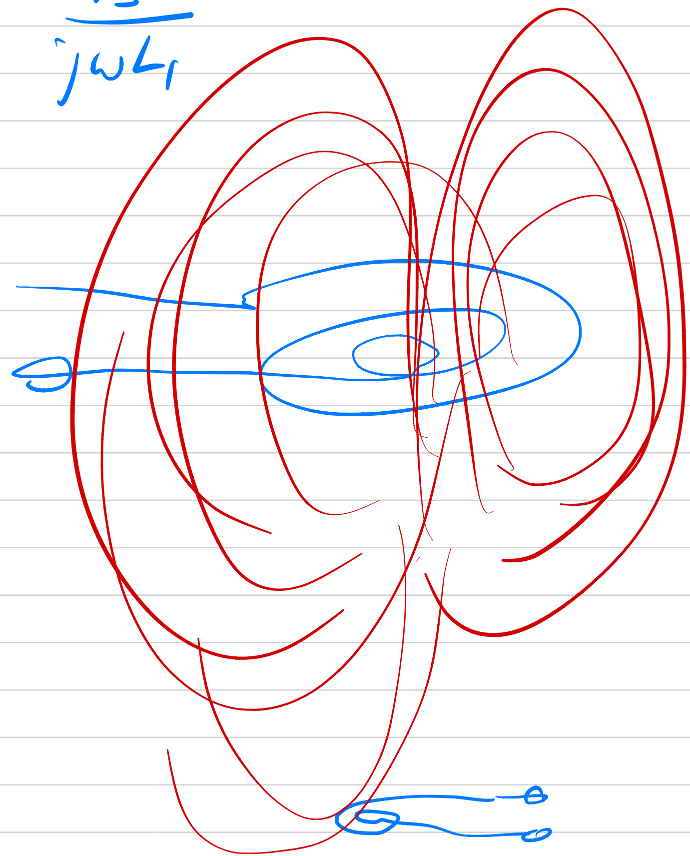
$$V_{oc} = j\omega M i_1 = j\omega M \frac{V_s}{j\omega L_1}$$

$$V_{oc} = \frac{M}{L_1} V_s$$

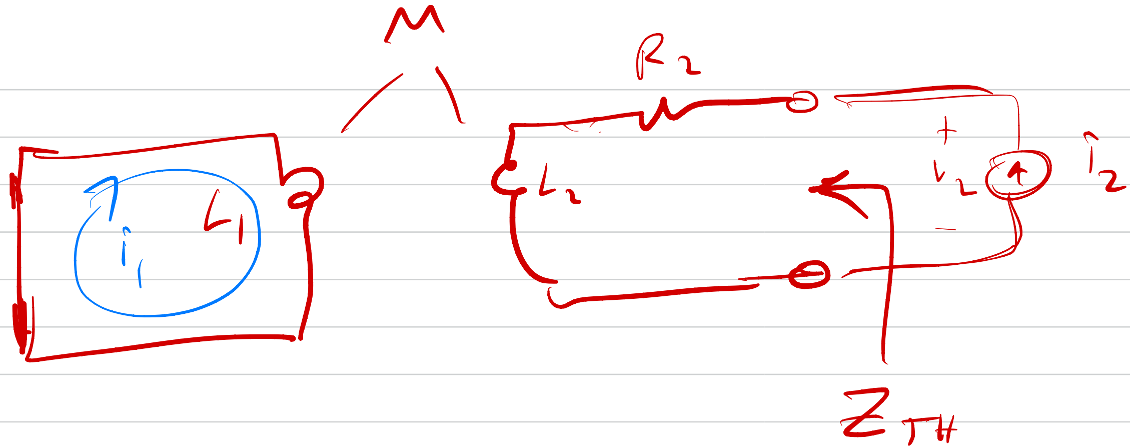
$$M = k\sqrt{L_1 L_2}$$

$$V_{oc} = k \cdot \sqrt{\frac{L_2}{L_1}}\, V_s$$

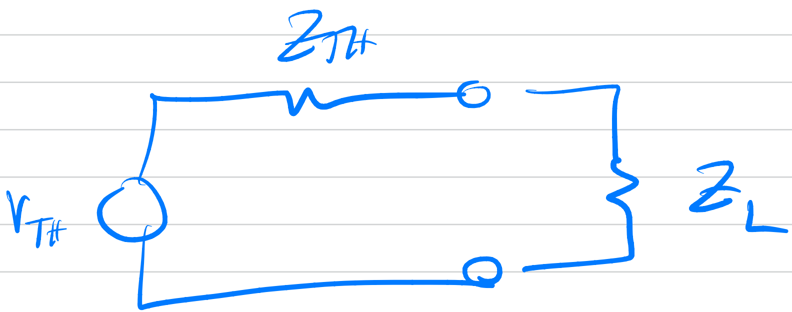
$k \ll 1$ $\qquad k = 10^{-3}$

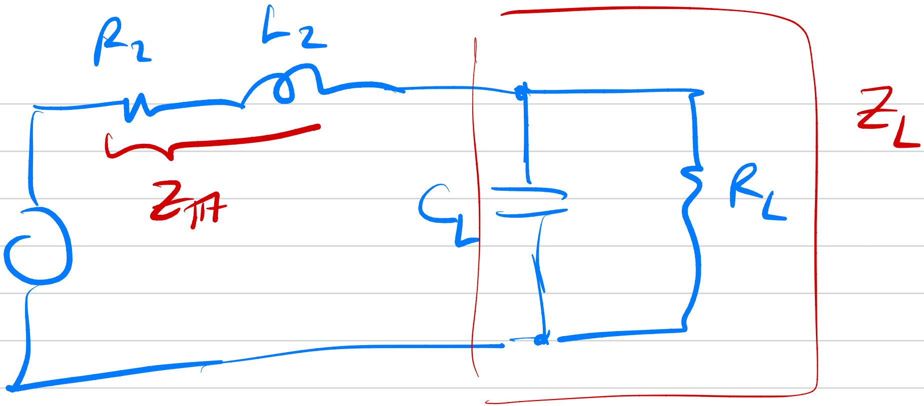


Z_{TH}



$$Z_{TH} \approx R_2 + j\omega L_2$$





$$Z_L = Z_{TH}^* = \frac{R_L}{1+j\omega R_L C_L} = \frac{R_L(1-j\omega R_L C_L)}{1+\omega^2 R_L^2 C_L^2}$$

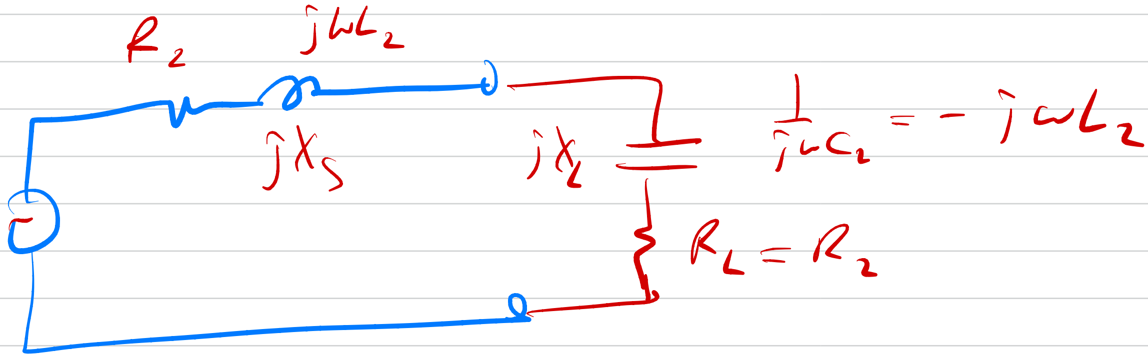
$$\frac{R_L}{1+\omega^2 R_L^2 C_L^2} = R_2$$

$$\frac{+\omega R_L^2 C_L}{1+\omega^2 R_L^2 C_L^2} = +\omega L_2$$

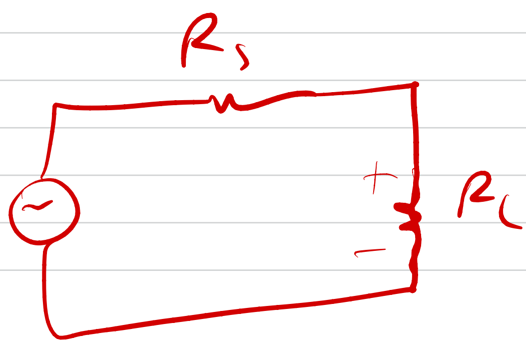
Reel Part

Im Part

Why not series?



$\Downarrow$



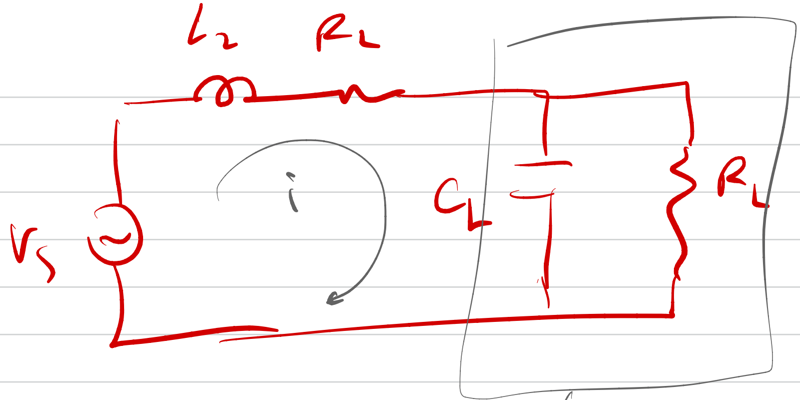
Valid under conjugate match at some ω

$$\frac{R_L}{1+\omega^2 R_L^2 C_L^2} = R_2 \qquad \frac{+\omega R_L^2 C_L}{1+\omega^2 R_L^2 C_L^2} = +\omega L_2$$

$$Q \triangleq \frac{\omega L_2}{R_2} = \frac{\omega R_L^2 C_L}{R_L} = \omega C_L R_L$$

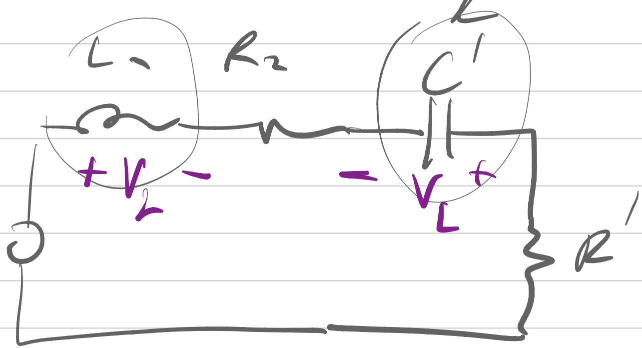
$$\frac{R_L}{1+Q^2} = R_2 \qquad \frac{\omega^2 C_L^2 R_L^2 \frac{1}{\omega C_L}}{1+Q^2} = \omega L_2$$

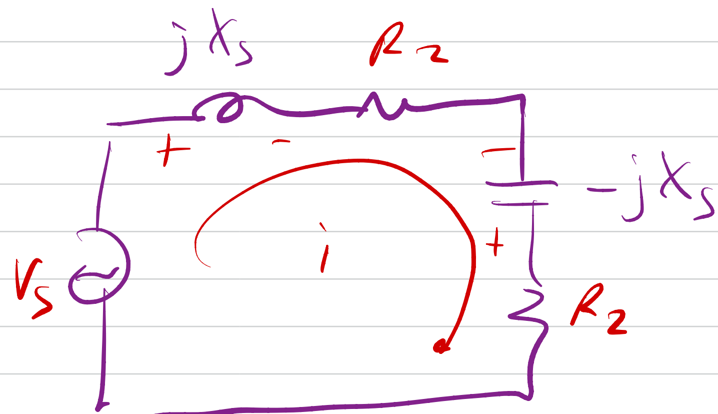
$$\boxed{R_L = R_2 (1+Q^2)} \qquad \boxed{\frac{1}{\omega C_L} = \omega L_2 \frac{(1+Q^2)}{Q^2}}$$



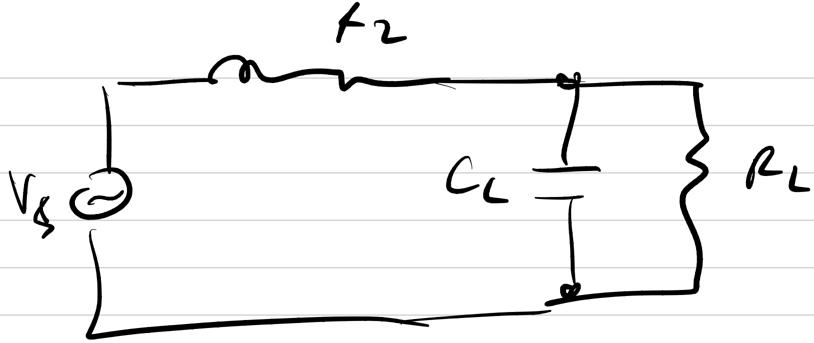
$$i = \frac{V_s}{2R_2}$$

$$X_L = -X_s$$





$$i = \frac{V_s}{2R_2} \quad \text{At} \quad \omega$$



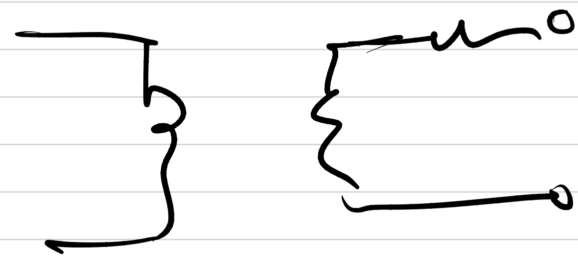
$$V_L = i \cdot Z_L = \frac{V_S}{2R_2} \cdot \frac{R_L}{1 + j\omega C_L R_L}$$

$$\left|\frac{V_L}{V_S}\right| = \frac{R_L}{2R_2}\sqrt{\frac{1}{1+\omega^2 C_L^2 R_L^2}}$$

$$= \frac{\cancel{R_2}(1+Q^2)}{2\cancel{R_2}} \frac{1}{\sqrt{1+Q^2}}$$

$$\left|\frac{v_L}{v_s}\right| = \frac{1}{2}\,\frac{1+Q^2}{\sqrt{1+Q^2}} = \frac{1}{2}\sqrt{1+Q^2}$$

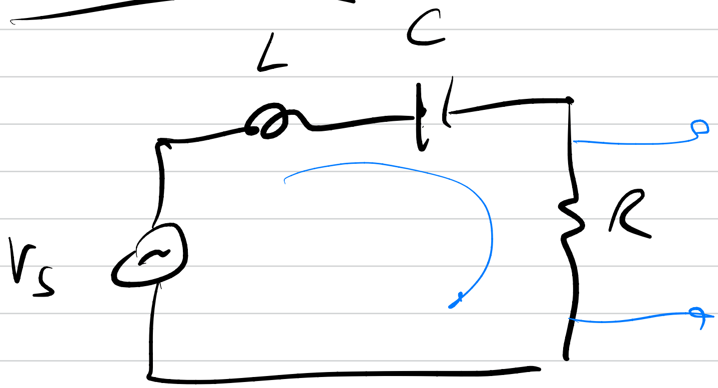
$$\boxed{Q = \frac{\omega L_2}{R_2}}$$



Say $Q \sim 100$

$$\left|\frac{v_L}{v_s}\right| \simeq \frac{1}{2}Q \sim 50\text{mV}$$

Resonance



$$Z = j\omega L + \frac{1}{j\omega C} + R$$

$$V_R = \frac{V_s}{Z} \cdot R \Rightarrow H_R = \frac{V_R}{V_s} = \frac{R}{Z} = \frac{R}{R + \frac{1}{j\omega C} + j\omega L}$$

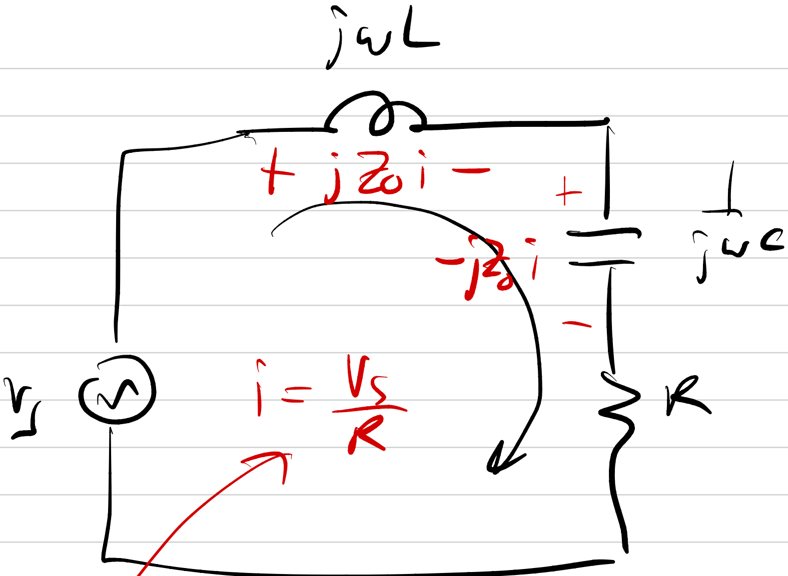
$$H_R(j\omega) = \frac{j\omega CR}{1 + j\omega RC + (j\omega L)(j\omega C)}$$

$$\boxed{H_R(j\omega) = \frac{j\omega RC}{(1 - \omega^2 LC) + j\omega RC}}$$ ✓ ✓

$$H_R(j\omega_0) = \frac{j\omega_0 RC}{\underbrace{(1-\omega_0^2 LC)}_{=0} + j\omega RC} = 1$$

$$\omega_0^2 LC = 1 \qquad \omega_0 = \sqrt{\frac{1}{LC}}$$

[Natural undamped resonant frequency]



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Z_L = j\omega_0 L = j\frac{L}{\sqrt{LC}} = j\underbrace{\sqrt{\frac{L}{C}}}_{Z_0} = jZ_0$$

$$Z_C = \frac{1}{j\omega_0 C} = \frac{\sqrt{LC}}{jC} = \frac{1}{j}\sqrt{\frac{L}{C}} = -j\sqrt{\frac{L}{C}}$$

$$= -jZ_0$$

at resonance

$$H_C(j\omega_0) = i \cdot \frac{1}{j\omega_0 C} = \frac{V_s}{R} \cdot \frac{1}{j\omega_0 C}$$

$$= V_s\,(-j)\,\frac{1}{\omega_0 RC}$$

$$= V_s\,(-j)\,\frac{\sqrt{LC}}{RC} \qquad Z_0 = \sqrt{\frac{L}{C}}$$

$$= V_s\,(-j)\left(\frac{Z_0}{R}\right)$$

$Z_0 = 1k\Omega \qquad R = 1\Omega$

Quality Factor

Resonance

$$\boxed{\omega_0 L = \frac{1}{\omega_0 C}}$$

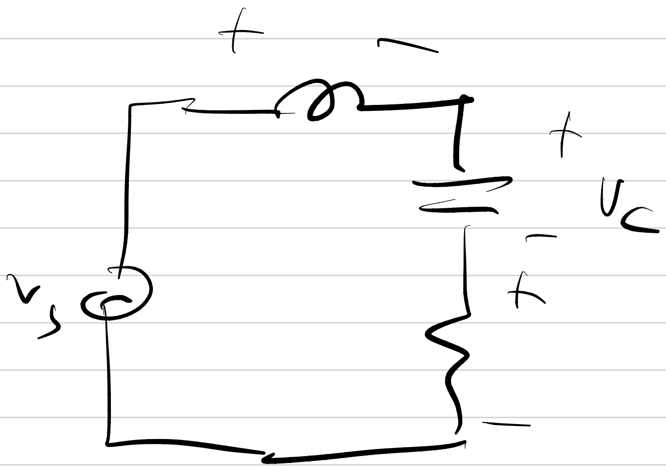
$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$

$$H_R(j\omega) = \frac{j\omega CR}{1 + j\omega RC + (j\omega L)(j\omega C)}$$

$$H_R(j\omega) = \frac{j\frac{\omega}{\omega_0}(\omega_0 RC)}{1 + j\frac{\omega}{\omega_0}(\omega_0 RC) + \left(j\frac{\omega}{\omega_0}\right)^2}$$

$$H_R(j\omega) = \frac{j\frac{\omega}{Q\omega_0}}{1 + j\frac{\omega}{Q\omega_0} + \left(j\frac{\omega}{\omega_0}\right)^2}$$

$$H_R(j\omega) = \frac{j\frac{\omega}{Q\omega_0}}{1 + j\frac{\omega}{Q\omega_0} + \left(\frac{j\omega}{\omega_0}\right)^2} = \frac{N(j\omega)}{D(j\omega)}$$



$$D(j\omega) = (j\omega + p_1)(j\omega + p_2) \cdots$$

Eigenvalues of the State-space rep. of the circuit

$$H_R(j\omega) = \frac{j\frac{\omega}{Q\omega_0}}{1 + \underbrace{j\frac{\omega}{Q\omega_0}}_{\text{real}} + \left(j\frac{\omega}{\omega_0}\right)^2}$$

$$\zeta = \frac{1}{2Q}$$

$$1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2 = 0$$

$$\omega_0^2 + \frac{s\omega_0}{Q} + s^2 = 0$$

$$s = -\frac{\omega_0}{2Q} \pm \frac{\sqrt{\left(\frac{\omega_0}{Q}\right)^2 - 4\omega_0^2}}{2}$$

$$\boxed{s^{+/-} = -\frac{\omega_0}{2Q} \pm \frac{\omega_0}{2}\sqrt{\left(\frac{1}{Q}\right)^2 - 4}}$$

$$\boxed{s^{+/-} = -\frac{\omega_0}{2Q} \pm \frac{\omega_0}{2}\underbrace{\sqrt{\left(\frac{1}{Q}\right)^2 - 4}}_{<0}}$$

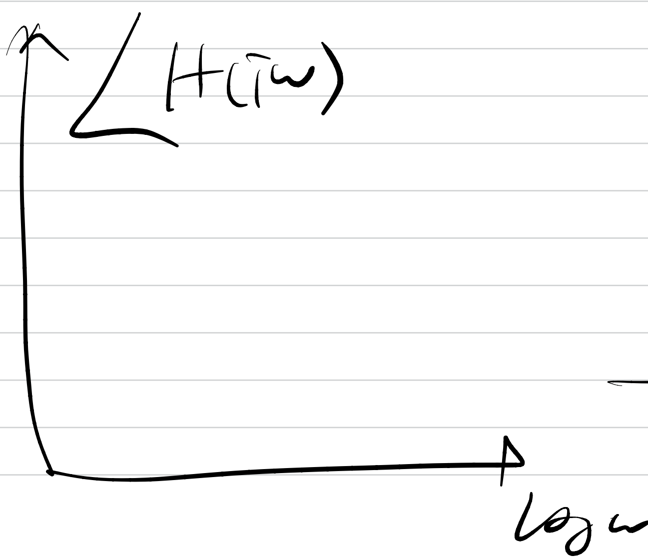
$$s^{+/-} = -\frac{\omega_0}{2Q} \pm \frac{\omega_0}{2} j\sqrt{4 - 1/Q^2}$$

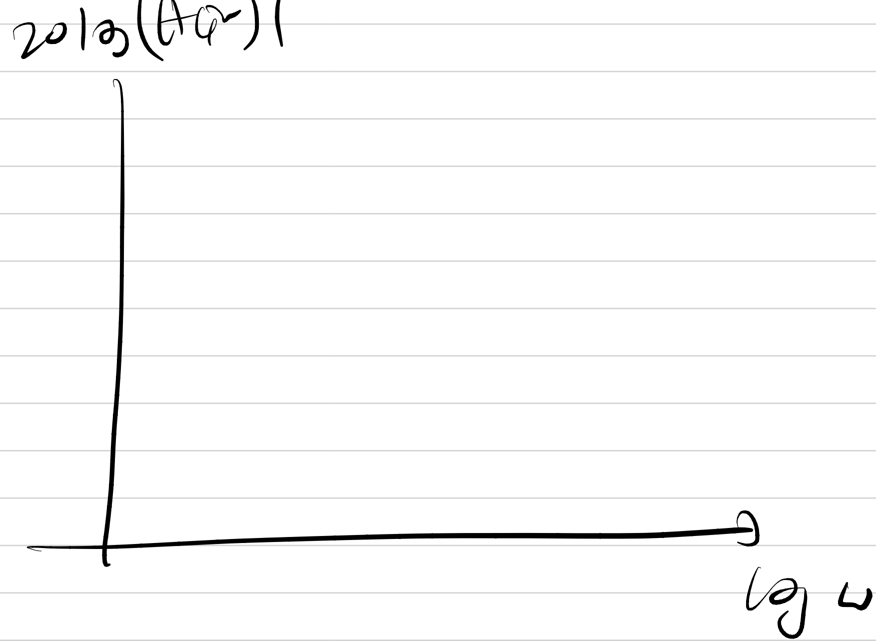
$$= -\frac{\omega_0}{2Q} \pm \omega_0 j \sqrt{1 - \underbrace{\frac{1}{(2Q)^2}}_{\zeta^2}}$$

$$= -\frac{\omega_0}{2Q} \pm \underbrace{\omega_0 j\sqrt{1 - \zeta^2}}$$

Bode Plots

$$10 \cdot \log\left\{ \left| H(j\omega) \right|^2 \right\} = 20 \log(H(j\omega))$$





$$H(j\omega) = \frac{N(j\omega)}{D(j\omega)} = \frac{(j\omega + z_1)(j\omega + z_2)\cdots}{(j\omega + p_1)(j\omega + p_2)\cdots}$$

$$\log|H(j\omega)| = \log \frac{|j\omega + z_1|\,|j\omega + z_2|\cdots}{|j\omega + p_1|\,|j\omega + p_2|\cdots}$$

$$= \log|z_1 + j\omega| + \log|z_2 + j\omega| + \cdots$$

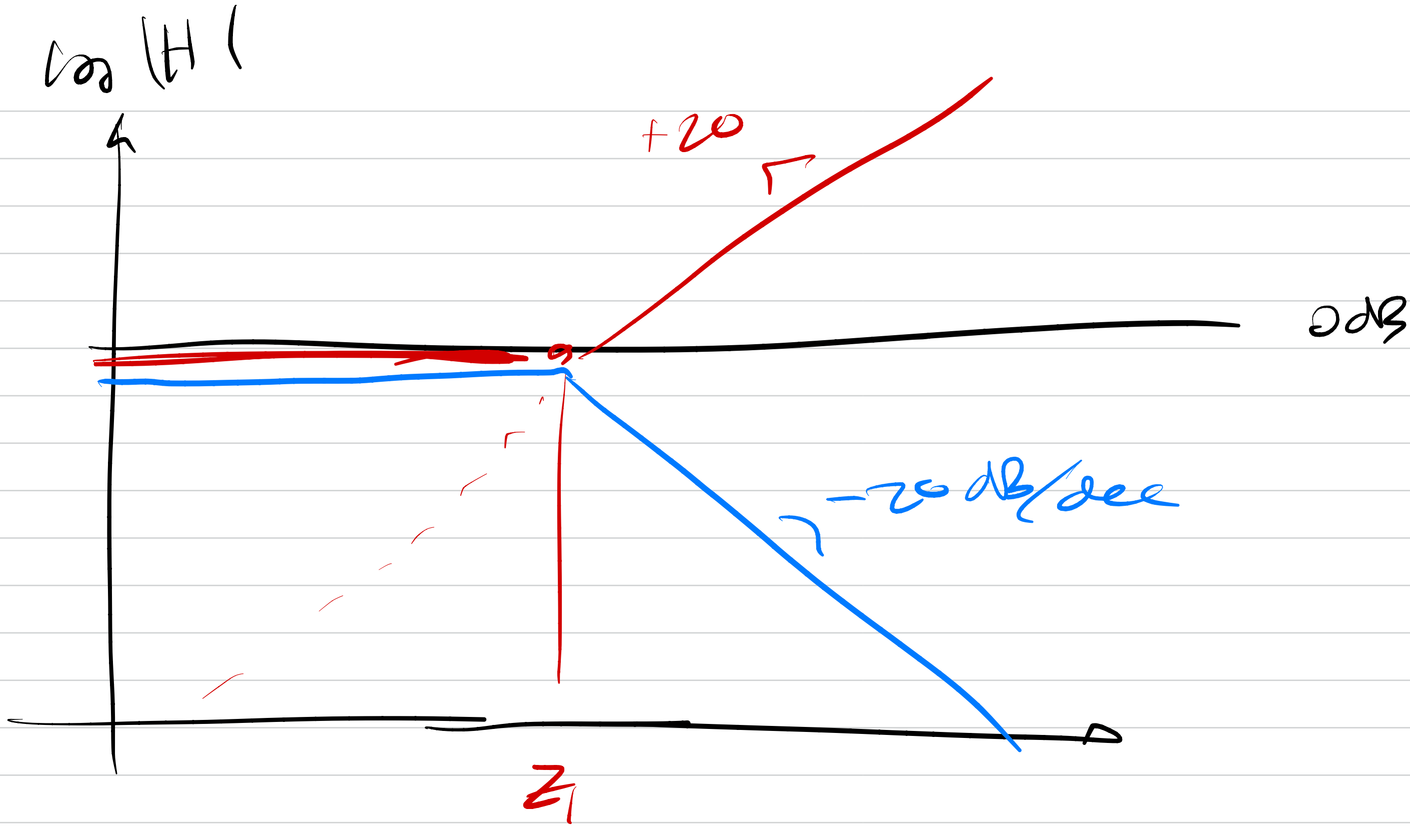
$$- \log|j\omega + p_1| - \log|j\omega + p_2| + \cdots$$

$$H(j\omega) = \frac{\left(1 + \frac{j\omega}{z_1}\right)\left(1 + \frac{j\omega}{z_2}\right) \cdots}{\left(1 + \frac{j\omega}{p_1}\right)\left(1 + \frac{j\omega}{p_2}\right) \cdots} K$$

$$20 \log \left| 1 + \frac{j\omega}{z_1} \right| = \begin{cases} 0 \text{ dB} & \frac{\omega}{z_1} \ll 1 \\ 20 \log \omega & \frac{\omega}{z_1} \gg 1 \end{cases}$$

$\frac{\omega}{z_1} \gg$

$$20 \log \left| \frac{\omega}{z_1} \right|$$


log |H|
+20
0dB
-20dB/dec
Z1