EECS 16B Designing Information Devices and Systems II
Summer 2020 UC Berkeley

Signals Review

1. DFT Properties

(a) Show that the k^{th} frequency component of a length N signal $x[n]$ can be written as

$$X[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

(b) Given the DFT $X[k]$ of a time domain signal $x[n]$, show that

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn}$$

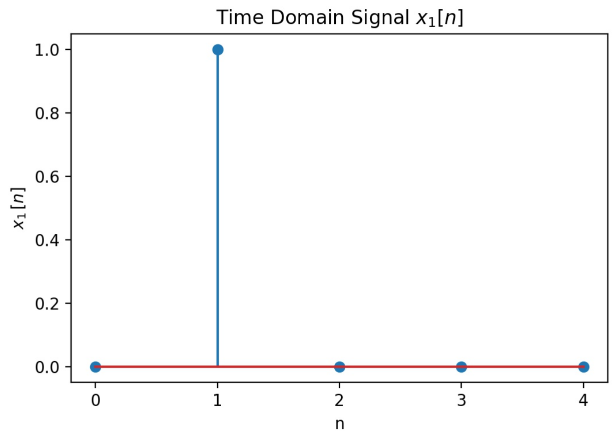
(c) Prove that if $x[n]$ is a real valued signal, $X[k] = \overline{X[N-k]}$.

(d) Prove that if $x[n]$ is real and $x[n] = x[N-n]$, then all of the DFT coefficients $X[k]$ are real.

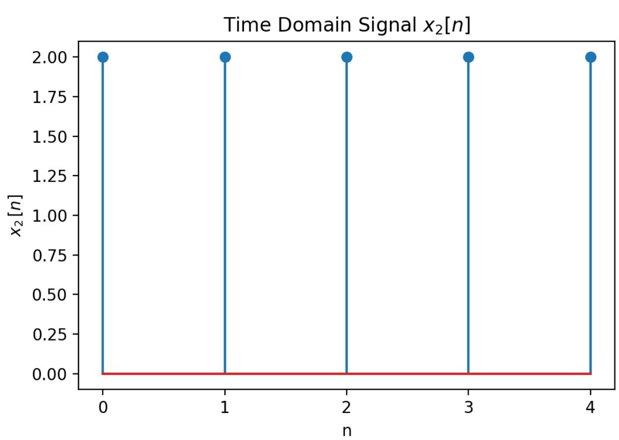
2. DFT Basics

Compute the 5 point DFT of the following signals

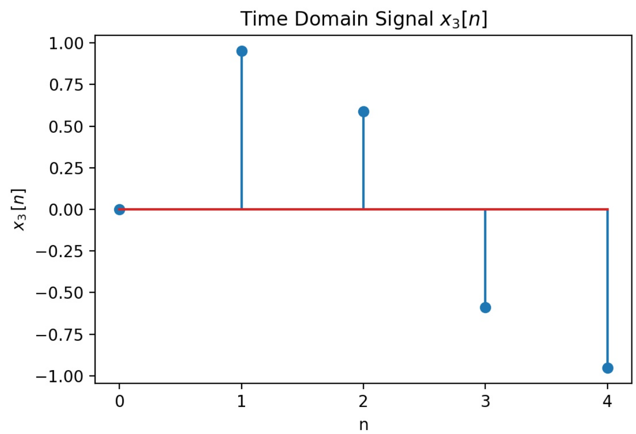
(a) $x_1[n] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}$.



(b) $x_2[n] = \begin{bmatrix} 2 & 2 & 2 & 2 & 2 \end{bmatrix}$.

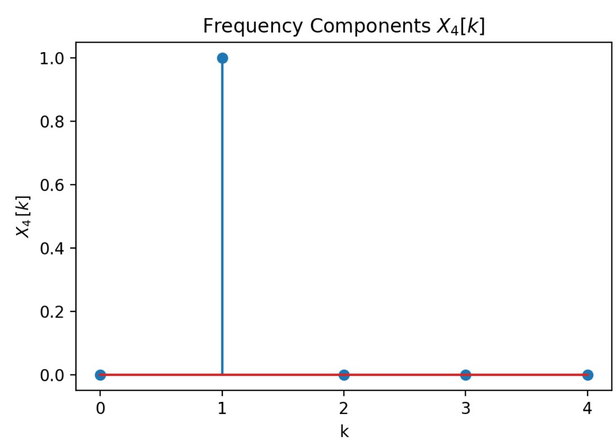


(c) $x_3[n] = \sin\left(\frac{2\pi}{5}n\right)$.

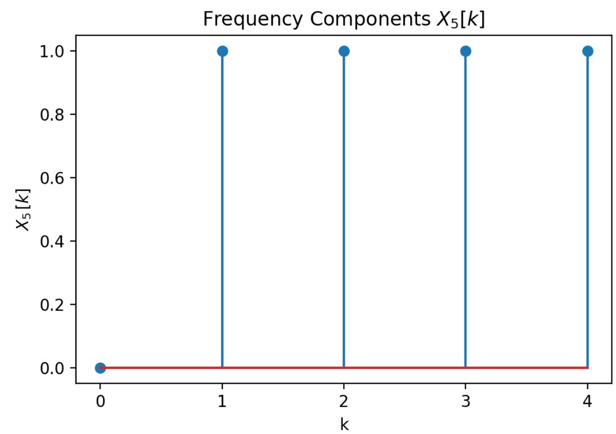


Now compute the 5 point inverse DFT given the following frequency components

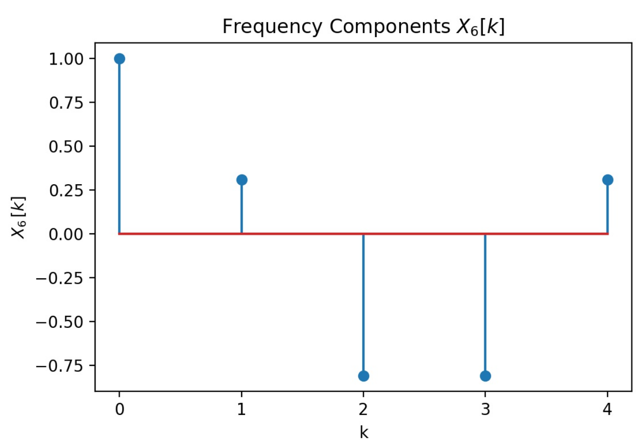
(d) $X_4[k] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}$.



(e) $X_5[k] = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \end{bmatrix}$.

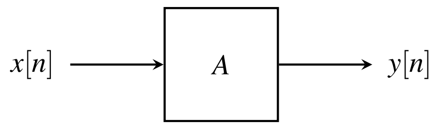


(f) $X_6[k] = \cos\left(\frac{2\pi}{5}k\right)$.



3. DFT and Finite Sequences (X points)

Consider a system $A\{\vec{x}\}$ which operates on length-8 sequences.



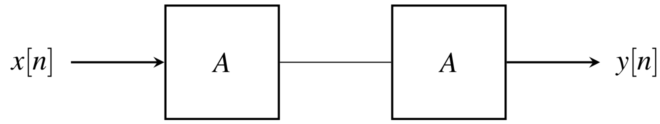
This system:

1) computes the DFT_8 of the sequence,
2) multiplies the first 4 elements ($k = 0,1,2,3$) by $-j$ and the next 4 elements ($k = 4,5,6,7$) by j, and
3) computes the IDFT_8 of the result.

(a) **Is the system linear?**

(b) The system is applied on an input sequence $x[n] = \sin\left(\frac{\pi}{4}n\right)$, $0 \leq n < 8$. **What is $y[n]$, the output of the system?** Full credit will only be given to the simplest expression.

(c) We apply two such systems in series to an *arbitrary sequence* $x[n]$, $0 \leq n < 8$:



Express $y[n]$ **in terms of** $x[n]$**.** Full credit will only be given to the simplest expression.

4. Integration by Convolution

Consider the following system that acts as a discrete-time integrator.

$$y[n] - y[n-1] = x[n] \tag{1}$$

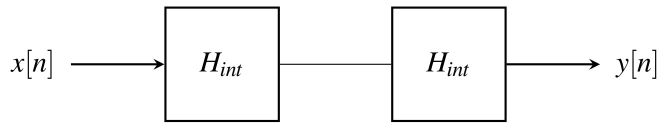
We will assume that $y[n] = 0$ for $n < 0$.

(a) **Show that this system is LTI.**

(b) **What is the system's imuplse response?**

(c) Suppose we input the unit step $x[n] = u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$. **What is the output** $y[n]$**?**

(d) Now let's create a new system of the following model



where each H_{int} represents one integrator system. **How can we express the input-output relationship of $x[n]$ and $y[n]$?**

(e) **What is the impulse response of this new system?**

(f) If we input $x[n] = u[n]$ to this new system, **what would the output $y[n]$ be?**
Hint: What is the integrator system doing? If you aren't sure, look back at part (c).

5. Stability of State Space Systems (X points)

Consider a discrete time state space system

$$\vec{x}[n+1] = \mathbf{A}\vec{x}[n].$$

For which of the following possible matrices **A** is the system stable? Explain your answers.

(a) **(X pts)**

$$\mathbf{A} = \frac{1}{4}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Stable? Yes / No
Explanation:

(b) **(X pts)**

$$\mathbf{A} = \frac{1}{2}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

Stable? Yes / No
Explanation:

For parts (c) and (d), consider a continuous time system

$$\frac{\mathrm{d}\vec{x}(t)}{\mathrm{d}t} = \mathbf{A}\vec{x}(t).$$

(c) **(X pts)**

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}$$

Stable? Yes / No
Explanation:

(d) **(X pts)** Recall that we are still considering the continuous time system.

$$\mathbf{A} = \begin{bmatrix} -2 & 1 & 0 & -1 \\ -1 & -2 & 1 & 0 \\ 0 & -1 & -2 & 1 \\ 1 & 0 & -1 & -2 \end{bmatrix}$$

Stable? Yes / No
Explanation:

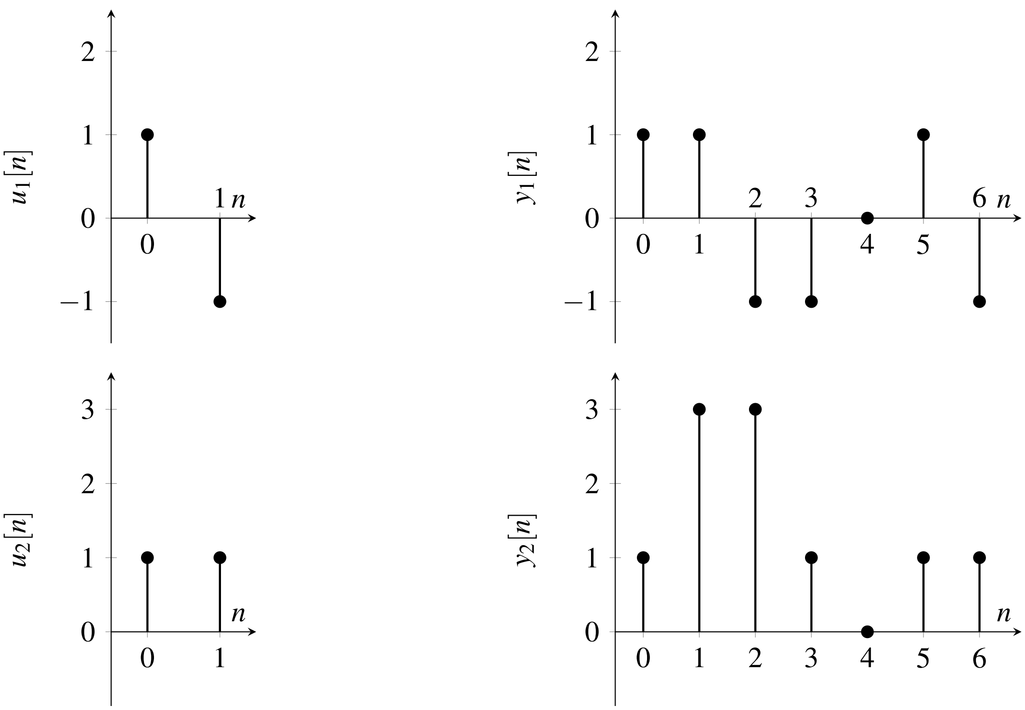
6. Signals and Systems (X points)

Consider a discrete time **observable** system

$$\vec{x}[n+1] = \mathbf{A}\vec{x}[n] + \mathbf{B}u[n]$$
$$y[n] = \mathbf{C}\vec{x}[n],$$

where $\mathbf{A} \in \mathbb{R}^{N\times N}$, $\mathbf{B} \in \mathbb{R}^{N\times 1}$, and $\mathbf{C} \in \mathbb{R}^{1\times N}$ are *unknown*. The system is in the state $\vec{x} = \vec{0}$ before any input is applied and is therefore LTI.

(a) **(X pts)** Given the following input-output pairs $u[n]$ and $y[n]$, what is the impulse response $h[n]$ of the system? Assume that the signals are 0 everywhere else.



(b) **(X pts) Is the system BIBO stable?**

(c) **(X pts)** Given the unit step input $u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$, the system's output eventually reaches a steady state value. **At what time step does the output reach the steady state value and what is the steady state value of the output?**

7. Sampling and Interpolation

Consider a frequency source that produces a signal

$$x(t) = \cos(2\pi f_0 t).$$

This signal is sampled with a sampling interval of T_s [sec] and reconstructed as $\tilde{x}(t)$ using sinc interpolation.

(a) **What are the sampling intervals that will result in a constant $\tilde{x}(t)$ for all t?**

(b) **How quickly must we sample $x(t)$ in order to get a perfect reconstruction?**

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