1 Complex Algebra

a) Express the following values in polar forms: $-1$, $j$, $-j$, $\sqrt{j}$, and $\sqrt{-j}$. Recall $j^2 = -1$.

b) Represent $\sin \theta$ and $\cos \theta$ using complex exponentials. (*Hint:* Use Euler’s identity $e^{j\theta} = \cos \theta + j\sin \theta$.)

c) For complex number $z = x + jy$ show that $|z| = \sqrt{z\bar{z}}$, where $\bar{z}$ is the complex conjugate of $z$. 
For the next two parts, let $A = 1 - j\sqrt{3}$ and $B = \sqrt{3} + j$.

d) Express $A$ and $B$ in polar form.

e) Find $AB, A\bar{B}, \frac{A}{\bar{B}}, A + \bar{A}, A - \bar{A}, \bar{AB}, \bar{A\bar{B}}$, and $\sqrt{B}$.

f) Show the number $A$ in complex plane, marking the distance from origin and angle with real axis.
g) Show that multiplying $A$ with $j$ is equivalent to rotating the magnitude of the complex number by $\pi/2$ or 90 degrees in the complex plane.

h) What are the roots of $z^2 = 1$? What about $z^3 = 1$? How many roots does $z^n = 1$ have? What is the general form for the solutions of $z^n = 1$?