1 Discrete time system responses

We have a system $x[k + 1] = \lambda x[k]$. For each $\lambda$ value plotted on the real-imaginary axis, sketch $x[k]$ with an initial condition of $x[0] = 1$. Determine if each system is stable.

![Diagram of the real-imaginary axis showing points A, B, C, D, E, F with annotations -1.5, -1, -0.5, 0.5, 1, 1.5]
2 Eigenvalues Placement in Discrete Time

Consider the following linear discrete time system

\[
\tilde{x}[t + 1] = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \tilde{x}[t] + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u[t] + \tilde{w}[t] \tag{1}
\]

(a) Is this system controllable?

(b) Is the linear discrete time system stable?

c) Derive a state space representation of the resulting closed loop system using state feedback of the form \( u[t] = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \tilde{x}[t] \)

d) Find the appropriate state feedback constants, \( k_1, k_2 \) in order the state space representation of the resulting closed loop system to place the eigenvalues at \( \lambda_1 = -\frac{1}{2}, \lambda_2 = \frac{1}{2} \)
e) Suppose that instead of \( \begin{bmatrix} 1 \\ 0 \end{bmatrix} u[t] \) in (1), we had \( \begin{bmatrix} 1 \\ 1 \end{bmatrix} u[t] \) as the way that the discrete-time control acted on the system. Is this system controllable from \( u[t] \)?

f) For the part above, suppose we used \( [k_1, k_2] \) to try and control the system. What would the eigenvalues be? Can you move all the eigenvalues to where you want? Give an intuitive explanation of what is going on.
3 Eigenvalue Placement

Consider the following linear discrete time system

\[
\tilde{x}(t + 1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -9 & -6 \end{bmatrix} \tilde{x}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)
\]

a) What are the eigenvalues of the \( A \) matrix? Is this system stable?

b) Using state feedback \( u(t) = K\tilde{x}(t) = \begin{bmatrix} k_0 & k_1 & k_2 \end{bmatrix} \tilde{x}(t) \) place the eigenvalues at \( 0, 1/2, -1/2 \).

c) Suppose we now have a limitation on how much our controller can amplify \( \tilde{x} \) and all of our \( k \) values must be in between \(-5\) and \(5\). Is it possible to pick a set of eigenvalues that will make the system stable?