Discussion 3C

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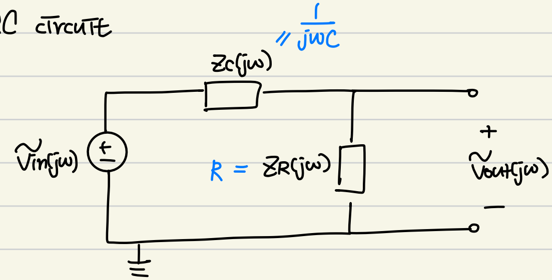
EECS 16B Su22

Q1. $H(j\omega) = \dfrac{\tilde{V}_{out}}{\tilde{V}_{in}}$ ← phasor domain

a) RC circuit



In phasor domain, we treat every element like a resistor
∴ can get $\tilde{V}_{out}$ using voltage divider

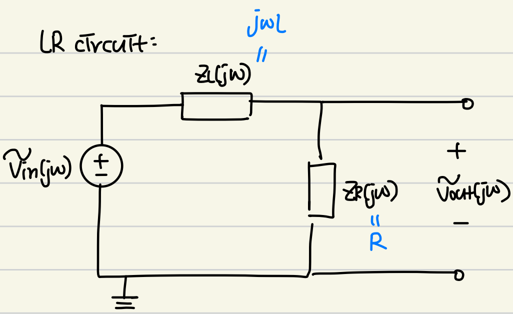
$$\tilde{V}_{out} = \underbrace{\boxed{\frac{Z_R}{Z_R + Z_C}}}_{= H(j\omega)} \tilde{V}_{in}$$

$$\therefore H(j\omega) = \frac{Z_R}{Z_C + Z_R} = \frac{R}{\frac{1}{j\omega C} + R} = \frac{j\omega RC}{1 + j\omega RC}$$

$$\left.\begin{aligned} \omega \to 0, &\quad |H(j\omega)| = 0 \\ \omega \to \infty, &\quad |H(j\omega)| \to 1 \end{aligned}\right\} \text{High Pass Filter}$$

(zero out the low frequency and maintains the high frequency)

b) LR circuit:

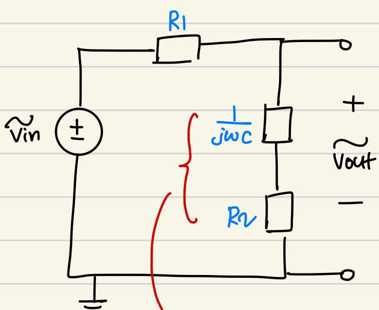


By voltage divider, $\tilde{V}_{out} = \frac{Z_R}{Z_L + Z_R} \tilde{V}_{in}$

$H(j\omega) = \frac{R}{j\omega L + R}$

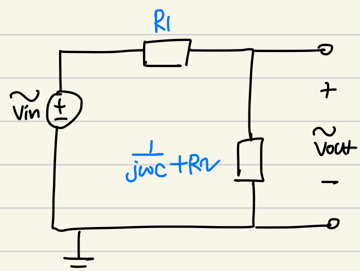
$\left.\begin{array}{l} \omega \to 0,\ |H(j\omega)| = 1 \\ \omega \to \infty,\ |H(j\omega)| \to 0 \end{array}\right\}$ Low-Pass Filter

c) RCR circuit:



In series ⊕ treat like resistors

$\Leftrightarrow$



$\therefore H(j\omega) = \frac{R_2 + \frac{1}{j\omega C}}{R_1 + R_2 + \frac{1}{j\omega C}} = \frac{j\omega R_2 C + 1}{j\omega (R_1 + R_2) C + 1}$

$\left.\begin{array}{l} \omega \to 0,\ |H(j\omega)| \to 1 \\ \omega \to \infty,\ |H(j\omega)| \to \frac{R_2}{R_1 + R_2} \end{array}\right\}$ No name for this filter

d) $R = \overset{1000}{\overset{/\!/}{1k\Omega}}$, $L = 25\mu H$, $C = \overset{10^{-5}}{\overset{/\!/}{10\mu F}}$, $\omega = 100\, rad/s$

From part a) : $H(j\omega) = \dfrac{j\omega RC}{1+j\omega RC}$

$V_{in}(t) = 12\sin(\omega_{in} t) = 12\sin(100t)$

① Convert to phasor domain

$V_o\cos(\omega t+\phi) \Longleftrightarrow \frac{V_o}{2}e^{j\phi}$

$V_{in}(t) = 12\sin(100t) = 12\cos(100t-\frac{\pi}{2})$ $\qquad \sin\theta = \cos(\theta - \frac{\pi}{2})$

$\therefore \tilde{V}_{in} = \frac{12}{2}e^{j\,-\frac{\pi}{2}} = 6e^{-j\frac{\pi}{2}}$

② Calculate $\tilde{V}_{out}$

$\tilde{V}_{out} = H(j\omega)\tilde{V}_{in}$

$$\tilde{V}_{out} = \left(\frac{j\omega RC}{1+j\omega RC}\right)\tilde{V}_{in} = \left(\frac{j\,10^2\cdot 10^3\cdot 10^{-5}}{1+j\,10^2\cdot 10^3\cdot 10^{-5}}\right)\tilde{V}_{in} = \frac{j}{\underbrace{1+j}_{\text{not in the polar form}}}\tilde{V}_{in}$$

③ Change $H(j\omega)$ into polar form

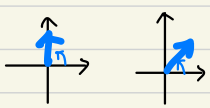
i.e. $H(j\omega) \rightarrow |H(j\omega)|e^{j\measuredangle H(j\omega)}$

1) magnitude

$$|H(j\omega)| = \left|\frac{j}{1+j}\right| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

2) phase

$$\measuredangle H(j\omega) = \measuredangle(j) - \measuredangle(1+j) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$



$$\Rightarrow H(j\omega) = \frac{1}{\sqrt{2}} e^{j\frac{\pi}{4}}$$

$$\tilde{V}_{out} = \left(\frac{1}{\sqrt{2}} e^{j\frac{\pi}{4}}\right)\left(6 e^{-j\frac{\pi}{2}}\right)$$

$$= 3\sqrt{2}\, e^{-j\frac{\pi}{4}}$$

④ Change back to time domain

$V_0 \cos(\omega t + \phi) \Longleftrightarrow \frac{V_0}{2} e^{j\phi}$ ← again use the formula from step①

$$\tilde{V}_{out} = \underbrace{3\sqrt{2}}_{\frac{V_0}{2}}\, e^{\overbrace{-j\frac{\pi}{4}}^{j\phi}}$$

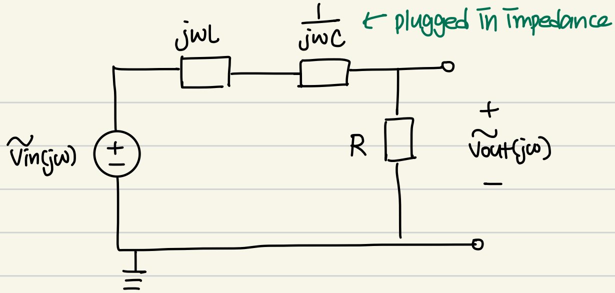
$$\Rightarrow V_{out}(t) = V_0 \cos(\omega t + \phi) = 6\sqrt{2} \cos\left(100t - \frac{\pi}{4}\right)$$

In summary,

$$\begin{cases} V_{in}(t) = V_0 \cos(\omega t + \phi) = 12 \cos\left(100t - \frac{\pi}{2}\right) \\ V_{out}(t) = 6\sqrt{2} \cos\left(100t - \frac{\pi}{4}\right) = \underbrace{\frac{1}{\sqrt{2}}}_{= |H(j\omega)|} \cdot 12 \cos\left(100t - \frac{\pi}{2} + \overbrace{\frac{\pi}{4}}^{= \angle H(j\omega)}\right) \end{cases}$$

$$\Rightarrow V_{out}(t) = \underbrace{|H(j\omega)|}_{\text{scaled by } |H(j\omega)|} V_0 \cos\left(\omega t + \phi + \underbrace{\angle H(j\omega)}_{\text{shifted by } \angle H(j\omega)}\right) \text{ at a given } \omega$$

Q2.



a) By voltage divider,

$$H(j\omega) = \frac{R}{j\omega L + \frac{1}{j\omega C} + R} = \frac{R}{R + j(\omega L - \frac{1}{\omega C})}$$

b) $$\underbrace{Z_{RLC}}_{\downarrow} = Z_R + Z_L + Z_C = j\omega L + \frac{1}{j\omega C} + R = \underbrace{R}_{A(\omega)} + j\underbrace{(\omega L - \frac{1}{\omega C})}_{X(\omega)}$$

Treat Impedance as resistance
→ add them up if they're in series

$X(\omega_n) = 0 \Rightarrow \omega L - \frac{1}{\omega C} = 0$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 LC = 1$$

$$\omega^2 = \frac{1}{LC}$$

$$\therefore \omega = \sqrt{\frac{1}{LC}} \quad (\omega \geq 0)$$

$$\therefore \omega_n = \frac{1}{\sqrt{LC}}$$

c) $H(j\omega) = \dfrac{R}{R + j(\omega L - \frac{1}{\omega C})}$

$$|H(j\omega)| = \left| \frac{R}{R + j(\omega L - \frac{1}{\omega C})} \right| = \frac{|R|}{|R + j(\omega L - \frac{1}{\omega C})|}$$

$$= \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

① $\omega = \omega_n$

$$|H(j\omega_n)| = \frac{R}{\sqrt{R^2 + (\omega_n L - \frac{1}{\omega_n C})^2}} = \frac{R}{\sqrt{R^2}} = \frac{R}{R} = 1$$

only term that's dependent on ω_n

$\omega_n L - \dfrac{1}{\omega_n C} = X(\omega_n) = 0$ from part (b)

② $\omega = \dfrac{\omega_n}{10}$

$$X\left(\frac{\omega_n}{10}\right) = X\left(\frac{1}{10\sqrt{LC}}\right)$$

$$= \frac{L}{10\sqrt{LC}} - \frac{10\sqrt{LC}}{C}$$

$$= \frac{\sqrt{L}}{10\sqrt{C}} - \frac{10\sqrt{L}}{\sqrt{C}} = \frac{\sqrt{L} - 100\sqrt{L}}{10\sqrt{C}} = -99 \cdot \frac{1}{10}\sqrt{\frac{L}{C}} = -9.9\sqrt{\frac{L}{C}}$$

③ $\omega = 10\omega_n$

$$X(10\omega_n) = X\left(\frac{10}{\sqrt{LC}}\right) = \frac{10L}{\sqrt{LC}} - \frac{\sqrt{LC}}{10C} = \frac{10\sqrt{L}}{\sqrt{C}} - \frac{\sqrt{L}}{10\sqrt{C}}$$

$$= \frac{100\sqrt{L} - \sqrt{L}}{10\sqrt{C}} = \frac{99}{10}\sqrt{\frac{L}{C}} = 9.9\sqrt{\frac{L}{C}}$$

In summary,

$$\begin{cases} \omega = \omega_n & X(\omega) = 0 \\ \omega = \frac{\omega_n}{10} & X(\omega) = -9.9\sqrt{\frac{L}{C}} \\ \omega = 10\omega_n & X(\omega) = 9.9\sqrt{\frac{L}{C}} \end{cases}$$

$$|H(j\omega)| = \frac{R}{\sqrt{R^2 + X(\omega)^2}}$$

$$X\left(\frac{\omega_n}{10}\right)^2 = X(10\omega_n)^2 = \left(9.9\sqrt{\frac{L}{C}}\right)^2 = 9.9^2\frac{L}{C}$$

$$\therefore \ \left|H\left(j\frac{\omega_n}{10}\right)\right| = |H(j\cdot 10\omega_n)|$$

$$= \frac{R}{\sqrt{R^2 + \underbrace{9.9^2\frac{L}{C}}_{>0}}} < 1$$

$$\Rightarrow \ |H(j\omega_n)| > |H(j\omega_n/10)| = |H(j10\omega_n)|$$

$\Rightarrow$ "lets through" or maintains signal at $\omega = \omega_n$, and attenuates every other signal.