Discussion 3D

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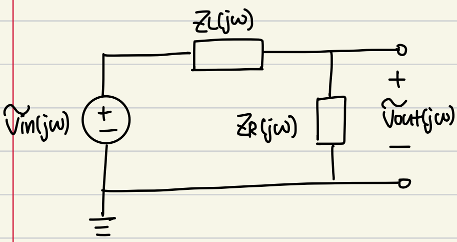
EECS 16B Su22

Q1.

a)



$$H(j\omega) = \frac{Z_R}{Z_R + Z_L} = \frac{R}{R + j\omega L}$$

$$|H(j\omega)| = \left|\frac{R}{R + j\omega L}\right| = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

$$\measuredangle H(j\omega) = \measuredangle R - \measuredangle(R + j\omega L)$$
$$= 0 - \text{atan2}(\omega L, R)$$
$$= -\text{atan2}(\omega L, R)$$

Magnitude: $\omega = 0$, $|H(j\omega)| = \frac{R}{\sqrt{R^2}} = 1$

$\omega \to \infty$, $|H(j\omega)| \to 0$

Phase: $\omega = 0$, $\measuredangle H(j\omega) = -\text{atan2}(0, R) = 0$

$\omega \to \infty$, $\measuredangle H(j\omega) = -\text{atan}(\omega L, R)$

$$= -\tan^{-1}\Big(\underbrace{\frac{\omega L}{R}}_{\to\infty}\Big) = -\frac{\pi}{2}$$

b) $H(j\omega) = \dfrac{R}{R+j\omega L}$

$|H(j\omega)| = \dfrac{R}{\sqrt{R^2+(\omega L)^2}} = \dfrac{1}{\sqrt{2}}$ at cutoff frequency $\omega = \omega_c$

$\Rightarrow (\omega_c L)^2 = R^2 \Rightarrow \omega_c L = R, \quad \omega_c = \dfrac{R}{L}$

$\dfrac{R}{\sqrt{R^2+R^2}} = \dfrac{R}{\sqrt{2R^2}} = \dfrac{1}{\sqrt{2}}$

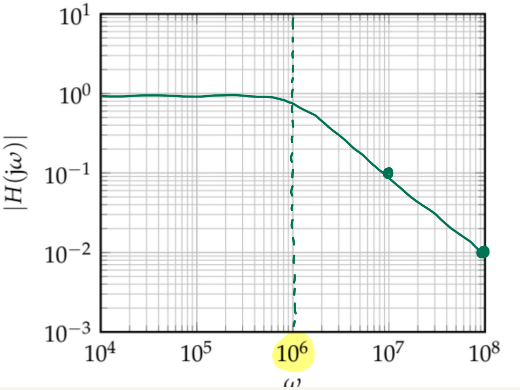
Plugging in given values,

$\omega_c = \dfrac{100}{100 \cdot 10^{-6}} = 10^6 \ \dfrac{rad}{s}$

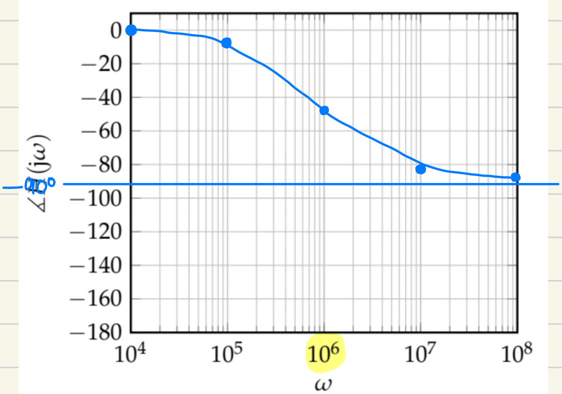
c)

ω	10^4	10^5	10^6	10^7	10^8
$\lvert H(j\omega)\rvert$	1.00	0.995	0.707	0.100	0.01
$\measuredangle H(j\omega)$	$-0.6°$	$-6°$	$-45°$	$-84°$	$-89°$

Plot of $\lvert H(j\omega)\rvert$ (for you to draw).



Plot of $\measuredangle H(j\omega)$ (for you to draw).



d) fig 1a

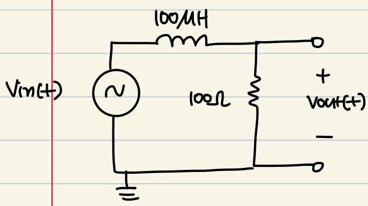
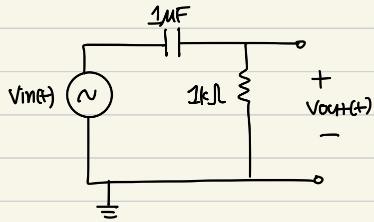
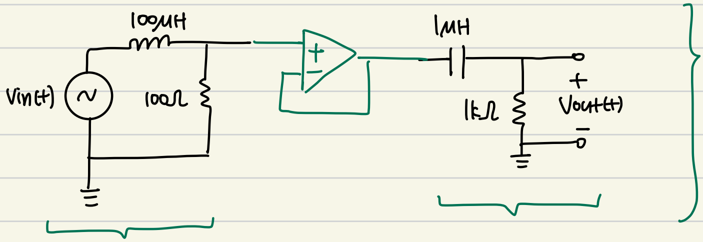


fig 3a



i) Connect w/ unity gain buffer



filter from part (a)

ii) magnitude plot

Recall:

$$\begin{cases} \text{1st filter} : & \tilde{V}_{mid} = H_1(j\omega)\tilde{V}_{in} \\ \text{2nd filter} : & \tilde{V}_{out} = H_2(j\omega)\tilde{V}_{mid} \end{cases} \Rightarrow \tilde{V}_{out} = \boxed{H_1(j\omega)\cdot H_2(j\omega)}\tilde{V}_{in}$$

transfer function of cascading filter

Convert transfer function into a polar form:

$$\tilde{V}_{out} = |H_1(j\omega)|e^{j\angle H_1(j\omega)} \cdot |H_2(j\omega)|e^{j\angle H_2(j\omega)} \cdot \tilde{V}_{in}$$

$$= |H_1(j\omega)||H_2(j\omega)|\, e^{j(\angle H_1(j\omega) + \angle H_2(j\omega))} \cdot \tilde{V}_{in}$$

$$V_0\cos(\omega t + \phi) \iff \frac{V_0}{2}e^{j\phi}$$

$$\Rightarrow V_{out}(t) = \underbrace{|H_1(j\omega)|\cdot|H_2(j\omega)|}_{|H_{total}(j\omega)|} V_0\cos(\omega t + \phi + \underbrace{\angle H_1 + \angle H_2}_{\angle H_{total}(j\omega)})$$

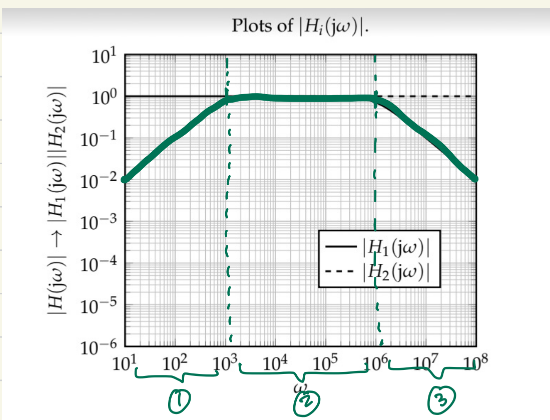


Figure 4: Log-log plot template for magnitude.

$|H_{total}(j\omega)| = |H_1(j\omega)|\cdot|H_2(j\omega)|$

① $|H_1| = 1$

$|H_{total}| = |H_2|$

② $|H_1| = |H_2| = 1$

$|H_{total}| = 1 \cdot 1 = 1$

③ $|H_2| = 1$

$|H_{total}| = |H_1|$

iii) phase plot

$\measuredangle H_{total}(j\omega) = \measuredangle H_1(j\omega) + \measuredangle H_2(j\omega)$

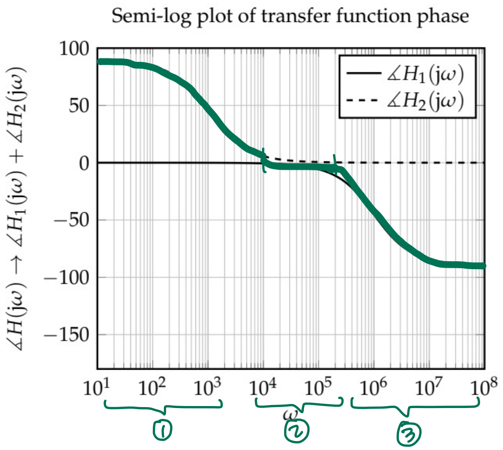


Figure 5: Plot template for phase.

① $\measuredangle H_1 = 0 \rightarrow \measuredangle H_{total} = \measuredangle H_2$

② $\measuredangle H_1 = 0$
$\measuredangle H_2 = 0$ $\rightarrow \measuredangle H_{total} = 0$

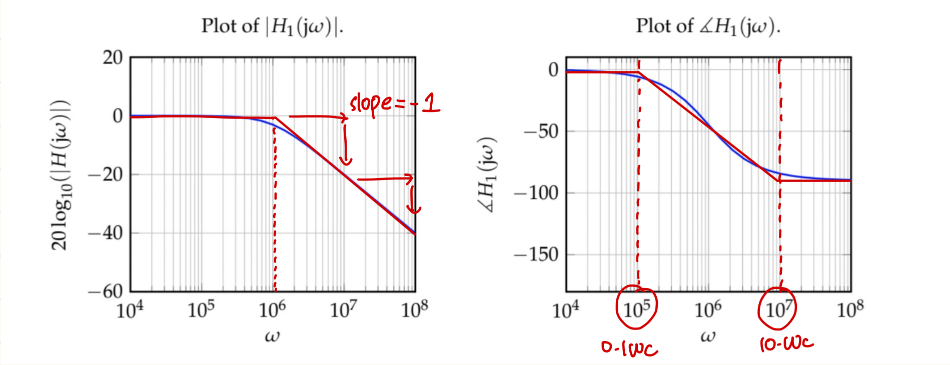
③ $\measuredangle H_2 = 0 \rightarrow \measuredangle H_{total} = \measuredangle H_1$

Q2.

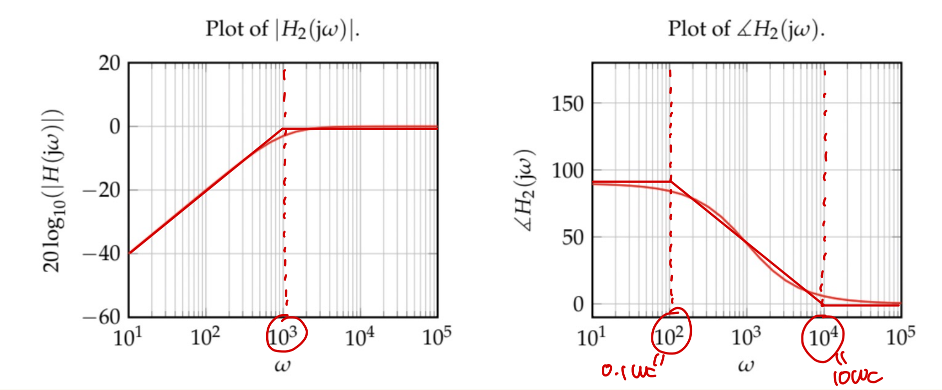
$L = 100\mu H$, $C = 1\mu F$, $R_1 = 100\Omega$, $R_2 = 1k\Omega$

a) Bode Plot ⇒ straight line approximation

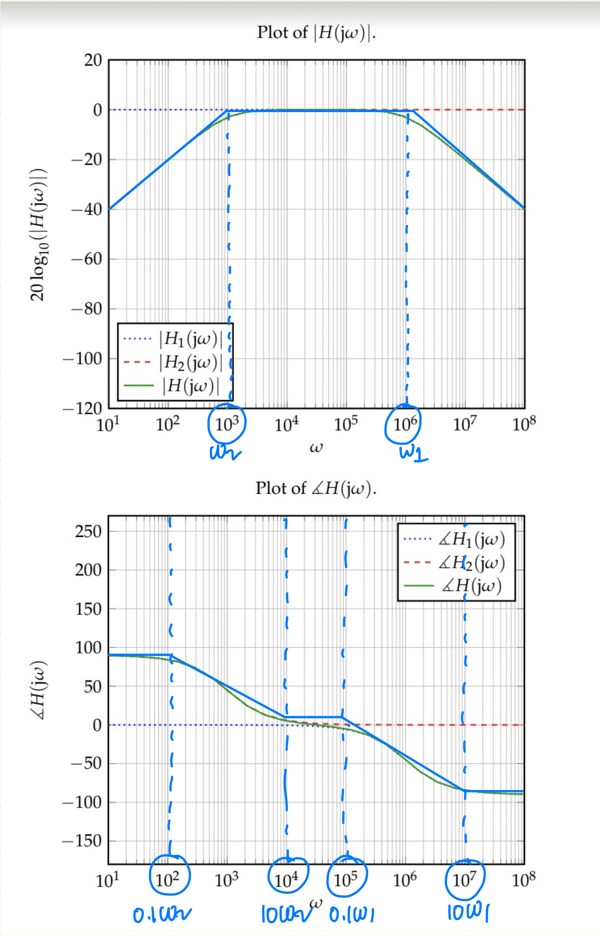
$\omega c = 10^6$ rad/s



b) $\omega c = 10^3$ rad/s



c)


Plot of $|H(j\omega)|$.
$20\log_{10}(|H(j\omega)|)$
$|H_1(j\omega)|$
$|H_2(j\omega)|$
$|H(j\omega)|$
ω
ω_2
ω_1
Plot of $\measuredangle H(j\omega)$.
$\measuredangle H(j\omega)$
$\measuredangle H_1(j\omega)$
$\measuredangle H_2(j\omega)$
$\measuredangle H(j\omega)$
$0.1\omega_2$
$10\omega_2$
$0.1\omega_1$
$10\omega_1$