

# Discussion 6B

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EECS16B S19

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Q1.

a)  $A = \begin{bmatrix} 0 & -1 \\ 3 & 0 \end{bmatrix}$

$$S = \begin{bmatrix} \cos \theta & \\ & \sin \theta \end{bmatrix} \quad | \quad 0 \leq \theta \leq 2\pi$$

$$AS = \begin{bmatrix} -\sin \theta & \\ & 3 \cos \theta \end{bmatrix} \quad | \quad 0 \leq \theta \leq 2\pi$$

$$\left\{ \begin{array}{l} \theta = 0 \rightarrow \begin{bmatrix} 0 \\ 3 \end{bmatrix} \\ \theta = \frac{\pi}{2} \rightarrow \begin{bmatrix} -1 \\ 0 \end{bmatrix} \\ \theta = \pi \rightarrow \begin{bmatrix} 0 \\ -3 \end{bmatrix} \\ \theta = \frac{3\pi}{2} \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{array} \right.$$

$\Rightarrow$  plot: ellipse going through  
 $(0, 3), (-1, 0), (0, -3), (1, 0)$

$$AS = (U \Sigma V^T) S$$

$\rightarrow$  see what operation that each of  $U, \Sigma, V$  does on the matrix

1)  $V^T S$

2)  $\Sigma (V^T S)$

3)  $U (\Sigma V^T S)$

< Jupyter Notebook for the rest of the points >

b)  $V^T \rightarrow$  orthonormal (columns are unit vector)  $\rightarrow$  cannot do scaling  $\Rightarrow$  rotate/reflect

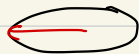
c)  $\Sigma \rightarrow$  scales vectors that have been transformed into  $V$  basis

d)  $U \rightarrow$  orthonormal  $\Rightarrow$  rotate/reflect

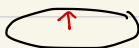
Note:  $V^T, U$  only does the rotation

$\Rightarrow$  all the scaling done by  $A$  is captured by  $\Sigma$

e)  $61\vec{u}_1 = \begin{bmatrix} 0 \\ -3 \end{bmatrix}$  semi-major axis



$62\vec{u}_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$  semi-minor axis



⊕ run through Jupyter Notebook