Discussion 6D

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EECS16B Su22

Q1.

a) Show that finding the top principal component is equivalent to maximizing $\sum_{i=1}^{n} \langle x_i, u \rangle^2$

Given: PCA solves $\underset{W \in \mathbb{R}^{d \times l}}{\operatorname{argmin}} \sum_{i=1}^{n} \| \vec{x_i} - \langle \vec{x_i}, \vec{u} \rangle \vec{u} \|^2 \quad (l=1)$

lets try to understand this equation first

$\rightarrow \sum_{i=1}^{n} \| \vec{x_i} - \langle \vec{x_i}, \vec{u} \rangle \vec{u} \|^2$

$$= \sum_{i=1}^{n} (\vec{x_i} - \langle \vec{x_i}, \vec{u} \rangle \vec{u})^T (\vec{x_i} - \langle \vec{x_i}, \vec{u} \rangle \vec{u})$$

$$= \sum_{i=1}^{n} \langle \vec{x_i}, \vec{x_i} \rangle - 2 \langle \vec{x_i}, \vec{u} \rangle^2 + \langle \vec{x_i}, \vec{u} \rangle^2 \underbrace{\langle \vec{u}, \vec{u} \rangle}_{=1 \text{ b/c } \|\vec{u}\|^2 = 1^2 \text{ (given)}}$$

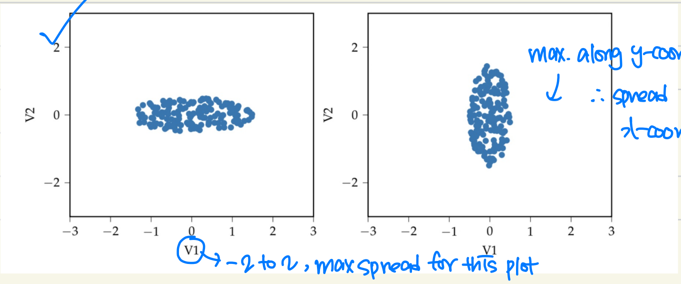
$$= \sum_{i=1}^{n} \|\vec{x_i}\|^2 - \langle \vec{x_i}, \vec{u} \rangle^2$$

$\vec{x_i}$: data points i.e. fixed $\rightarrow$ cannot change

$$\therefore \operatorname{argmin} \sum_{i=1}^{n} \|\vec{x_i}\|^2 - \langle \vec{x_i}, \vec{u} \rangle^2 = \operatorname{argmin} \sum_{i=1}^{n} - \langle \vec{x_i}, \vec{u} \rangle^2 = \max \sum_{i=1}^{n} \langle \vec{x_i}, \vec{u} \rangle^2$$

b) To find the valid plot: consider part (a) $\rightarrow$ maximize $\sum_{i=1}^{n} \langle \vec{x_i}, \vec{u} \rangle^2$

$\Rightarrow$ maximize the sum of squares of the $\underbrace{x\text{-coordinates}}_{V1}$ of the plot



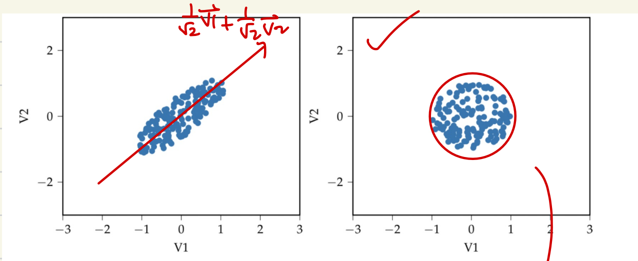
max. along y-coordinates $\downarrow$ $\therefore$ spread across the x-coordinates is not max

V1 $\rightarrow$ −2 to 2, max spread for this plot c)

or sum of squares

max spread is along the line

$\frac{1}{\sqrt{2}}\vec{v_1} + \frac{1}{\sqrt{2}}\vec{v_2}$



circle

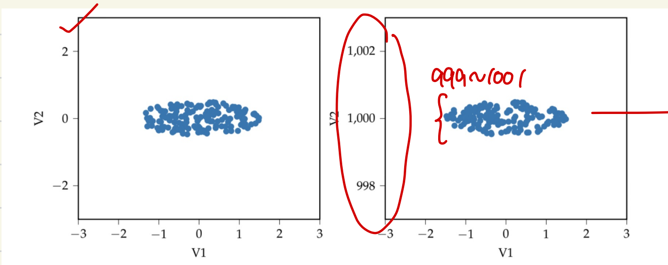
spread same across along any axis

$\Rightarrow$ plausible $\vec{v_1}$ to be 1st principal component

↓

Its spread can be considered "max"

d)



999~1001

→ sum of squares max. along $\vec{v_2}$ not $\vec{v_1}$