1. Linear Algebra Review

For the following matrices, find the following properties:

i. What is the column space of the matrix?

ii. What is the null space of the matrix?

iii. What are the eigenvalues and corresponding eigenspaces for the matrix?

(a) \[
\begin{bmatrix}
2 & 4 \\
0 & 3
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
1 & -2 \\
2 & -4
\end{bmatrix}
\]
2. KVL/KCL Review

Use Kirchhoff’s Laws on the circuit below to find \( V_x \) in terms of \( V_{in}, R_1, R_2, R_3 \).

![Example Circuit](image)

**Figure 1:** Example Circuit

(a) Recall Node Voltage Analysis (NVA). Determine \( V_x \) by labeling the circuit and writing equations to solve a system of equations in node voltages.

(b) In EECS16A, you learned you can simplify analysis by replacing series or parallel resistors with equivalents and memorizing common circuit design blocks. Determine \( V_x \) by leveraging resistor equivalence and recognition of a design block.
(c) As a check, as $R_3 \to \infty$, what is $V_x$ for what you found in (a) and (b)? The $V_x$'s of each part should approach the same value. What is the name we used for this type of circuit?
3. Current Sources And Capacitors

(Adapted from EECS16A Fall 20 Disc 9A.)

(HINT: Recall charge has units of Coulombs (C), and capacitance is measured in Farads (F). Also, \(1 \text{ F} = \frac{1 \text{ C}}{1 \text{ V}}\). It may also help to note metric prefix examples: \(3 \mu \text{F} = 3 \times 10^{-6} \text{F}\).)

Given the circuit in fig. 2, find an expression for \(v_{\text{out}}(t)\) in terms of \(I_S, C, V_0,\) and \(t\), where \(V_0\) is the initial capacitor voltage at \(t = 0\).

![Figure 2: A current source attached to a capacitor.](image)

Then plot the function \(v_{\text{out}}(t)\) over time on the graph below for each set of conditions, detailed below.

Use the values \(I_S = 1 \text{ mA}\) and \(C = 2 \mu \text{F}\).

1. Capacitor is initially uncharged \(V_0 = 0\) at \(t = 0\).
2. Capacitor has been charged with \(V_0 = 1.5 \text{ V}\) at \(t = 0\).
3. (PRACTICE) Swap this capacitor for one with half the capacitance \(C = 1 \mu \text{F}\), which is initially uncharged \(V_0 = 0\) at \(t = 0\).

(HINT: Recall the calculus identity \(\int_a^b f'(x) \, dx = f(b) - f(a)\), where \(f'(x) = \frac{df}{dt}\).)
4. (OPTIONAL) Op-Amp Summer

Consider the following circuit (assume the op-amp is ideal):

![Op-amp Summer Circuit](image)

**Figure 3: Op-amp Summer**

What is the output $V_o$ in terms of $V_1$ and $V_2$? You may assume that $R_1$, $R_2$, and $R_f$ are known.